Phil 50 – Introduction to Logic

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Midterm Sample Questions

The midterm is scheduled for April 25th, 2014 during normal class time. It will consist of a subset of the following questions, roughly 10-15 questions. The answers to the questions that follow can be found in the slides and in the notes (both available on the course website). Have a look at the textbook (chapter 2, especially sections 2.1–2.6 and section 2.9) to get the general picture. Please feel free to email me or the TAs if anything is not clear.

Week 1: Preliminaries

1. What is logic about? (Do not simply repeat what's in the slides, but share your own experience of doing logic during the first three weeks of this course.)
2. What is an argument?
3. What is *modus ponens*? What is *modus tollens*?
4. What is the difference between a deductively valid argument and an inductively valid argument?
5. What is the difference between syntax and semantics?
6. What is the liar paradox?
7. Explain how the liar paradox can be used to argue that there are true contradictions.
8. Give an argument for the claim that from the contradiction anything follows.
9. What is the difference between classical and intuitionistic logic?

Week 2: Syntax and Semantics of Propositional Logic

Monday slides

10. Give an inductive definition of the formulas of the propositional language.
11. Give an inductive definition of the number of brackets in the formulas of the propositional language.

12. Give an inductive definition of the number of 2-place connectives in the formulas of the propositional language.
13. Give an inductive definition of a function that assigns 1 or 0 to all formulas of the propositional language.
14. Complete the following iff-statements:
    (a) $V \models \psi$ iff ...
    (b) $V \not\models \psi$ iff ...
15. Write up the truth table expressing the meaning of the connective $\vee$.
16. Write up the truth table expressing the meaning of the connective $\wedge$.
17. Write up the truth table expressing the meaning of the connective $\neg$.
18. Suppose $V(\varphi) = 1 - V(\psi)$. Determine the truth value of $(\varphi \wedge \psi)$.
19. Suppose $V(\varphi) \geq V(\psi)$. Determine the possible truth values of $\neg(\varphi \vee \neg\psi)$.
20. Explain why the semantics of propositional logic (as defined in this course) assumes the principle of bivalence.

WEDNESDAY SLIDES

21. Write up the truth table expressing the meaning of the connective $\rightarrow$.
22. Give an example of a truth-functional connective. Explain your choice.
23. Give an example of a non-truth-functional connective. Explain your choice.
24. Give an example of a 3-place connective.
25. Is "$+$" in "$2 + 2 = 4$" a connective? Is it a truth-functional connective?
26. What are two tenets of logical atomism?
27. Can a valuation satisfy the constraint that $V(p) = 1 - V(q)$?

FRIDAY SLIDES

28. Complete the following iff-statements:
    (a) $\models \psi$ iff ...
    (b) $\varphi_1, \varphi_2, \ldots, \varphi_k \models \psi$ iff ...
    (c) $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$ iff ...
29. What does it mean to say that $\varphi_1, \varphi_2, \ldots, \varphi_k \models \psi$ can hold vacuously? Explain and give an example.
30. Is it possible for $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$ to hold vacuously? Explain.
31. Is it possible that $p \models q$?
32. Suppose $p \models \varphi$. Does it follow that $p, q \models \varphi$ for any atomic formula q? Explain.
33. Suppose $p \models \neg p$. Does it follow that $p \models q$ for any atomic formula q? Explain.
34. Give an example of a formula that is always true and show semantically that the formula you've given is, in fact, always true.
35. Give an example of a formula that is always false and show semantically that the formula you've given is, in fact, always false.
36. Give an example of a formula that is sometimes (but not always) true and show semantically that the formula you've given is, in fact, sometimes (but not always) true.
37. Show that $\varphi \to \varphi$ and $(\neg\varphi \to \varphi) \to \varphi$ are equivalent.
38. Show that $\varphi \to \psi$ and $\neg\psi \to \neg\varphi$ are equivalent.
39. Show how the connective $\to$ con be defined in terms of $\neg$ and $\vee$.
40. Show how the connective $\wedge$ can be defined in terms of $\neg$ and $\vee$.
41. Show how the connective $\vee$ can be defined in terms of $\wedge$ and $\neg$.
42. Show how the connective $\to$ in terms of the connective $\wedge$ and $\neg$.
43. Show how the connective $\neg$ can be defined in terms of $\perp$ and $\to$.

WEEK 3: DERIVATIONS IN PROPOSITIONAL LOGIC

MONDAY SLIDES

44. State the derivation rules for $\wedge$ and the derivation rules for $\to$
45. Assume $\varphi \vdash \psi$. Construct a derivation establishing that $\vdash \varphi \to \psi$.
46. Construct a derivation of $(\varphi \wedge \psi) \to (\psi \wedge \varphi)$
47. Construct a derivation of $((\varphi \wedge \psi) \to \sigma) \to (\psi \to (\varphi \to \sigma))$

WEDNESDAY SLIDES

48. State the rule RAA and the rule $\bot$.
49. Construct a derivation of $\neg(\varphi \wedge \neg\varphi)$
50. Construct a derivation of $\varphi \to \neg\neg\varphi$
51. Construct a derivation of $\neg\neg\neg\varphi \to \neg\varphi$
52. Construct a derivation of $\neg\neg\varphi \to \varphi$
53. Construct a derivation of $\varphi \vee \neg\varphi$
54. Construct a derivation of $\varphi \to (\neg\varphi \to (\neg\varphi \vee \sigma))$
55. Which of the above (schemata of) formulas will the intutionistic logician accept? Explain.
56. Which derivation rule formalizes proof by contradiction?
57. Give an example of a proof or reasoning by contradiction.
58. Explain why the following does not have the structure of a proof by contradiction, although it does contain a contradiction. Which rule has been applied to derive $\neg\varphi$?

$$\frac{\begin{matrix}\varphi \\ \vdots \\ \bot\end{matrix}}{\neg\varphi}$$

FRIDAY SLIDES

59. What is the difference between rule $I\vee$ and rule $\vee E$? State both rules. Explain in what sense $I\vee$ is an introduction rule, while $\vee E$ is an elimination rule.

60. Which derivation rule formalizes proof (or reasoning) by cases?

61. Give an example of a proof (or reasoning) by cases.

62. Complete the following iff-statements:

    (a) $\vdash \psi$ iff ....

    (b) $\varphi_1, \varphi_2, \ldots, \varphi_k \vdash \psi$ iff ...

63. Is it possible for $\varphi_1, \varphi_2, \ldots, \varphi_k \vdash \psi$ to hold vacuously? Explain.

64. Give a statement of completeness of proposition logic.

65. Give a statement of soundness of propositional logic.

66. Is the following equivalent to the statement of completeness or soundness of propositional logic? *If* $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$*, then* $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$. Explain.

67. Is the following equivalent to the statement of completeness or soundness of propositional logic? *If* $\varphi_1, \varphi_2, \ldots, \varphi_k \not\models \psi$*, then* $\varphi_1, \varphi_2, \ldots, \varphi_k \nvdash \psi$. Explain.