On the Existence of Possible Worlds and the Nature of Necessity

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Where We Are

Last Class

- Modal Predicate Logic (and theory of reference)
- The nature of possible worlds
- Actualism vs. possibilism

This Class

- Actualism vs. Possibilism (continued)
- Variaties of Necessity
- Metaphysical necessity and modal logic

Next Class

- de re vs. de dicto modality
- essences
- rigidity and direct reference
- trans-world identity and personal identity

Recap: Actualism vs. Possibilism

Actualism (Plantinga):
At most one world is actual, namely the actual world.

Possibilism or **Modal Realism** (Lewis):
Each possible world is actual.

More Careful Definition of Possibilism

T1:
There are more worlds than actually exist.
Being is wider than actuality.

T2:
Actuality is an indexical notion (=relative to the world we are in).

T1+T2: Each world is actual.

Possibilism: Possibilities Exist (Argument by Lewis)

Fact 1a	Things might be otherwise than they are.
Fact 1b	There are many *ways things could have been* besides the way they are.
Fact 2	Taken at its face value, the above sentence is an existential quantification.
Definition	way things could have been $=$ possible world
Conclusion	Possible worlds (besides the one we live in) exist.
Intended Conclusion	Possible worlds (besides the one we live in) are actual.

Actualism: a Reply

- The fact that ordinary language allows speakers to existentially quantify over possible worlds does not entail that possible worlds exist—the way language works need not mirror the way reality is.

Possibilism: Counter-reply by Lewis

- We need not take existential quantifiers in natural language at their *face value*, but why shouldn't we?
- An argument for not taking them at their face value is needed.
- Such an argument needs to show that
  (1) taking them at their face value leads to trouble, and
  (2) not taking them at their face value doesn't lead to trouble.
- Option (2) fails.

Possibilities Cannot Be but Quantifiers over PWs

Alternatives:

- Possibility is analyzed in terms of consistency
  - Consistent is a sentence which could be true (circular theory)
  - Consistent is a sentence whose denial is not a theorem (incorrect theory)
- Possibility are quantifiers over maximally consistent set of . . .
  - How do we explain consistency?

Conclusion:

Option (2) leads to trouble, and so:

i) It's best to take existential quantification at its face value.

ii) Possible worlds cannot be reduced to other entities

Challenges to Actualism from SQML

SQML = Simplest Quantified Modal Logic

Semantics:

- Two fixed non-empty domains:
  W (possible worlds) and D (individuals).
- Interpretation function I for constants, variables and predicates (relative to worlds).

Proof System:

Axioms for First Order Logic (e.g., axioms for quantifiers)

N $\vdash \varphi$ implies $\vdash \Box\varphi$.

K $\Box(\varphi \rightarrow \psi) \rightarrow \Box\varphi \rightarrow \Box\varphi$

T $\Box\varphi \rightarrow \varphi$

5 $\Diamond\varphi \rightarrow \Box\Diamond\varphi$

First Challenge to Actualism from SQML

$w \Vdash \Diamond\varphi$ iff **there is a** w' such that $w' \Vdash \varphi$.

The truth-condition of $\Diamond$-formulas entails the existence of non-actualized possible worlds.

Two lines of response:

- SQML talks about abstract entities (but what are they?).
- SQML has no effective existential commitment.
  It is a only a formal tool.

Second Challenge Challenge to Actualism from SQML

SQML proves $\Diamond\exists x\varphi \rightarrow \exists x\Diamond\varphi$ (equivalent to $\forall x\Box\varphi \rightarrow \Box\forall x\varphi$)

From: It is possible that there is an x such that x is an Alien.

To: There is an x such that it is possible that it is an Alien.

Problem:

- Not only *possibly* there are Alines, but there are individuals in this world such they can be Aliens.

Replies:

- It is a problem only for essentialists.
- We may drop the requirement of the same domain of individuals for each possible worlds.

Third Challenge to Actualism from SQML

SQML proves $\forall x \Box \exists y (y = x)$.

- $\exists y (y = b)$ means b exists.
- $\forall x \Box \exists y (y = x)$ means everything necessarily exists.

Problem:

- "Everything necessarily exists" is a possibilist claim.

Reply:

- That every being has necessary existence is utterly wrong.
- We may drop the requirement of the same domain of individuals for each possible worlds.

A Moderate Possibilism (Stalnaker)

Four actualist claims:

- T1 Possible worlds exist.
- T2 Possible worlds are defined along the lines "I and my surroundings".
- T3 Actuality is indexical.
- T4 Possible worlds are irreducible entities.

Moderate Possibilism

T1 + T3 (only semantical indexicality of actuality) + T4.

An Example of a Related Metaphysical Dispute

Consider the notion of **change**.
E.g., from S being the case to $\neg S$ being the case.

Suppose $S \in w_1$ and $\neg S \in w_2$.

Actualism: from w_1 being actual to w_2 being actual.

Possibilism: both w_1 and w_2 obtain, so change is an illusion.

Cf. Aristotele vs. Megarians (a Socratic School) about the reality of change and the act/potence distinction
(in Aristotle, *Metaphysics*, Book IX).

Plan

Variaties of Necessity

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If $p \rightarrow q$, and p is the case, then q is the case (logical necessity)

$7 + 2 = 9$ (mathematical necessity)

Red id a color (conceptual necessity)

Pain is awful (conceptual necessity?)

Each person is conscious, at least for some time (metaphysical necessity)

The same body cannot be red and white at the same time (metaphysical necessity?)

Bodies do not move faster than the speed of the light (physical necessity).

One should not kill anyone (normative necessity)

Related but Different Notions than Metaphysical Necessity

- unrevisability of ungiveupbility (matter of practice or customs)
- self-evidence (epistemic notion)
- a prioricity (epistemic notion)
  - 'I exists' (a priori but contigent)
  - knowable a priori by omniscient mind = necessary?

Definitions of Necessities (mostly following Kit Fine)

A proposition is . . . necessary iff

Logical its truth is preserved under any substitution of its non-logical component.

Mathematical its truth holds in virtue of mathematical objects.

Conceptual its truth holds in virtue of the identity of concepts.

Metaphysical its truth holds in virtue of the identity of things (or the essence of things).

Normative its truth holds in virtue of a normative or axiological system.

Caution in Defining Necessities!

'Necessarily φ' was defined as

> φ is true in virtue of some properties of some objects.
>
> **But:** How can such a definition account for the **modality** in 'necessarily φ'?

Two <u>different</u> questions:

- How do modalities (as ways of being) arise?
  Why do we need them?
  Look what happens if $\Box\varphi \leftrightarrow \varphi$.
- Is there only one modality?

Modal Reductionism and Monism

1. Modal reductionism is the claim that a type of necessity is reducible to another type necessity.
   E.g., normative necessity is reducible to metaphysical necessity.
2. Modal monism is the claim that all types of necessity are reducible to one type.
   E.g., naturalism in modal terms is the claim that there is only natural necessity.

Strategy for modal reductionism and monism:

Let $\Box_m$ be the only permitted necessity. Then, a different necessity, call it $\Box_d$, can be reduced to $\Box_m$ as follows:

$$\Box_d\varphi \text{ iff } (\Box_m\psi) \wedge \chi \leftrightarrow \Box_d\varphi.$$

Necessity $\Box_d$ is explained in terms of necessity $\Box_m$ and some non-modal sentence χ.

Irreducibility of Metaphysical to Natural Necessity

Claim: It is not true that, if x is a natural necessity then x is a metaphysical necessity.

Argument: It is a natural necessity that $F = k\frac{m_1 \times m_2}{d^2}$. However, it is metaphysically possible that $F = k\frac{m_1 \times m_2}{d^3}$.

Counter-argument: The symbols m_1 and m_2 in the second formula may not refer to bodies endowed with mass, but with schmass, and hence $F = k\frac{m_1 m_2}{d^2}$ would also be a metaphysical necessity.

Counter-counter-argument:
It is a natural necessity that there is no schmass (but problem of uninstantiated kinds).
It is a metaphysical possibility that there is schmass.

Plan

Metaphysical necessity and modal logic

Metaphysical Necessity and the Accessibility Relation R

The meaning of the operator '$\Box$' can be specified by defining the conditions that R satisfies.

Claim:
If $\Box$ is to be interpreted as **metaphysical necessity**, then R should be **reflexive**, **transitive** and **symmetrical**.

Argument for Reflexivity

R is reflexive iff wRw, for any $w \in W$.

If something is metaphysically necessary, it should obtain in the actual world.
Reflexivity on R validates the axiom $\Box\varphi \rightarrow \varphi$ (exercise).

Can this axiom apply to physical necessity?
And to deontic necessity?

Argument for Transitivity

R is transitive iff $wRv \wedge vRu \rightarrow wRu$, for any $w, v, u \in W$.

Does it make sense that φ is metaphysically necessary at w and not necessary at v? Metaphysical necessity should be evenly distributed. So, if something is metaphysically necessary, it should be metaphysically necessary that it is metaphysically necessary. **Warning:** this argument may be misleading.

Transitivity on R validates the axiom $\Box\varphi \rightarrow \Box\Box\varphi$ (exercise).

Can this axiom apply to physical necessity?
And to deontic necessity?

Argument for Symmetry

R is symmetric iff $wRv \rightarrow vRw$, for any $w, v \in W$

Suppose φ is true. Then, φ should be possible. Further, φ should be necessarily possible.
(there cannot be something that is metaphysically possible at w and not metaphysically possible at w'.)

Symmetry on R validates the axiom $\varphi \rightarrow \Box\Diamond\varphi$ (exercise).

Can this axiom apply to physical necessity?
And to deontic necessity?

R Becomes an Equivalence Relation

R is en equivalence relation iff R is reflexive, transitive and symmetric.

Simplification: $w \models \Box\varphi$ iff $w' \models \varphi$ for any $w' \in W$.

But: the equivalence relation R is not universal.

Consequence: Metaphysical necessity is still relative.

Suggestion:
Should we require that R be total ($wRw' \vee w'Rw$ for any $w, w' \in W$)?

Intuitive Principles for Metaphysical Modalities

T : $\Box\varphi \rightarrow \varphi$ (also: $\varphi \rightarrow \Diamond\varphi$)

4 : $\Box\varphi \rightarrow \Box\Box\varphi$ (also: $\Diamond\Diamond\varphi \rightarrow \Diamond\varphi$)

B : $\varphi \rightarrow \Box\Diamond\varphi$ (also: $\Diamond\Box\varphi \rightarrow \varphi$)

E : $\Diamond\Box\varphi \rightarrow \Box\varphi$ (also: $\Diamond\varphi \rightarrow \Box\Diamond\varphi$)

Principle E is validated by R being Euclidean, i.e., $w_1 Rv \wedge wRu \rightarrow vRu$ for all w, v, u.

Equivalence Relation or Reflexivity Plus Euclidicity

Claim: If a relation is reflexive and Euclidean, then it is also symmetric and transitive.

Proof

Symm Assume wRv. We have wRw by reflexivity. By Euclidean R, we have vRw.

Trans Assume $wRv \wedge vRu$. By R symmetric, we have vRw. By Euclidean R, we have wRu.

Upshot

Option 1 Justify principles:
$\Box\varphi \rightarrow \varphi$ (reflexivity),
$\Box\varphi \rightarrow \Box\Box\varphi$ (transitivity), and
$\varphi \rightarrow \Box\Diamond\varphi$ (symmetry).

Option 2 Justify principles:
$\Box\varphi \rightarrow \varphi$ (reflexivity) and
$\Diamond\Box\varphi \rightarrow \Box\varphi$ (Euclidicity).

How Many "Effective" Modalities are we Left with?

Claim: The "effective" modalities we are left with are $\Box$ and $\Diamond$

Argument: These principles are valid:

$\Box\varphi \leftrightarrow \Box\Box\varphi$

$\Diamond\varphi \leftrightarrow \Diamond\Diamond\varphi$

$\Diamond\Box\varphi \leftrightarrow \Box\varphi$

$\Box\Diamond\varphi \leftrightarrow \Diamond\varphi$

The above principles suggest that we can always erase the outermost modality, except the last one.