PHIL 50 – INTRODUCTION TO LOGIC

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HOMEWORK – WEEK #6

1 HEALTHCARE [30 POINTS]

Consider the following argument:

> (1) The Obama's healthcare bill does *not* accomodate everybody's demands, and in contrast, (2) all healthcare bills that do accomodate everybody's demands foster social cohesion. Therefore, (3) the Obama's healthcare bill does not foster social cohesion.

Now, please do the following:

(a) Translate statements (1), (2), and (3), in set-theoretic notation. [*Hint*: the Obama's healthcare bill can simply be treated as one object, call it ob. In addition, let HB be the set of healthcare bills, A the set of what accommodates everybody's needs, and FC the set of what fosters social cohesion. Use operations on sets and set membership to carry out the set-theoretic translation.]

(b) Check whether the argument is valid using a set-theoretic argument or give a counterexample if the argument is invalid.

(c) Translate statements (1), (2), and (3) using the language of predicate logic. [*Hint*: Let ob be the constant symbol for Obama's health care bill; let HB be the predicate symbol for the attribute of being a healthcare bill; let A be the predicate symbol for the attribute of accommodating everybody's needs; let FC be the predicate symbol for the attribute of fostering social cohesion. Use connectives and quantifiers to carry out the translation.]

2 MISMATCHES [20 POINTS]

Formalize in predicate logic the following chunks of sentences:

(a) Liv likes Ron. Ron likes Debbie. Debbie likes Liv. There is no one who is liked by someone they like. But everyone is liked by someone. And everyone likes someone. But there is no one whom everbody likes.

(b) Some Americans are poor. Some Americans are rich. All Americans are either rich or poor, not both. Some rich Americans give to poor Americans. Some rich Americans do not give to poor Americans. If there were no poor American to whom some rich Americans did not give, then no American would be poor and no American would be rich. (*As for the last sentence, translate its antecedent and consequent separately. Then, put them together by using the material implication. Treat the verbs "were" and "would be" as if they were in the indicative mood, and treat the verb "did not give" as if it were in the present tense.*)

3 TRUTHS AND ARROWS [35 POINTS]

Consider the following situations:

Situation 1 : $\sharp \rightleftarrows \clubsuit$ *Situation 2* : $\triangle \longrightarrow \heartsuit$

Let's assume that:
- the relation symbol 'R_1" refers to the arrow-relation in *Situation 1*;
- the relation symbol "R_2" refers to the arrow-relation in *Situation 2*;
- the constants "*sharp*" and "*club-suit*" refer to the objects $\sharp$ and $\clubsuit$ respectively;
- the constants "*triangle*" and "*heart-suit*" refer to the objects $\triangle$ and $\heartsuit$ respectively.

Remark 1: A formula such as $R_1(\textit{sharp}, \textit{club-suit})$ should be understood as saying that the arrow (in *Situation 1*) goes from the object $\sharp$ to the object $\clubsuit$. Similarly, $R_2(\textit{triangle}, \textit{heart-suit})$ should be understood as saying that the arrow (in *Situation 2*) goes from $\triangle$ to $\heartsuit$.

Remark 2: Note an important difference between *Situation 1* and *Situation 2*. Relative to *Situation 1*, the formula $R_1(\textit{sharp}, \textit{club-suit})$ and the formula $R_1(\textit{club-suit}, \textit{sharp})$ are both true, because the arrow goes both directions. Instead, relative to *Situation 2*, the formula $R_2(\textit{triangle}, \textit{heart-suit})$ is true, but the formula $R_2(\textit{heart-suit}, \textit{triangle})$ is not true. The arrow only goes one direction.

Remark 3: When you are checking (relative to *Situation 1*) the truth of a formula with the universal quantifier, e.g. $\forall x R_1(x, \textit{sharp})$, you should consider all objects in *Situation 1*, namely $\sharp$ and $\clubsuit$. Similarly, when you are checking (relative to *Situation 2*) the truth of a formula with the universal quantifier, e.g. $\forall x R_2(x, \textit{triangle})$, you should consider all objects in *Situation 2*, namely $\triangle$ and $\heartsuit$.

Now check whether the following formulas are true relative to the given situation:

(a) $R_1(\textit{club-suit}, \textit{sharp}) \rightarrow \neg R_1(\textit{sharp}, \textit{club-suit})$ relative to *Situation 1*

(b) $\neg(R_2(\textit{heart-suit}, \textit{triangle}) \vee \neg R_2(\textit{triangle}, \textit{heart-suit})$ relative to *Situation 2*

(c) $\exists x(R_1(x, \textit{sharp})) \wedge \exists x(R_1(x, \textit{club-suit}))$ relative to *Situation 1*

(d) $\forall x R_1(x, \textit{sharp})$ relative to *Situation 1*

(e) $\exists x \exists y(R_2(x, y)) \wedge \exists y \exists x(R_2(y, x))$ relative to *Situation 2*

(f) $\forall x \exists y R_1(x, y)$ relative to *Situation 1*

(g) $\exists x \forall y R_1(x, y)$ relative to *Situation 1*

Explain your answers as carefully as possible. In the case of quantified formulas, please rephrase the formula in natural language so that you demonstrate you have understood what the formula means.

4 VAL OF INTERSECTION AND THE INTERSECTION OF VAL [15 POINTS]

This is the same set up as in the last exercise of homework 5. Let Γ and Δ be sets of formulas. Let $Val(\Gamma)$ be the set of valuations which make true all the formulas in Γ. More precisely, $Val(\Gamma) = \{V |$ *for all* φ*, if* $\varphi \in \Gamma$*, then* $V(\varphi) = 1\}$. And similarly for $Val(\Delta)$, so that $Val(\Delta) = \{V |$ *for all* φ*, if* $\varphi \in \Delta$*, then* $V(\varphi) = 1\}$. Now, please do following:

(a) Find a counterexample to the claim that $Val(\Delta \cup \Gamma) = Val(\Delta) \cup Val(\Gamma)$

(b) Show that $Val(\Delta \cup \Gamma) \subseteq Val(\Delta) \cap Val(\Gamma)$