PHIL 50 – INTRODUCTION TO LOGIC

MIDTERM EXAM - SOLUTIONS

Please answer the following questions as clearly as you can. You have 50 minutes, so you should take roughly 5 minutes per question. You might want to start by answering the questions that come more easily to you and reserve the rest of the time for the questions that you find most difficult. Good luck!

1. What is the difference between a deductively valid argument and an inductively valid argument?

   SOLUTION. A deductively valid argument is such that whenever its premises are true, its conclusion is always true. An inductively valid argument is such that whenever its premises are true, its conclusion is most likely true.

2. Suppose $V(\varphi) = 1 - V(\psi)$. Determine the truth value of $(\varphi \wedge \psi)$.

   SOLUTION. Given the constraint that $V(\varphi) = 1 - V(\psi)$, the formulas φ and ψ will always have different truth values, and in particular, the formulas φ and ψ will not both have truth value 1. Hence, $(\varphi \wedge \psi)$ will not take true value 1, but truth value 0.

3. Explain why the semantics of propositional logic (as defined in this course) assumes the principle of bivalence.

   SOLUTION. In the semantics of propositional logic every formula gets assigned the value 1 or 0. The principle of bivalence states that every statement is either true or false. The value 1 and 0 corresponds to truth and falsity.

4. Give an example of a non-truth-functional connective. Explain your choice.

   SOLUTION. The connective 'because' is not truth functional. Consider the table:

φ	*because*	ψ
1	?	1

   Even when both φ andf ψ are true, it is not clear whether the statement 'φ because ψ' is true or false. Suppose that today is a beautiful day and that water is H20. Now, the statements 'today is a beautiful today' and 'water is H20' are both true. But what about 'today is a beautiful day because water is H20'? It is neither true nor false, or at least the truth value of the statement containing 'because' is not a function of the truth values of the statements 'today is a beautiful today' and 'water is H20.'

5. Show how the connective $\wedge$ can be defined in terms of $\neg$ and $\vee$.

   SOLUTION. The formula $\varphi \wedge \psi$ is equivalent to $\neg(\neg\varphi \vee \neg\psi)$ as the following shows:

φ	$\wedge$	ψ	$\neg$	$(\neg$	φ	$\vee$	$\neg$	$\psi)$
1	1	1	1	0	1	0	0	1
1	0	0	0	0	1	1	1	0
0	0	1	0	1	0	1	0	1
0	0	0	0	1	0	1	1	0

6. Suppose $p \models \varphi$. Does it follow that $p, q \models \varphi$ for any atomic formula q? Explain.

   SOLUTION. Yes, it follows. We want to establish that if p and q are both true, then φ is true. So, let's suppose that p and q are both true, and let's establish that φ is also true. Now, if both p and q are true, we should conclude that p alone is true. But, by assumption, we have that $p \models \varphi$, so φ must be true, and this is what we wanted to establish.

7. Construct a derivation of $\neg(\varphi \wedge \neg\varphi)$

   SOLUTION.

$$\dfrac{\dfrac{\dfrac{[\varphi\wedge\neg\varphi]^1}{\neg\varphi}\wedge E \quad \dfrac{[\varphi\wedge\neg\varphi]^1}{\varphi}\wedge E}{\bot}\to E}{\neg(\varphi\wedge\neg\varphi)}\to I^1$$

8. Construct a derivation of $\neg\neg\varphi \to \varphi$

   SOLUTION.

$$\dfrac{\dfrac{\dfrac{[\neg\neg\varphi]^1 \quad [\neg\varphi]^2}{\bot}\to E}{\varphi}\ RAA^2}{\neg\neg\varphi\to\varphi}\to I^1$$

9. Construct a derivation of $\varphi \to (\neg\varphi \to (\neg\varphi \vee \sigma))$

   SOLUTION.

$$\dfrac{\dfrac{\dfrac{\dfrac{[\neg\varphi]^1\quad[\varphi]^2}{\bot}\to E}{\neg\varphi\vee\sigma}\ \bot}{\neg\varphi\to(\neg\varphi\vee\sigma)}\to I^1}{\varphi\to(\neg\varphi\to(\neg\varphi\vee\sigma))}\to I^2$$

10. Is the following equivalent to the statement of completeness or soundness of propositional logic? *If* $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$*, then* $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$. Explain.

    SOLUTION. Soundness is the statement that if $\varphi_1, \varphi_2, \ldots, \varphi_k \vdash \psi$, then $\varphi_1, \varphi_2, \ldots, \varphi_k \models \psi$, or in the contrapositive form, if $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$, then $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$. So, the given statement is equivalent to soundness of propositional logic.