## PHIL 50 – INTRODUCTION TO LOGIC

### MARCELLO DI BELLO – STANFORD UNIVERSITY

### HOMEWORK – WEEK #4 – SOLUTIONS

## 1 MIDTERM WARM UP [60 POINTS]

The following six questions are similar to the questions you will get in the upcoming midterm on April 25th. My advice is that you answer these questions as you prepare for the midterm.

1. Explain how the liar paradox can be used to argue that there are true contradictions.

   SOLUTION. Consider the sentence

   (S) "this sentence is not true".

   Now, if (S) is true, then (S) is not true. If (S) is not true, then (S) is true. So, (S) is true and (S) is not true. This is a contradiction. This can be taken to suggest that insofar as we entertain sentences such as (S) in our minds, contradictions exist (at least in our minds).

2. Suppose $V(\varphi) \geq V(\psi)$. Determine the possible truth values of $\neg(\varphi \vee \neg\psi)$.

   SOLUTION. Consider the truth table for the formula in question:

| $\neg$ | $(\varphi$ | $\vee$ | $\neg$ | $\psi)$ |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |

   We want to make sure that $V(\varphi) \geq V(\psi)$, and this constraint is satisfied only in the first, second, and fourth lines of the table. In each of those lines, the formula in question takes value 0. So, given the constraint that $V(\varphi) \geq V(\psi)$, the formula $\neg(\varphi \vee \neg\psi)$ can only be false.

3. Can a valuation satisfy the constraint that $V(p) = 1 - V(q)$?

   SOLUTION. Yes, a valuation can satisfy the constraint that $V(p) = 1 - V(q)$. Let $V(p) = 1$ and let $V(q) = 0$, so that $V(p) = 1 - V(q)$. Of course, there could be another valuation $V'$ such that e.g. $V'(p) = V'(q) = 1$ so that the constraint $V'(p) = 1 - V'(q)$ would fail for $V'$. So, with $p$ and $q$, the constraint that $V(p) = 1 - V(q)$ will be satisfied by some valuations, but not by all valuations. (Note, instead, that with $\varphi$ and $\neg\varphi$, the constraint $V(\varphi) = 1 - V(\neg\varphi)$ should be satisfied by all valuations, because $\neg\varphi$ is the negation of $\varphi$.)

4. Show how the connective $\to$ can be defined in terms of $\wedge$ and $\neg$.

   The formula $\varphi \to \psi$ is equivalent to $\neg(\varphi \wedge \neg\psi)$ as the following truth table shows:

| $\varphi$ | $\to$ | $\psi$ | | $\neg$ | $(\varphi$ | $\wedge$ | $\neg$ | $\psi)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | | 1 | 0 | 0 | 1 | 0 |

5. Construct a derivation of $\neg\neg\neg\varphi \to \neg\varphi$ (preferably without using $RAA$)

   SOLUTION. Here is the required derivation:

$$\dfrac{\dfrac{\dfrac{[\neg\neg\neg\varphi]^3 \qquad \dfrac{\dfrac{[\varphi]^1 \quad [\neg\varphi]^2}{\bot}\,{\to}E}{\neg\neg\varphi}\,{\to}I^2}{\bot}\,{\to}E}{\neg\varphi}\,{\to}I^1}{\neg\neg\neg\varphi \to \neg\varphi}\,{\to}I^3$$

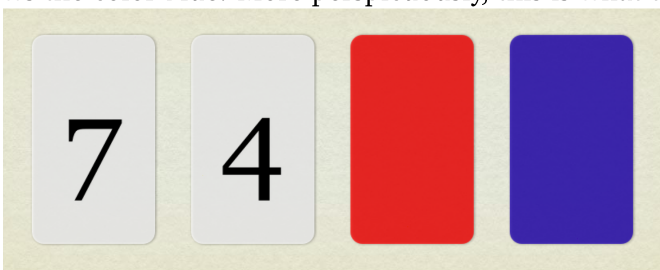
6. Is the following equivalent to the statement of completeness or soundness of propositional logic? *If* $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$*, then* $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$. Explain.

   SOLUTION. Soundness means that if $\varphi_1, \varphi_2, \ldots, \varphi_k \vdash \psi$, then $\varphi_1, \varphi_2, \ldots, \varphi_k \models \psi$. We can formulate soundness also in the contrapositive form as the statement that if $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$, then $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$. So the given statement does not conform to the statement of soundness.

   On the other hand, completeness means that $\varphi_1, \varphi_2, \ldots, \varphi_k \models \psi$, then $\varphi_1, \varphi_2, \ldots, \varphi_k \vdash \psi$. We can formulate completeness also in the contrapositive form as the statement that if $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$, then $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$. So, the given statement is equivalent to completeness.

## 2 THE WASON'S SELECTION TASK REDONE [40 POINTS]

The Wason's selection task is a well-known psychological experiment involving four cards. Here is how it works. First, the participants in the experiment are told that the four cards have a number on one face and a color on the other face. Next, the four cards are displayed in front of the participants, with each card showing only one face. One card shows an odd number; one card shows an even number; one card shows the color red; and one card shows the color blue. More perspicuously, this is what the participants are given:




Finally, the participants are asked to turn as few cards as possible in order to check whether the following if-then statement is correct:

*If the card shows an even number on one face, then its opposite face is red.*

If we interpret the if-then statement in question as containing a material implication, the correct answer would be that one should turn the card showing blue and the card showing the even number. There is no need to turn the card showing red or the card showing an odd number. The vast majority of participants, however, give a different answer. It is not clear why this happens, and there are various explanations in the literature. For more background on the Wason's selection task, please read carefully section 2.12 (chapter 2, pp. 2-34 and 2-35) of the textbook *Logic in Action*. Now, do the following:

(a) Pick two friends and ask them individually to perform Wason's selection task as described above. Pick two friends who have not seen the Wason's selection task before and who have not taken logic before. Record their answers, but please emphasize that there is no right or wrong answer, so that your friends won't feel intimidated.

(b) Once you have recorded the answers, please ask your friends why they gave the answer they gave. Summarize their explanation as clearly as you can. Feel free to ask follow-up questions if this helps you become clear about why they answered the way they did. A few written paragraphs for each friend are sufficient.

SOLUTION. There is not one correct solution for this exercise. It is important that the answers and the explanations of the participants are reported carefully and clearly.