

PROBABILITY SAMPLE EXAM #2 - CRITICAL REASONING - PHI 169

- (a) What is the difference between $P(A|B)$ and $P(B|A)$? Please explain by giving a couple of illustrative examples.
- (b) If $P(B) = P(A)$, does it follow that $P(A|B) = P(B|A)$? Please explain with simple mathematical reasoning.
- (c) Suppose you want to know what your chances are to succeed at X , where X is something that matters to you and you want to do (say, graduate, make one billion dollars, write a novel, win the nobel prize, whatever). Suppose you learn from a very reliable source that those who succeed at X are very often of type Y , that is, $P(\text{a person is of type } Y | \text{a person succeeds at } X)$ is very high. You are a person of type Y (say, introvert or extrovert or shy or ... whatever). What is your reaction? Could you then be reassured that you are very likely to succeed at X since you are of type Y ? Please explain and show me the logic of your argument.
- (d) You are deciding whether to go to school A or B . They cost the same, are equally prestigious and you have been accepted to both. You care mostly about a highly paying job and it turns out that most of those who have highly paying jobs in your town studied at school A . That is, $P(\text{person studied at school } A | \text{person has highly paying job})$ is higher than $P(\text{person studied at school } B | \text{person has highly paying job})$. Is this a good reason to pick school A over B ? Please explain and show me the logic of your argument.
- (e) Luveko and Eresia are happy living together. They do a regular check-up one day and it turns out they both test positive for a rare disease which could cause serious health complications as they get older. They both start to get worried, but they try to remain calm and seek more information. Here is what they find out. The test is 99% reliable, but the disease is quite rare. It affects overall only 1% of the population. However, as it turns out, the disease affects males more frequently than females, and in particular, it affects males with blue eyes forty times as often as females with brown eyes. Luveko is a male with blue eyes, while Eresia is a female with brown eyes. We know the disease in question affects 20% of males with blue eyes. Should Luveko and Eresia still be worried? How likely are they to have the disease given that they tested positive? Please answer this question using Bayes' theorem. Make sure you show me the logic of your argument.

NB: What is most important here is that you show me your reasoning as clearly as you can. You must demonstrate you fully understand how to use conditional probabilities and Bayes' theorem effectively. Do not be too worried about the numerical calculations. Of course, try to get your calculations right, but the most important thing is that you get the logic of the argument right.