Where We Are

Yesterday's Class

- ▶ Actualism vs. Possibilism
- ▶ Varieties of necessities and metaphysical necessity
- ▶ Metaphysical necessity in modal logic formulas

Today's class

- ▶ Metaphysical necessity in modal logic formulas (continued)
- ▶ De re vs. de dicto
- ▶ Essences
- ▶ Transworld identity and personal identity

Caution: Which 'Things' Are Said to Be Necessary?

Throughout last class, we tacitly assumed that necessity applies to:

- sentences (=expressions that can be true or false), or
- propositions (=what is expressed by sentences), or
- states of affairs (=what is referred to by sentences).

For we characterized the meaning of the box '$\Box$' in construction such as '$\Box\varphi$'.

Recap: Principles for Metaphysical Modalities

T : $\Box\varphi \to \varphi$ (also: $\varphi \to \Diamond\varphi$)

4 : $\Box\varphi \to \Box\Box\varphi$ (also: $\Diamond\Diamond\varphi \to \Diamond\varphi$)

B : $\varphi \to \Box\Diamond\varphi$ (also: $\Diamond\Box\varphi \to \varphi$)

E : $\Diamond\Box\varphi \to \Box\varphi$ (also: $\Diamond\varphi \to \Box\Diamond\varphi$)

Principle E is validated by R being Euclidean, i.e.,
$wRv \land wRu \to vRu$ for all w, v, u.

Equivalence Relation as Reflexivity Plus Euclidicity

R is an equivalence relation iff

i) R is reflexive, symmetric and transitive, or

ii) R is reflexive and Euclidean.

Claim: If R is reflexive and Euclidean, then R is also symmetric and transitive (hence, R is an equivalence relation).

Proof

Symm Assume wRv. We have wRw by reflexivity. By Euclidean R, we have vRw.

Trans Assume $wRv \wedge vRu$. By R symmetric, we have vRw. By Euclidean R, we have wRu.

Two Ways To Capture Metaphysical Necessity

Option 1 Justify principles:
$\Box\varphi \to \varphi$ (reflexivity),
$\Box\varphi \to \Box\Box\varphi$ (transitivity), and
$\varphi \to \Box\Diamond\varphi$ (symmetry).

Option 2 Justify principles:
$\Box\varphi \to \varphi$ (reflexivity) and
$\Diamond\Box\varphi \to \Box\varphi$ (Euclidicity).

Question: Which one is more intuitive?

Claim: The "effective" modalities we are left with are $\square$ and $\lozenge$

Argument: These principles are valid:
$\square\varphi \leftrightarrow \square\square\varphi$
$\lozenge\varphi \leftrightarrow \lozenge\lozenge\varphi$
$\lozenge\square\varphi \leftrightarrow \square\varphi$
$\square\lozenge\varphi \leftrightarrow \lozenge\varphi$

The above principles suggest that we can always erase the outermost modality, except the last one.

Shortening Metaphysical Frames

Fact: The "effective" modalities are only $\Box$ and $\Diamond$.

Claim: The truth of any metaphysical formula ($=$ formula with metaphysical modalities) is preserved under shortening of the model.

Intuitive Argument: Let $\overbrace{\Diamond \ldots \Diamond}^{k \text{ times}} \varphi$ be true at w_0:

$$w_0 \longrightarrow w_1 \longrightarrow \ldots \longrightarrow w_k \varphi$$

By **Fact**, we have $\overbrace{\Diamond \ldots \Diamond}^{k \text{ times}} \varphi \leftrightarrow \Diamond \varphi$. So we only need:

$$w_0 \longrightarrow w_1 \varphi$$

Plan

De re/de dicto distinction

The Locus of Necessity

1. Some beings are necessary, e.g., God (if it exists).
2. Some features of things are necessary—they are essential features.
3. Necessity lies in the way we speak about things (not in the things themselves).

Three Claims about the Locus of Necessity

C1: There is no God. Everything is contingent and transitory.
(e.g., Leopardi, Nietzsche, Post-modernism)

C2: There are no essences.
(e.g., Hume, Nietzsche, Existentialism, Buddism)

C3: Necessity is given by the linguistic game we are in.
(e.g., Wittgenstein, Quine)

The Three Claims in Modal Logic

C1: There is no individual that exists in every possible world.

C2: The modality *de re* does not make sense.

C3: The modality *de dicto* makes sense.

We will focus on *de re* vs. *de dicto* modalities.

De re vs. de dicto

Observe the position of the modal operator '$\Box$' indicating necessity:

De re: x is necessarily P.
Someone necessarily wins the race.
$\exists x \Box Win(x)$.

De Dicto: Necessarily x is P.
Necessarily someone wins the race.
$\Box \exists x Win(x)$.

The truth-conditions for the *de re*-sentence are different from those for the *de dicto*-sentence:

- The former requires that an individual x necessarily has the property *Win* (=*Win* is an **essential feature** of x).
- The latter is based on the fact that a race has to have a winner.

De re Modalities and the Necessity of Essences

De re-sentences can be used to express essential features or properties of individuals.

So they can be used to characterize essences.

Essence = the set of all essential features or properties that an individual has.

De dicto Modalities and the Necessity of Linguistic Games

Objects are specified depending on the way we talk about them (=our linguistic game).

Our ontology is relative to language.

E.g., the red objects are those we call 'red objects'.

So, it is unreasonable to think that 'red-ness' can be predicated essentially (=necessarily, in a *de re* way) of red objects.

We can only say that, necessarily (=given the linguistic game we are in) red objects are red.

De re vs. De dicto – Barcan Formulas

$\forall x \Box Fx \rightarrow \Box \forall x Fx$ (not valid).

Counter-example:

Suppose there is a world w where each individual (at that world) is a cat. There is nothing else at w. So, we can say that at w everything is essentially an animal (provided we cannot think of cats being rocks or whatever). Thus, $w \models \forall x \Box Animal(x)$.

However, suppose that at v there are also mountains. So at v, it is false that everything is an animal. Thus, $w \not\models \Box \forall x \Box Animal(x)$.

Key-point:

Not every possible world has the same domain of individuals.

$\Box \forall x Fx \rightarrow \forall x \Box Fx$ (not valid).
Counter-example:
Clearly every individual exists at the world in which it exists. So, necessarily everything exists. Thus, $w \models \Box \forall x Exist(x)$.
However, some individual may not exist in some possible world. So, it is false that everything necessarily exists. Thus, $w \not\models \forall x \Box Exist(x)$.

Doubt:
But existence is not a predicate (Kant).
Can we rephrase the counter-example while using a "real" predicate?

Plan

The notion of essence

Essence – Some Basics

Essence can be defined as

i) that in virtue of which a thing is the thing it is.

ii) the collection of all non-accidental (=essential) properties.
(cf. distinction between accidental vs. non-accidental properties)

Distinction between the essence of a genus and the essence of an individual. We will be concerned with <u>essences of individuals</u> only.

Why is the Notion of Essence so Controversial?

Difficulties:

Epistemological:
It is difficult to know what the essence of, say, Socrates is.
Socrates could have been a different person, e.g., Turkish, a
mathematician, very handsome.
What properties (we know) are essential to him?

Moral: If there are essential properties, an individual is not free to
realize himself (see existentialism: existence precedes essence).

Steps For Formalizing the Notion of Essence

Step 1   Formalizing the notion of essential property

Step 2   Collecting the essential properties P, i.e. $E := \bigwedge_i P_i$

Problem   An essence E might apply to more than one individual.

Solution   Distinction between
- Essence and
- Hecceity (hecceitas, see Duns Scotus).

Essential Properties Represented in Modal Logic

Some options to express the fact that 'P' is essential to 'a':

1. $\exists x(x = a \wedge \Box Px)$, or
2. $\Box Pa$, or
3. $\forall x \Box(x = a \rightarrow Px)$, or
4. $\Box \exists x(x = a \rightarrow Pa)$, or

Which one to select?

$\exists x(x = a \land \Box Px)$

Given x, which is identical to a, x has P in every possible world.

This definition does not work if

- there is a world w such that $g(x) \notin D_w$. For we would have that $w \not\models Px$, and so $\Box Px$ would come out false.
- we want to describe the essence of individuals which do not exist in the actual world. For $\exists x(x = a)$ would always be false.

$\Box Pa$

a has P in every possible world

Again, this definition does not work if there is a world w such that $I_w(a) \notin D_w$.

$\forall x \Box (x = a \rightarrow Px)$

For every x, if x is identical to a, necessarily x has P.

This definition works out fine, and
  i) it requires a *de re* modality;
  ii) it requires the constant 'a' to refer *rigidly* (why?).

Does it make a difference if we write $\forall x (x = a \rightarrow \Box Px)$?
Yes. Any property P will be essential to individuals that do not exist in the actual world (why?).

$\Box\exists x(x = a \rightarrow Pa)$

Necessarily *a* has *P*, provided *a* exists.

This definition works out fine, and

  i) it does not require *de re* modalities;

  ii) but it does require the constant '*a*' to refer *rigidly* (why?).

Questions:

  - Do we have $\forall x\Box(x = a \rightarrow Px)$ iff $\Box\exists x(x = a \rightarrow Pa)$?

  - Does it make a difference if we write $Pa \land \Box\exists x(x = a \rightarrow Pa)$?

Two Assumptions while Defining Essential Properties

A Formalizing the notion of essence with the notion of necessity.

B Using constants *qua* rigid designators.

Problem A Formalizing the notion of essence using necessities leads to troubles.

Problem B What is the metaphysical basis for rigid designators?

Against Assumption A (essences via necessity)

Consider the most common definition of essential property:

$$P \text{ is essential to } a \text{ iff } \Box \exists x(x = a \to Pa)$$

Claim: The property '*being distinct from the Eiffel Tower*' is essential to Edgar.

Proof:
Let '*e*' denote Edgar and '*t*' the Eiffel Tower. Let $D(x) := x \neq t$ (=being distinct from the Eiffel Tower).

Clearly, we have $e \neq t$. Further, we have $\forall x \forall y(x \neq y \to \Box x \neq y)$ (why?). By universal instantiation, $e \neq t \to \Box e \neq y)$. By modus ponens, $\Box e \neq t$. By $D(x) := x \neq t[e/x]$, we have $\Box D(e)$. *A fortiori*, we have $\Box(\exists x(x = e) \to D(e))$.

Where Is the Problem?

Claim: The property '*being distinct from the Eiffel Tower*' is essential to Edgar.

Problem: The property '*being distinct from the Eiffel Tower*'
- has nothing to do with Edgar and his essence.
- is an extrinsic property of Edgar. That is:

  intuitively, it applies to Edgar only if there is the Eiffel Tower, which need not be. So, it should not be essential to Edgar. (Actually, it should not be even necessary to Edgar.)

Upshot: Two different notions:
- x has P necessarily $\not\Leftrightarrow$ x has P essentially.
  (both or only one direction fails?)

Against Assumption B (essences via rigidity)

First Some background on rigidity.

Then The problem with rigidity and the notion of essence.

Rigidity of Constants or Proper Names

Many authors (e.g., Kripke) have argued that names should refer rigidly, within metaphysical contexts:

Rigiditiy: Given the constant c, for any $w, w' \in W$, we have $I_w(c) = I_{w'}(c)$, provided that c has a denotation in both w and w'.

Consequence

Rigidity validates the principle: $a = b \rightarrow \Box(a = b)$.

Argument for the Rigidity of Proper Names

Arg1: Sentences (1) and (2) have different truth-values:

  (1) Socrates might not have been the lover of Alcibiades
  (2) Socrates might not have been Socrates

  Hence, names behave differently than definite descriptions.

Arg2: I can refer to Socrates even if I do not know any of his properties;
  or I can think of Socrates not being a philosopher, not being Greek, etc., and still refer to him.
  Hence, the name 'Socrates' refers to the <u>bare individual</u> Socrates, who is the same in every possible world.

  Objection: Socrates might not have been (called) Socrates.

  Reply: Yes, but one still refers to Socrates, even if he is called Aristotle. So, there is a crucial distinction between **metaphysical reference** (which is rigid) and **epistemic reference** (which is not rigid).

Rigidity of Names and Direct Reference

The claim that proper names refers **rigidly** was advanced by Kripke together with the claim that names refer **directly**, and not by means of definitive descriptions (as in Russel's theory).

- According to Russell, the logical form of a proper name is a *definitive description* (= being so-and-so) that *uniquely* refers to an individual.
- The claim that names refer directly is based on J. S. Mill's claim that proper names have denotation, but not connotation.

Plantiga has argued that we can still claim that names refer rigidly, but not directly.

Key-point:
Direct reference $\nRightarrow$ rigidity.

Direct Reference is False (Plantinga)

The sentence '*Socrates is a number*'

is necessarily false (why?). This amounts to the problem whether I can still refer to Socrates, while thinking of him as a number. It seems, I cannot.

Thus, 'Socrates' expresses the property 'being a non-number'.

Plantinga concludes that
(C1) **proper names express essential properties**.

From (C1), it follows that
(C2) **proper names refer rigidly** (why?).

The problem: Rigidity and Essences

Fact 1 We can express that '*P*' is essential to '*a*' without using *de re* modalities, but on the assumption that names such as '*a*' refer rigidly.

Fact 2 However, Plantinga claimed that the rigidity of proper names is based on the fact that they refer to essences.

Problem Isn't there a form of circularity?
(rigidity is based on essences, and essences on rigidity)