SEMANTICS of the Propositional Language



Evaluating Formulas

How do we know if a given formula φ is **true** or **false**?

- We need the **truth-values** of the basic propositions p, q, r, ... that appear in φ .
- We need to know the **meaning** of \neg , \land , \lor , \rightarrow and \leftrightarrow .

Valuation Functions

This encodes the **principe of bivalence**. For every atomic propositions is assigned value 1 or 0.

Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

Example: assume $P = \{p, q\}$.

There are **four** different valuations (**four** different situations):

$$V_{1}(\boldsymbol{p}) = 1 \quad V_{1}(\boldsymbol{q}) = 1$$
$$V_{2}(\boldsymbol{p}) = 1 \quad V_{2}(\boldsymbol{q}) = 0$$
$$V_{3}(\boldsymbol{p}) = 0 \quad V_{3}(\boldsymbol{q}) = 1$$
$$V_{4}(\boldsymbol{p}) = 0 \quad V_{4}(\boldsymbol{q}) = 0$$

How MANY Valuations Functions?

With one atomic proposition, there are **two** possible valuations.

With **two** atomic propositions, there are **four** possible valuations.

With **three** atomic propositions, there are **2^3=8** possible valuations.

With **n** atomic propositions, there are **2^n** possible valuations.