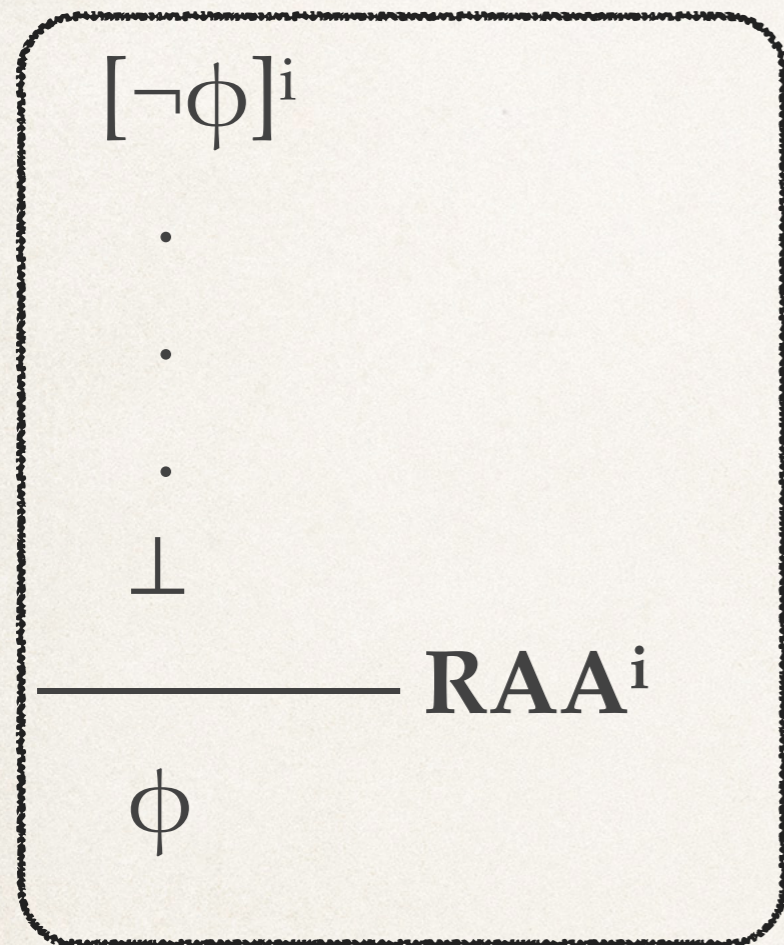


Proof by Contradiction

(reductio ad absurdum)



The idea behind this form of reasoning is that you can **establish a positive claim ϕ by showing that the negation of ϕ leads to a contradiction.**

This is a form of **indirect proof** because you do not establish ϕ directly but by showing that its negation implies a contradiction.

Establishing $\vdash (\neg\neg\phi \rightarrow \phi)$

$$\begin{array}{c} [\neg\neg\phi]^1 \quad [\neg\phi]^2 \\ \hline \perp \qquad \rightarrow\text{E} \\ \hline \text{RAA}^2 \\ \phi \\ \hline \rightarrow\text{I}^1 \\ (\neg\neg\phi \rightarrow \phi) \end{array}$$

The formula $\neg\neg\phi \rightarrow \phi$ says that **two negations make an affirmation.**

The derivation of $\neg\neg\phi \rightarrow \phi$ crucial rests upon **RAA**

Intuitionistic logic

Those who deny **RAA** or principles like $\neg\neg\phi\rightarrow\phi$ are called **intuitionistic logicians**.

They believe that in mathematics there should be no indirect proofs, but only direct (“constructive”) proofs.



Brouwer