PHI 169 – CRITICAL REASONING – SAMPEL EXAM – SOLUTIONS

(a) *Solution*: We need to determine $Pr(CrimeVictim)$. By the definition, for any set A, we have

$$Pr(A) = \frac{\#\text{ elements in } A}{\#\text{ elements in } \Omega}.$$

In our case, Ω is the set of people living in Orange Beach, Alabama in 2015. So,

$$Pr(CrimeVictim) = \frac{\#\text{ elements in } CrimeVictim}{\#\text{ elements in } PeopleOrangeBeach}.$$

The set $PeopleOrangeBeach$ counts 5,000 elements. The set $CrimeVictim$ is the set of all people who were a crime victim. This set counts 100 elements. So,

$$Pr(CrimeVictim) = \frac{100}{5,000} = 0.02 = 2\%.$$

So the probability that if you lived in Orange Beach in 2015, you were the victim of a crime is 2%. Since Aristeo was living there and we don't now anything more about him, this is also the probability that Aristeo was the victim of a crime. The same probability of being the victim of a crime would apply to any other resident.

(b) *Solution:* The question is about *conditional probability*, that is, we want to know the value of $Pr(CrimeVictim|Drug)$. By definition, for any set A and B, we have

$$Pr(A|B) = \frac{\#\text{ elements in } A \cap B}{\#\text{ elements in } B}.$$

In our case,

$$Pr(CrimeVictim|Drug) = \frac{\#\text{ elements in } CrimeVictim \cap Drug}{\#\text{ elements in } Drug}.$$

The set $Drug$ counts 50 elements and $CrimeVictim \cap Drug$ counts 5 elements. Therefore,

$$Pr(CrimeVictim|Drug) = \frac{5}{50} = 0.1 = 10\%$$

Since Aristeo was living there and was involved in the drug business, and we are not told anything more about him, 10% is also the probability that Aristeo was the victim of a crime. The same 10% probability of being the victim of a crime would apply to any other resident who was involved in the drug business.

(c) *Solution:* We want to know the value of $Pr(CrimeVictim|Drug^c)$, that is,

$$Pr(CrimeVictim|Drug^c) = \frac{\#\text{ elements in } CrimeVictim \cap Drug^c}{\#\text{ elements in } Drug^c}.$$

this should be 5,000-50=4,950

We know that the set $Drug^c$ counts 1,000-50=4,950 elements. We also know that the set $CrimeVictim \cap Drug^c$ counts 100-5=95 elements. Therefore,

$$Pr(CrimeVictim|Drug^c) = \frac{95}{4,950} \approx 0.019 = 1.9\%$$

Since Avital was living there but was not involved in the drug business, and we are not told anything more about her, this is also the probability that Avital was the victim of a crime. The same 1.9% probability of being the victim of a crime would apply to any other resident who was not involved in the drug business. Clearly, Aristeo is more at risk of being the victim of a crime than Avital, 10% v. 1.9%.

(d) *Solution:* We want to know the value of $Pr(Drug|CrimeVictim)$, that is,

$$Pr(Drug|CrimeVictim) = \frac{\#\text{ elements in } Drug \cap CrimeVictim}{\#\text{ elements in } CrimeVictim}.$$

We know that the set $CrimeVictim$ counts 100 elements. We also know that the set $Drug \cap CrimeVictim$ counts 5 elements. Therefore,

$$Pr(Drug|CrimeVictim) = \frac{5}{100} \approx 0.02 = 2\%$$

oops, this should be 5/100=0.05=5%

The answer is different from (b) where we were asked about $Pr(CrimeVictim|Drug)$, while here we are asked about $Pr(Drug|CrimeVictim)$. The two are not the same.

(e) *Solution:* People in race D are—by far—more likely to end up in prison because

$$Pr(Prison|A) = \frac{500{,}000}{25{,}000{,}000} = 0.002 = 0.2\%$$
$$Pr(Prison|B) = \frac{700{,}000}{200{,}000{,}000} = 0.0035 = 0.35\%$$
$$Pr(Prison|C) = \frac{1{,}500{,}000}{70{,}000{,}000} = 0.02142857142 \approx 2\%$$
$$Pr(Prison|D) = \frac{100{,}000}{400{,}000} = 0.25 = 25\%$$

However, it is people in race C you are more like to find in prison. Since the total prison population is

$$500,000 + 700,000 + 1,500,000 + 100,000 = 2,800,000,$$

we have

$$Pr(A|Prison) = \frac{500{,}000}{2{,}800{,}000} = 0.17857142857 \approx 17\%$$
$$Pr(B|Prison) = \frac{700{,}000}{2{,}800{,}000} = 0.25 = 25\%$$
$$Pr(C|Prison) = \frac{1{,}500{,}000}{2{,}800{,}000} = 0.53571428571 \approx 54\%$$
$$Pr(D|Prison) = \frac{100{,}000}{2{,}800{,}000} = 0.03571428571 \approx 0.35\%$$

While race C is more frequently encountered in prison, it is people in race D who are worse off. If you are one of them, you are more likely than anyone else to end up in prison. There are very few of them and this explains why not many of them are in prison, after all, compared to other races.