What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a formula **relative to ONE Valuation**

 $V \vDash \psi$

How can we evaluate a formula relative to ALL valuations?

How MANY Valuations Functions?

With one atomic proposition, there are **two** possible valuations.

With **two** atomic propositions, there are **four** possible valuations.

With **three** atomic propositions, there are **2^3=8** possible valuations.

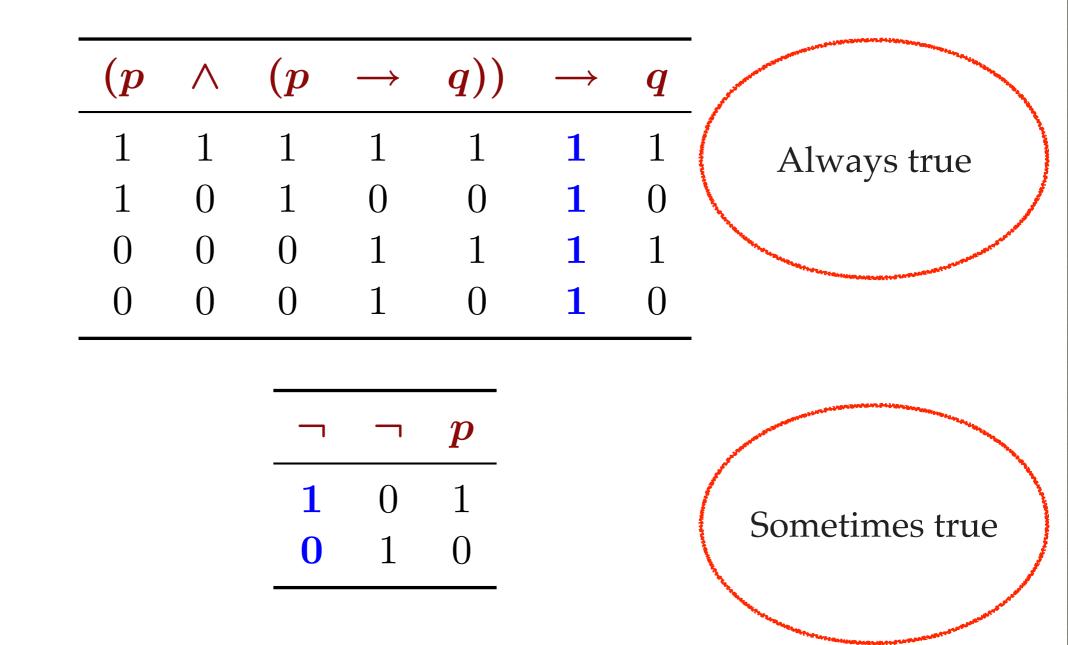
With **n** atomic propositions, there are **2^n** possible valuations.

<i>(p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	\boldsymbol{q}
1		1		1		1
1		1		0		0
0		0		1		1
0		0		0		0

<i>(p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	$oldsymbol{q}$
1		1	1	1		1
1		1	0	0		0
0		0	1	1		1
0		0	1	0		0

(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	$oldsymbol{q}$
1	1	1	1	1		1
1	0	1	0	0		0
0	0	0	1	1		1
0	0	0	1	0		0

(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	\boldsymbol{q}
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0



Classification of Formulas

• Those that are never true (contradiction):

 $p \wedge (\neg p), \ldots$

• Those that can be true (**satisfiable**):

 $(\neg p) \lor q, \ldots$

• Those that are always true (valid, tautology):

 $(p \land (p
ightarrow q))
ightarrow q, \ldots$

If the formula φ is valid, we write $\models \varphi$

The expression $V \vDash \phi$ means that ϕ is true relative to ONE valuation. Instead, the expression $\vDash \phi$ means that ϕ is true relative to ALL valuations.