PHIL 50 – INTRODUCTION TO LOGIC

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HOMEWORK – WEEK #8 – DUE WEDNESDAY MAY 28 BECAUSE MONDAY IS MEMORIAL DAY

1 ELLIS ISLAND [40 POINTS]

You are at Ellis Island in NY at the beginning of the 20th century and lots of immigrants are arriving from Europe. You are in a room with 1,000 immigrants in which:

(F1) each of the 1,000 immigrants in the room is either Irish or Italian;

(F2) at least one of the 1,000 immigrants is Irish; and

(F3) for any two different immigrants taken from the 1,000 immigrants in the room, at least one of the two is Italian.

Now, please do the following:

(a) Translate (F1), (F2), and (F3) into predicate logic. Assume your domain consists only of the 1,000 immigrants in the room. There are no chairs, cats, saunas, rivers, etc.

(b) Determine how many immigrants are Italian and how many are Irish. Explain.

(c) Suppose your answer to (b) is n Italian immigrants and m Irish immigrants. Show that any model in which the numbers of Irish immigrant is greater than m makes false (F3), or at least, it makes false your translation of (F3) in predicate logic. (One extra assumption you might need here is that no immigrant can concurrently be Irish and Italian. This assumption is contained in (F1) which says that each immigrant is either Irish or Italian, so that no immigrant can be both.)

2 DOES GOD REALLY EXIST?[60 POINTS]

This exercise discusses a (failed) attempt to formalize Anselm's argument for the existence of God in predicate logic. There are a couple of preliminary parts you should read carefully. The third part describes what you are expected to do in the exercise.

2.1 ANSELM'S ARGUMENT

Anselm is a Medieval philosopher who formulated a proof for the existence of God. The proof goes (roughly) as follows. Let God be the being than which nothing greater can be

conceived. Arguably, God can exist in thought, in the sense that we can think of such a being in our minds. The interesting question, though, is whether God exists in reality as well. Anselm argued that it did. How? To begin with, suppose for contradiction that

(1) x is such that nothing greater than x can be conceived

and yet

(2) x does not exist in reality, only in thought.

Now, Anselm notes, we can conceive of some y greater than x as follows. Let y satisfy the definition of God, and also, let y exist both in reality and in thought. It seems that y is greater than x because y is just like x, and in addition, y also exists in reality. (The hidden assumption here is that existence in reality gives more "greatness" to a being than mere existence in thought). So, we have arrived at the conclusion that

(3) we can conceive of something, namely y, which is greater than x.

But (3) contradicts (1). If we blame (2) for the contradiction, we can conclude that

(not-2) x also exists in reality, not only in thought.

Since x satisfies the definition of God, it follows that God exists—*in reality!*

2.2   A FORMALIZATION IN PREDICATE LOGIC

We now want to formalize Anselm's argument in predicate logic. Anselm begins by giving a definition of God. Let $\gamma(x)$ be a placeholder for 'x is God'. Following Anselm, we should fill in $\gamma(x)$ with a formula expressing the idea that nothing greater than x can be conceived. To this end, let's introduce a two-place predicate $>$ for the two-place relation *better-than*. One possibility for filling in the formula $\gamma(x)$ is as follows:

$$\gamma(x) \text{ is an abbreviation of the formula } \forall y(\neg(y > x)).$$

The formula $\forall y(\neg(y > x))$ says that for all y, no y is greater than x. This captures (roughly) the idea that nothing greater than x can be conceived (or can exist in thought).

With this definition in place, Anselm distinguishes between existence in thought and existence in reality. To this end, let us introduce a predicate R for "*existence in reality*". You should not be confused between the statement $\exists x Px$ which means "there exists (in thought) an x that is P" (where P is some predicate), and the statement $\exists x(Px \land Rx)$ which means "there exists (in thought) an x that is P and x exists **in reality**". We are interested in whether God exists in reality, not simply in thought. More precisely, we want to know whether

$$\exists x(\gamma(x) \land R(x)).$$

Anselm's argument rests on two crucial premises. One premise is that there exists (in thought) a being than which nothing greater can be conceived, as follows:

$$\exists x \gamma(x).$$

Note that the above formula does not presuppose that there is an x such that $\gamma(x)$ and x exists in reality. This is precisely what's at issue. The other premise is that Existence is Better than Non-existence. Here's one way to formalize it:

$$\forall x \forall y ((\gamma(x) \wedge Rx) \wedge (\gamma(y) \wedge \neg R(y)) \to x > y).$$

The above formula says that between a God-like entity x which exists in reality and a God-like entity y which does not exist in reality, x is greater than y.

It would be great if there was a derivation of $\exists x(\gamma(x) \wedge R(x))$ from premises $\exists x \gamma(x)$ and $\forall x \forall y ((\gamma(x) \wedge Rx) \wedge (\gamma(y) \wedge \neg R(y)) \to x > y)$. As you'll learn in the next section, this derivation does not exist. This need not mean that Anselm was wrong. Our formalization of his argument might not be good enough, after all.

2.3 JUGGLING WITH GOD'S EXISTENCE

Equipped with such divine formalization, now please do the following:[1]

(a) Construct a (very simple) model that makes false $\exists x(\gamma(x) \wedge R(x))$ while making true both $\exists x \gamma(x)$ and $\forall x \forall y ((\gamma(x) \wedge Rx) \wedge (\gamma(y) \wedge \neg R(y)) \to x > y)$.

(b) Construct a derivation of $\forall y \neg(\gamma(y) \wedge R(y))$ from the assumptions:

(A1) $\forall y \forall x (((\gamma(y) \wedge R(y)) \wedge (\gamma(x) \wedge \neg R(x))) \to y > x)$

(A2) $\exists x(\gamma(x) \wedge \neg R(x))$

(c) Construct a derivation of $\forall y \neg(\gamma(y) \wedge \neg R(y))$ from the same assumptions as in (b) except that (A2) is replaced by (A2′) $\exists y(\gamma(y) \wedge R(y))$

(d) Put (b) an (c) together. What do they tell you about God's existence (insofar as our formalization goes)?

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[1]**NB:** You can always use the fact that $\gamma(x)$ is an abbreviation of the formula $\forall y(\neg(y > x))$. So, whenever you have the formula $\gamma(x)$ simply replace it with $\forall y(\neg(y > x))$. If you have the formula $\gamma(z)$, then do the same but use z instead of x, and similarly for other variables.