## **Bayesian Networks for Judges**

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School of Law - University of Lisbon - Al for Judges

## Agenda

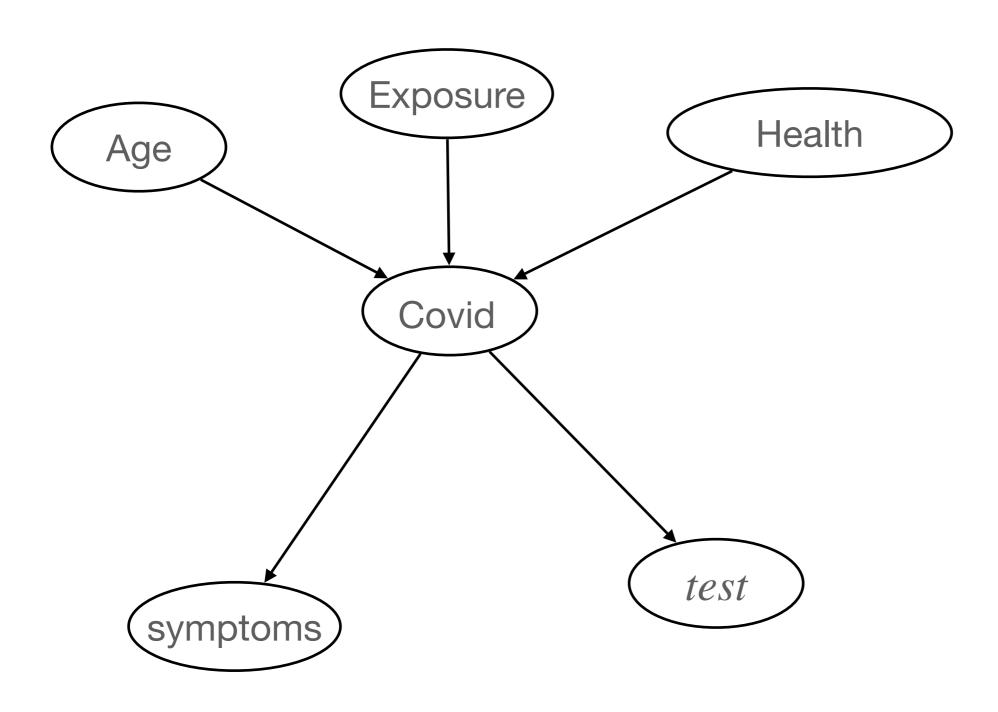
**PART I**: What Are Bayesian Networks?

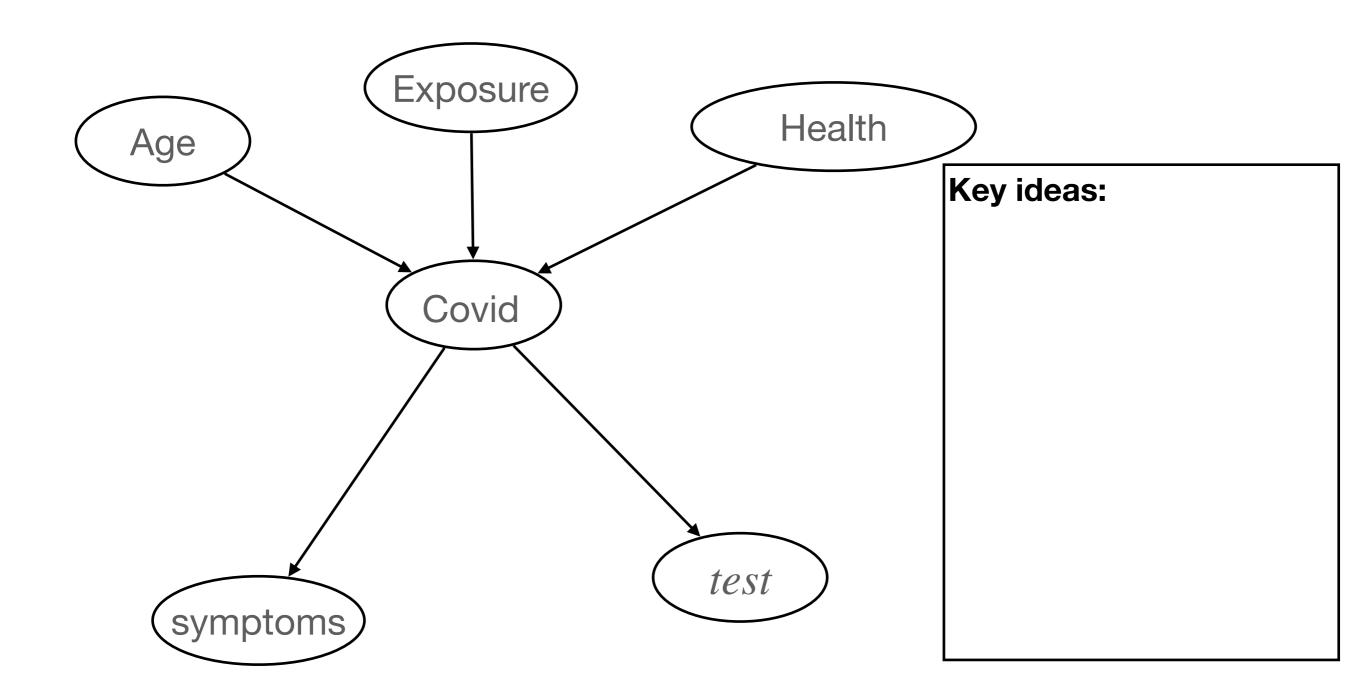
PART II: Group Exercise and Discussion

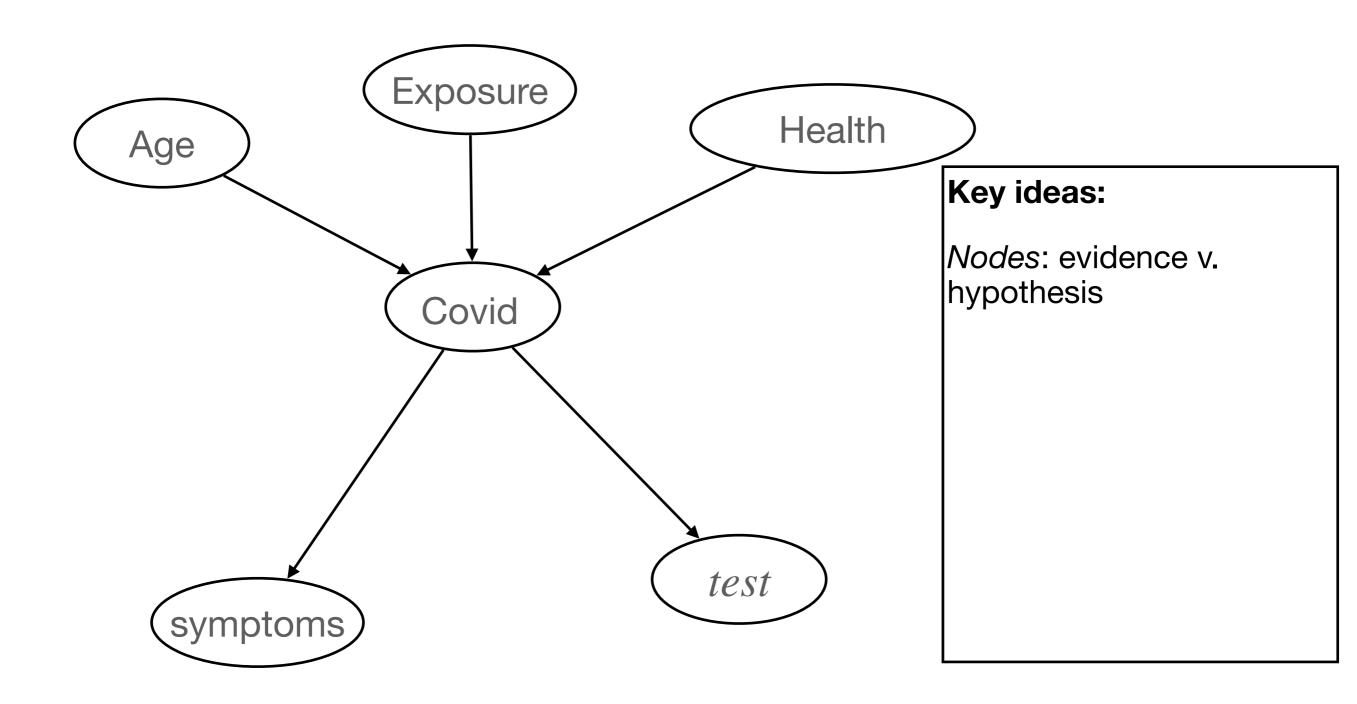
PART III: Analyzing a Legal Case Using Bayesian Networks

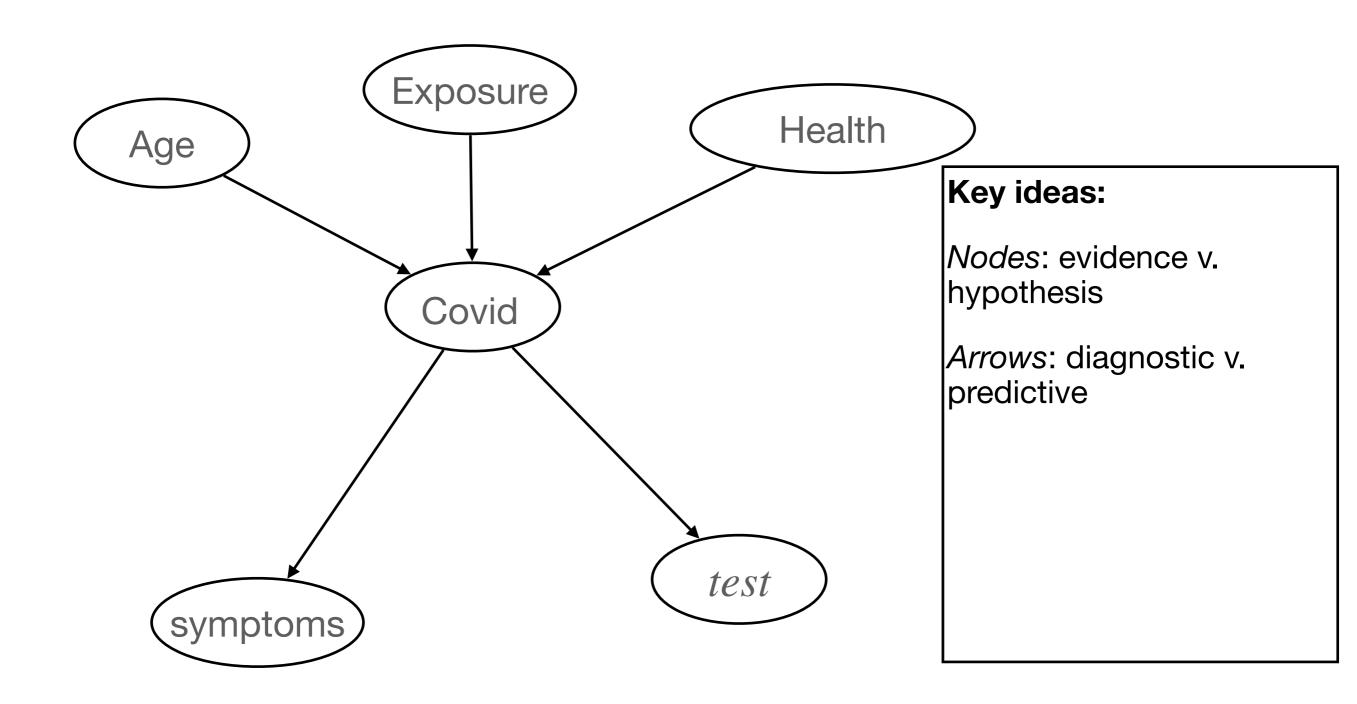
## **PART I**

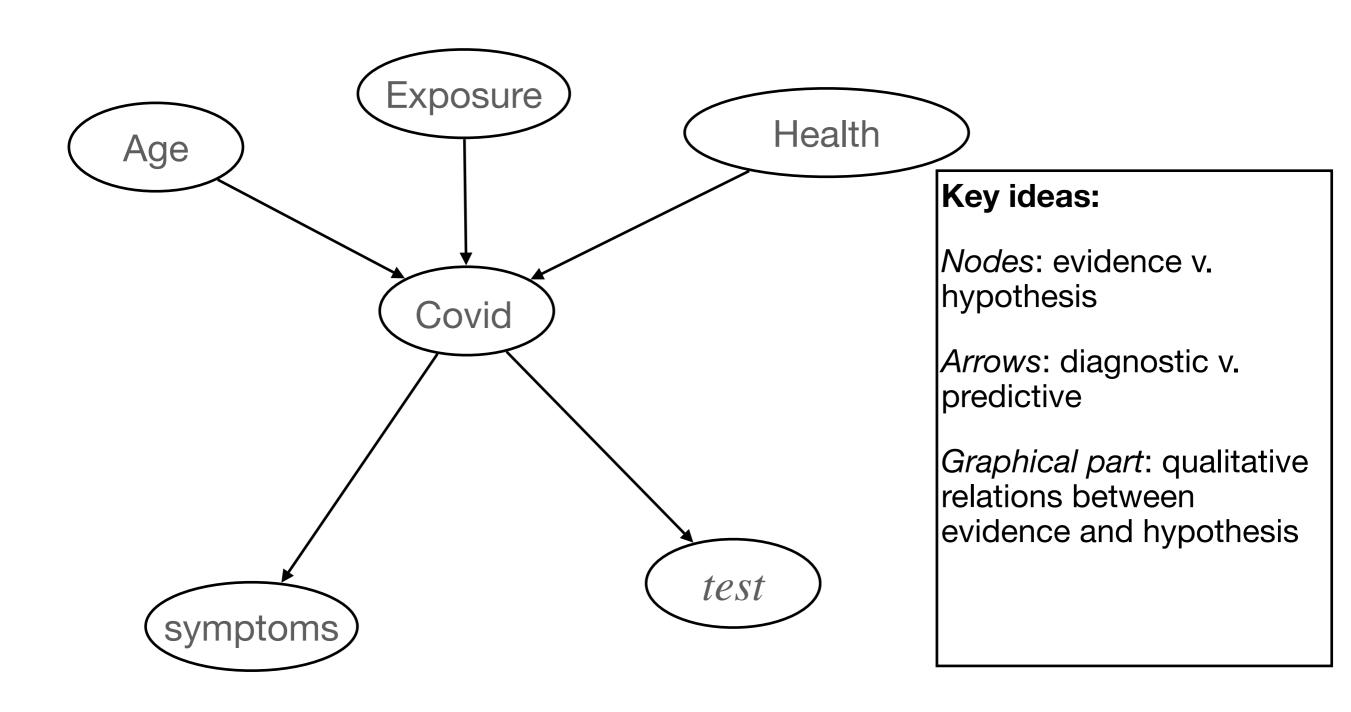
What Are Bayesian Networks?

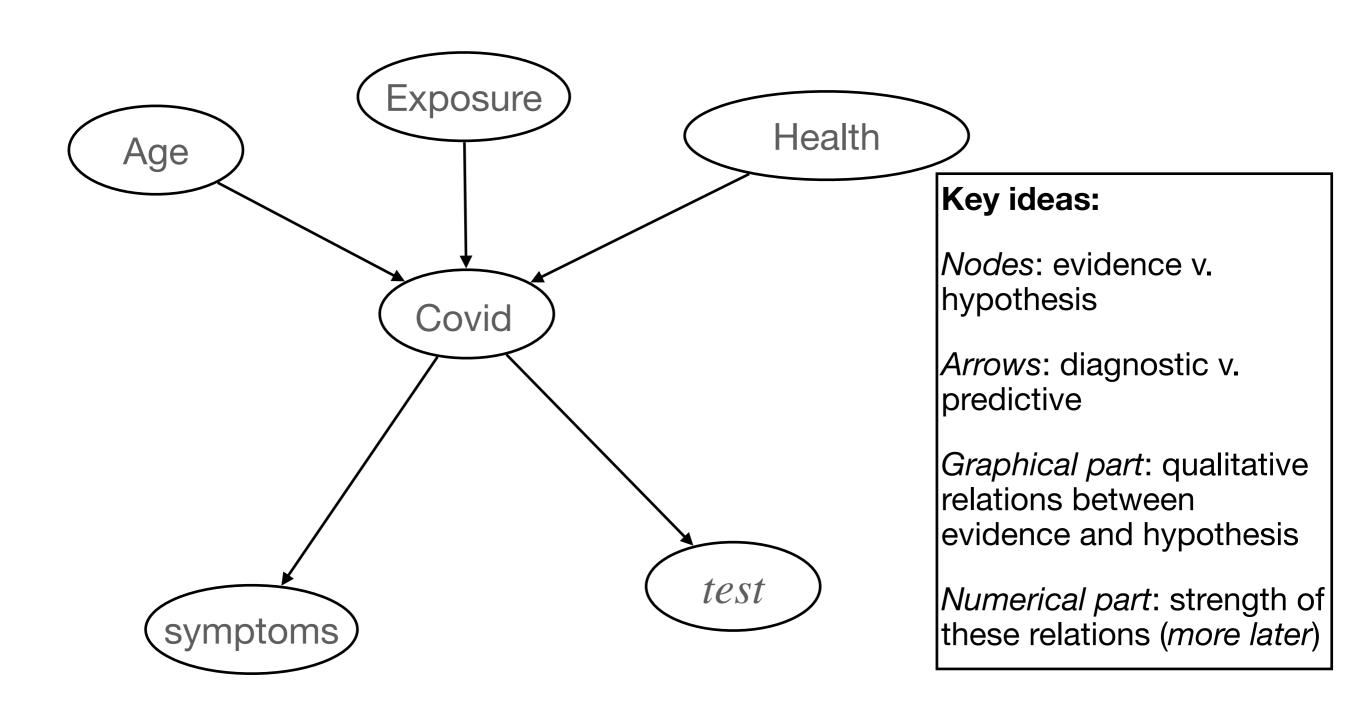




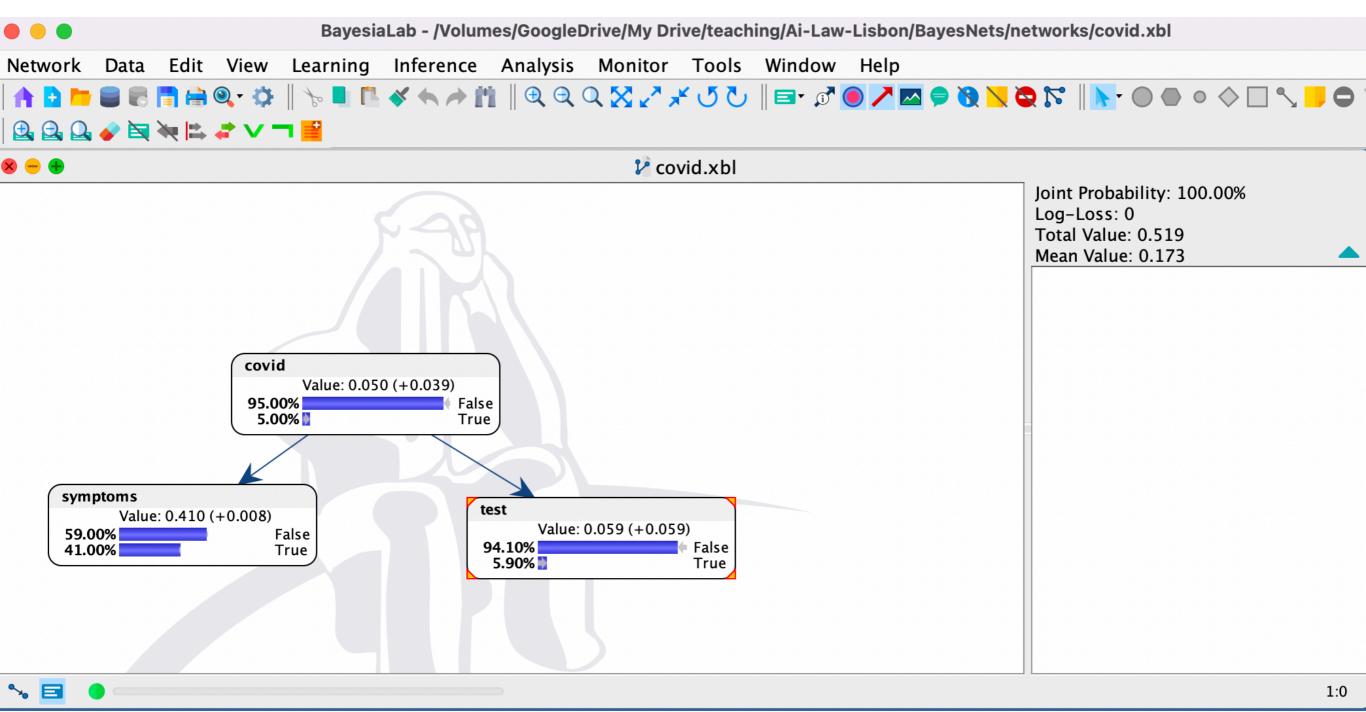




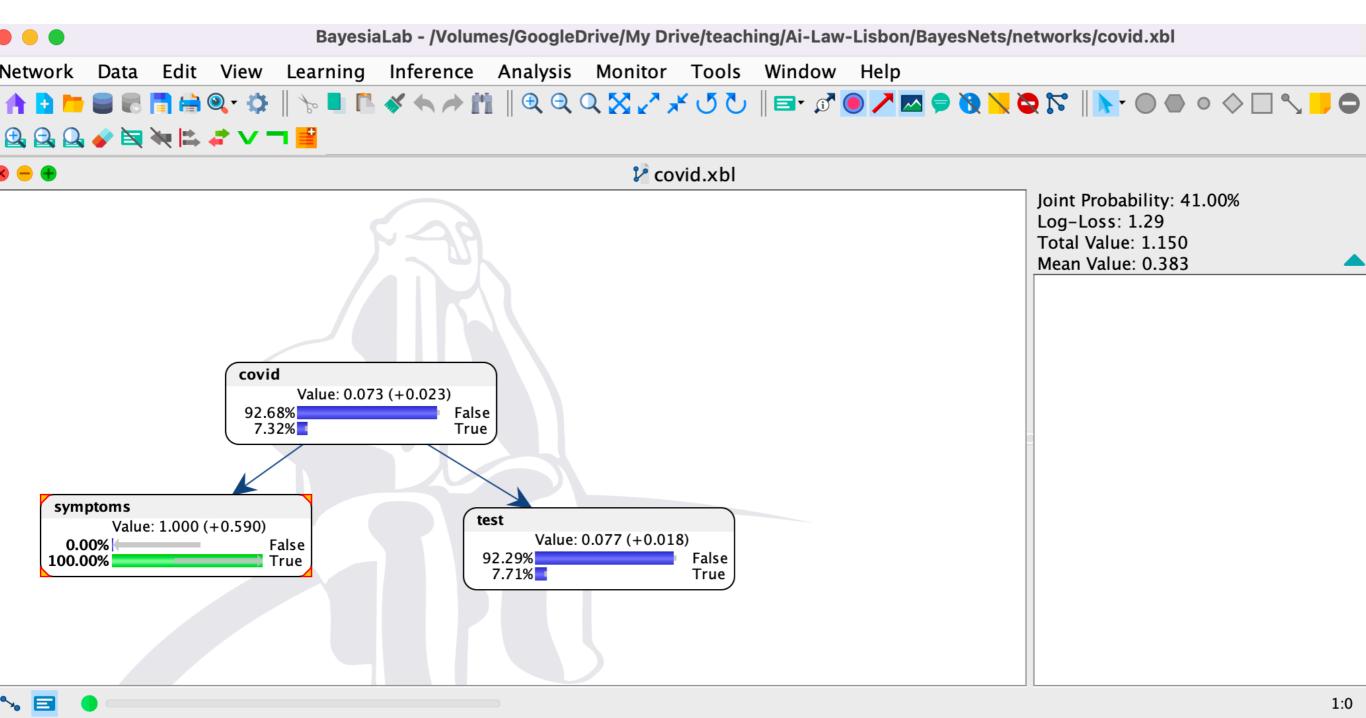




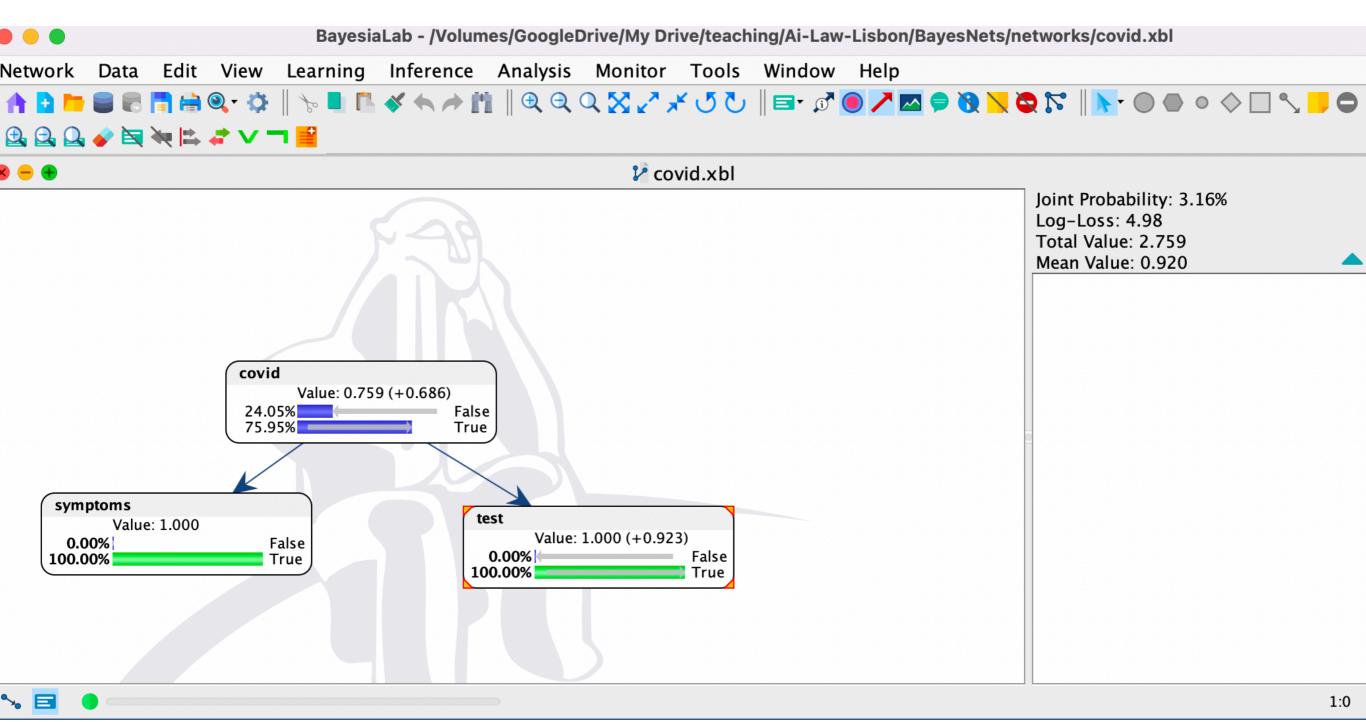
## (1) Bayesian Network with BayesiaLab



## (2) Bayesian Network with BayesiaLab



## (3) Bayesian Network with BayesiaLab



## Graphical Components: Nodes, Arrows and Idioms

### (1) Graphical Components of a Bayesian Network

#### Nodes

```
Each node represents possible states of the world

"defendant killed victim" //
"defendant did not kill victim"

"defendant had a motive" //
"defendant did not have a motive"
```

"gun powder found on defendant" //

"witness testifies they saw defendant near crime scene" // "witness testifies they did not see defendant near crime scene"

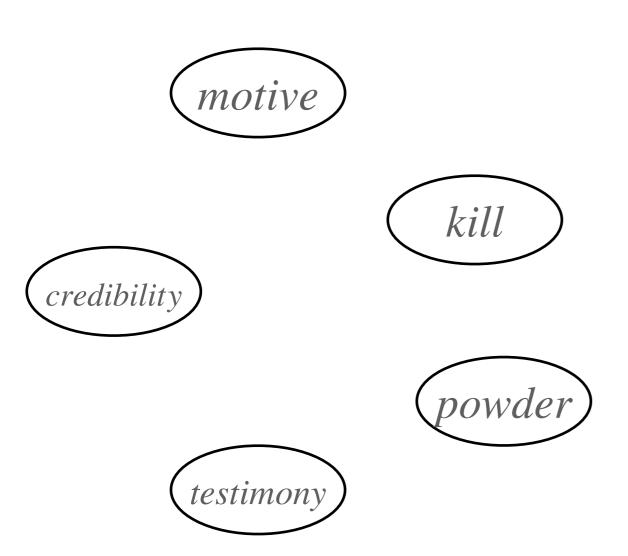
"gun powder not found on

defendant"

"witness is credible" // "witness is not credible"

#### (1) Graphical Components of a Bayesian Network

#### **Nodes**



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Each node represents possible states of the world
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"defendant killed victim" //
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"witness testifies they saw defendant near crime scene" // "witness testifies they did not see defendant near crime scene"

"witness is credible" // "witness is not credible"

### (2) Graphical Components of a Bayesian Network

#### **Arrows**

As a first approximation, think of **arrows** as *directions of causal influence* (though this interpretation is debated):

Whether or not the defendant had a motive to kill influences whether or not the defendant killed the victim

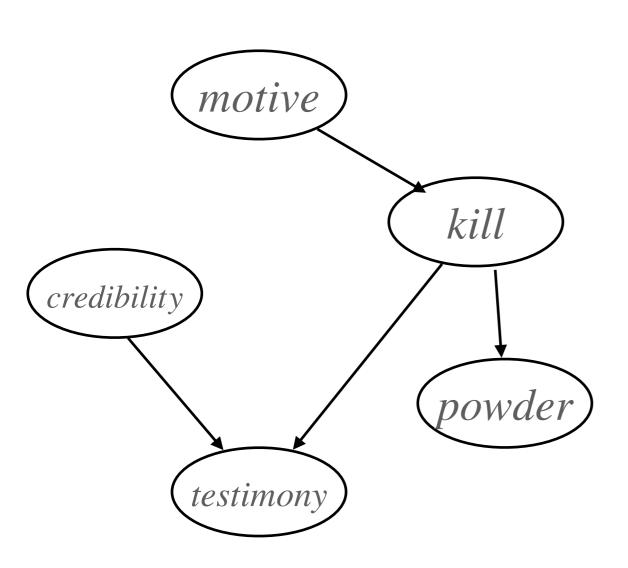
Whether or not the defendant killed the victim influences whether or not gunpowder was found on defendant

Whether or not the defendant killed the victim influences what the witness saw

Whether or not the witness is credible influences what the witness says

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As a first approximation, think of **arrows** as *directions of causal influence* (though this interpretation is debated):

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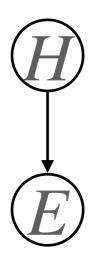
Whether or not the defendant killed the victim influences what the witness saw

Whether or not the witness is credible influences what the witness says

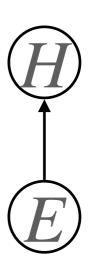
(3a) Graphical Components of a Bayesian Network Idioms (=basic graphical structures)

Hypothesis / one piece of evidence

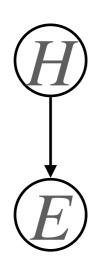
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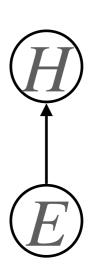
Hypothesis / one piece of evidence

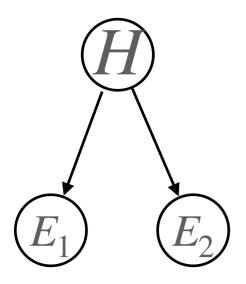


## (3a) Graphical Components of a Bayesian Network Idioms (=basic graphical structures)

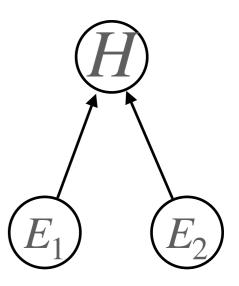


Hypothesis / one piece of evidence





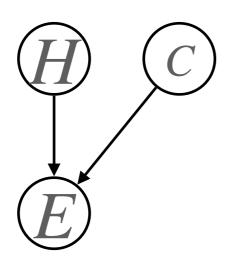
Hypothesis / two piece of evidence



## (3b) Graphical Components of a Bayesian Network Idioms (=basic graphical structures)

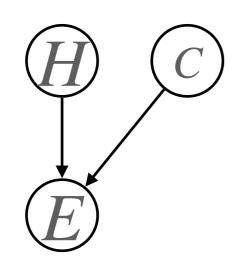
Evidence /
Hypothesis *plus* **Credibility** 

## (3b) Graphical Components of a Bayesian Network Idioms (=basic graphical structures)

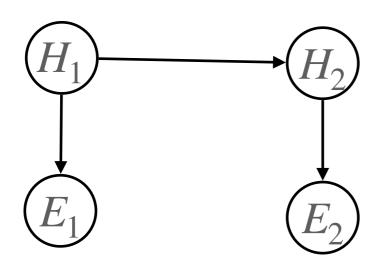


Evidence / Hypothesis *plus* **Credibility** 

## (3b) Graphical Components of a Bayesian Network Idioms (=basic graphical structures)



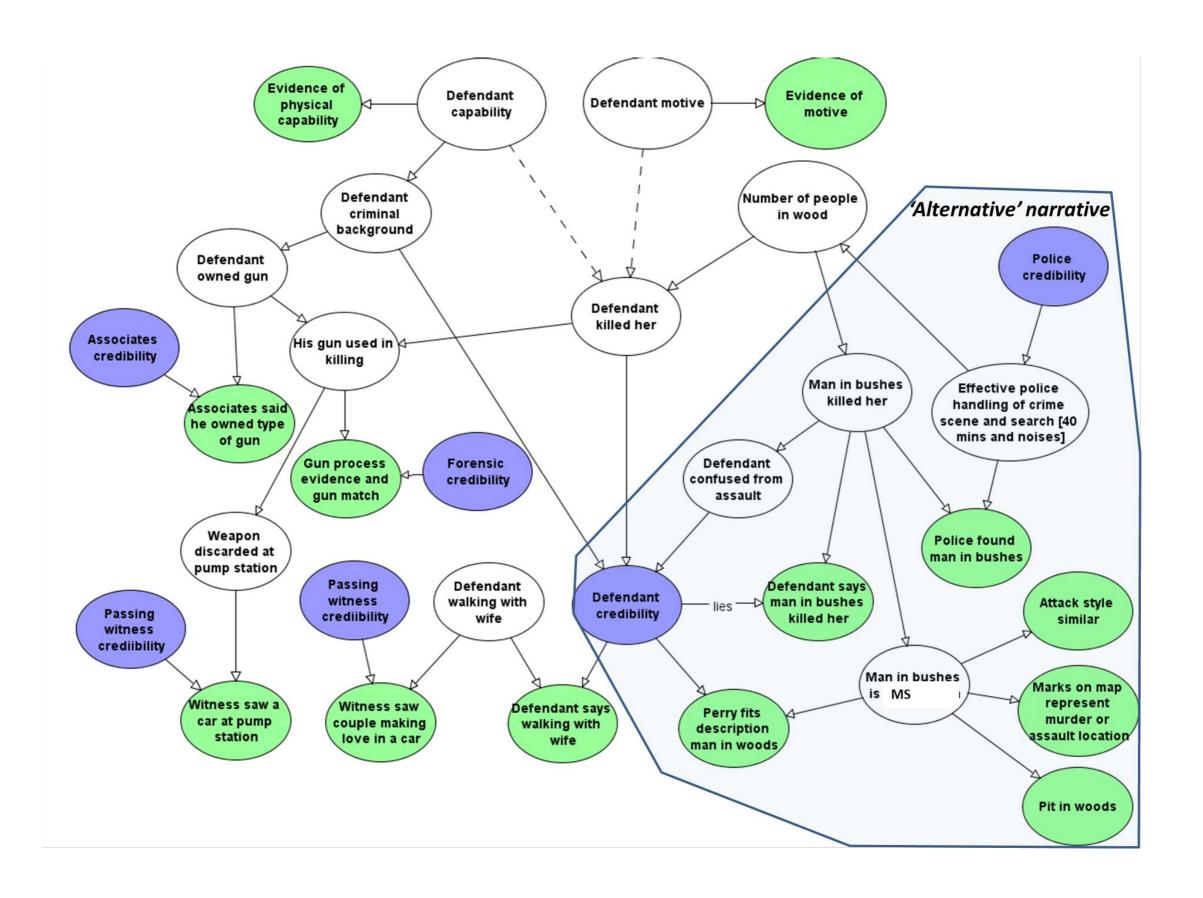
Evidence / Hypothesis *plus* **Credibility** 



#### Rebuttal:

hypotheses H1 and H2 are incompatible

## Basic Idioms Can Be Combined and Form More Complex Graphs

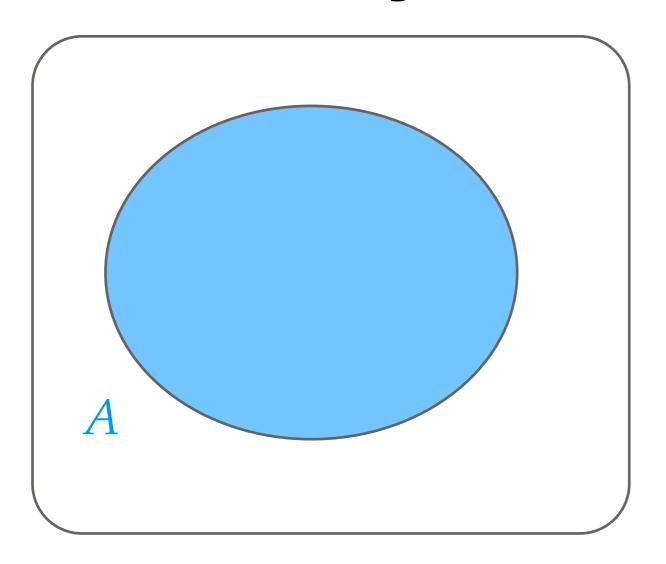


# Numerical Component: Probability Tables

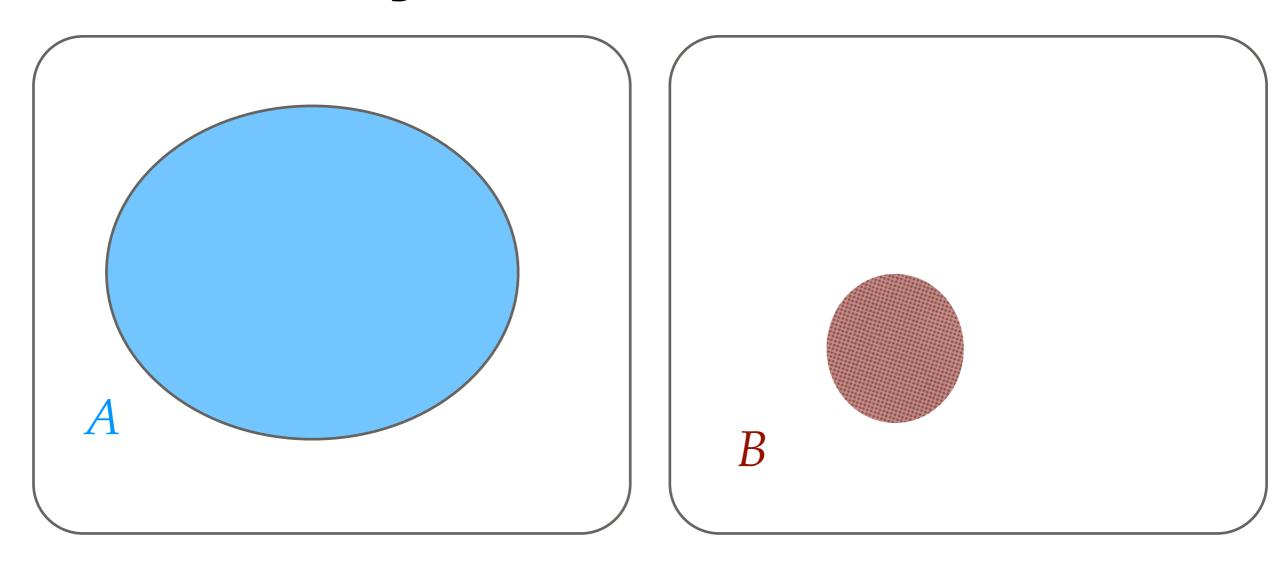
### Before we get into that...

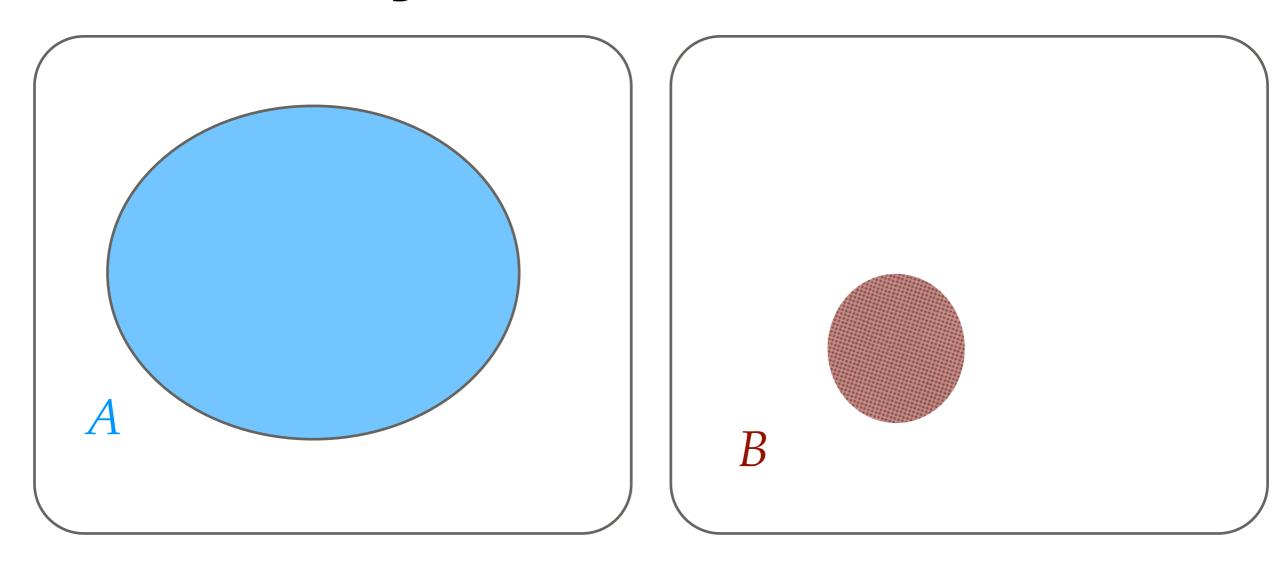
## Two preliminary topics:

- Conditional probability
- Bayes' theorem (see also handout)



Pr(A)



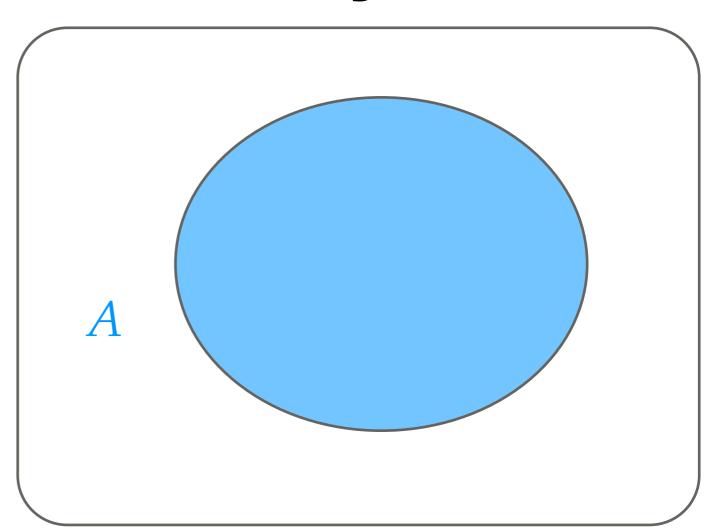


## (1) Conditional Probability

```
Pr(B|A)=
proportion of
area A that is
  also B =
 Pr(B & A) /
    Pr(A)
```

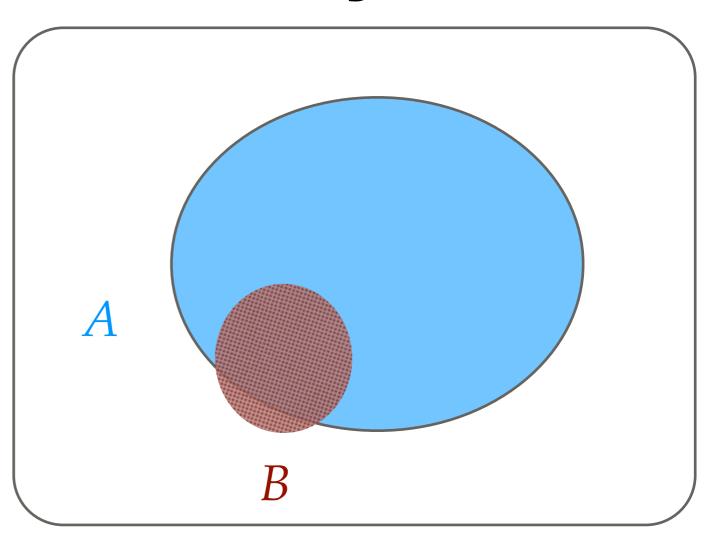
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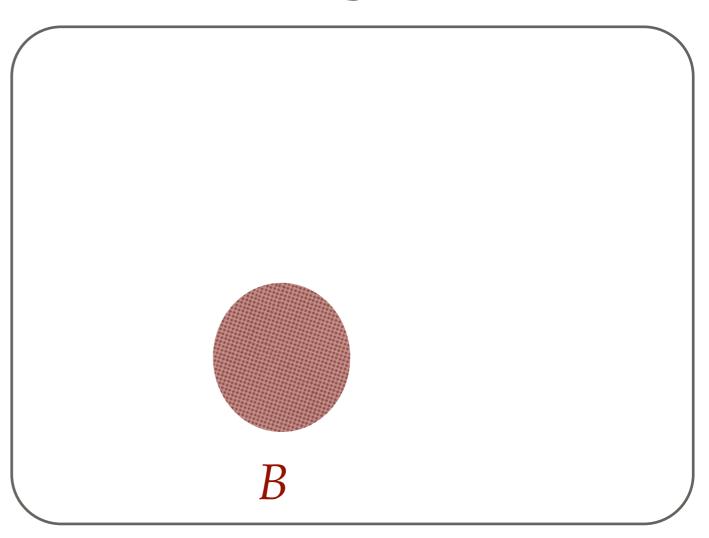


## (2) Conditional Probability

```
Pr(A|B)=
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    Pr(B)
```

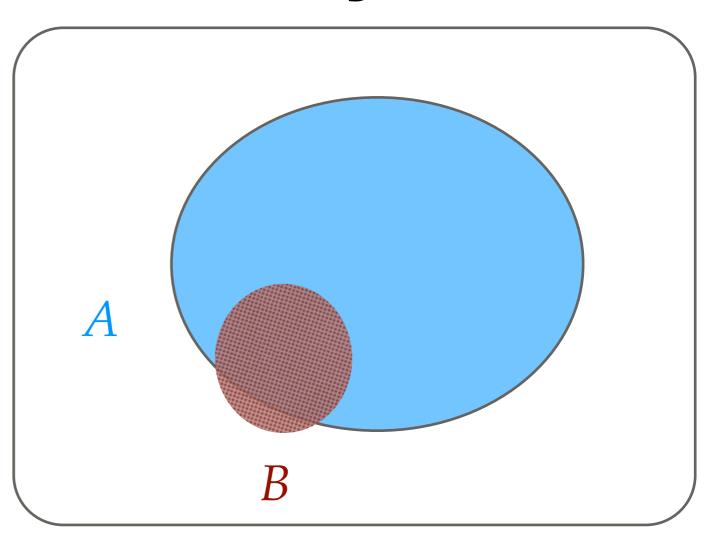
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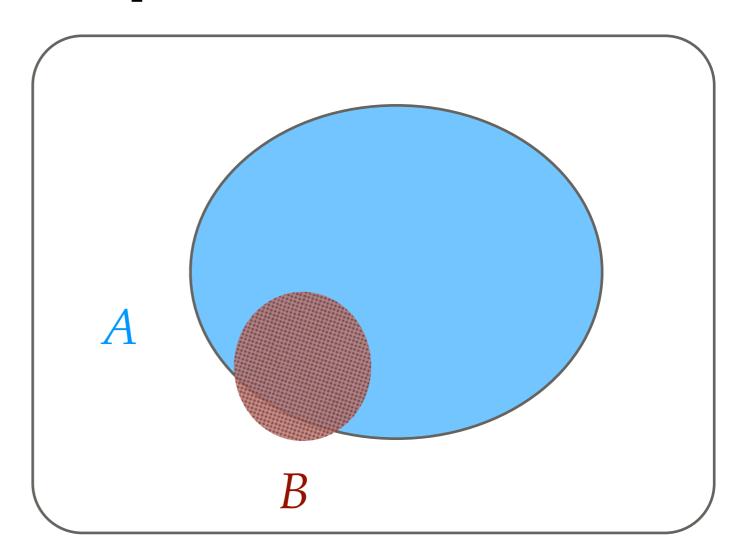
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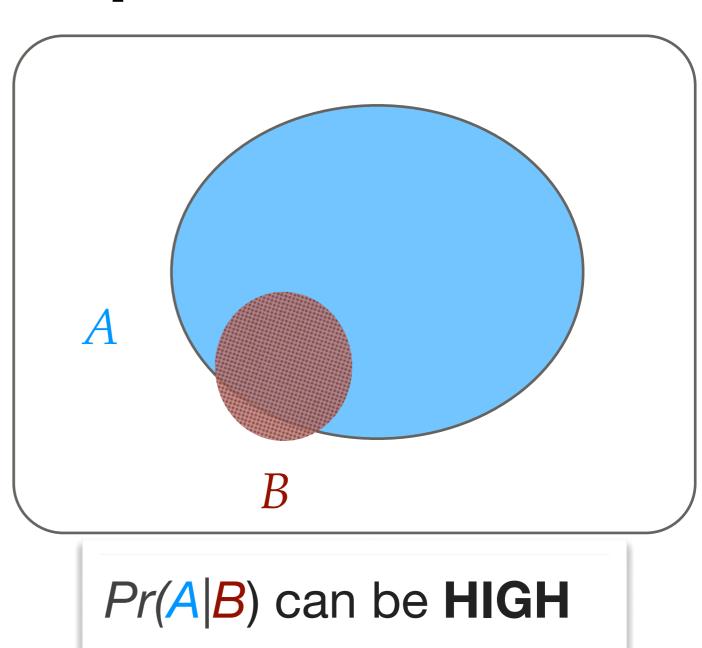
## A Difference to Keep in Mind

Pr(B|A)
versus
Pr(A|B)



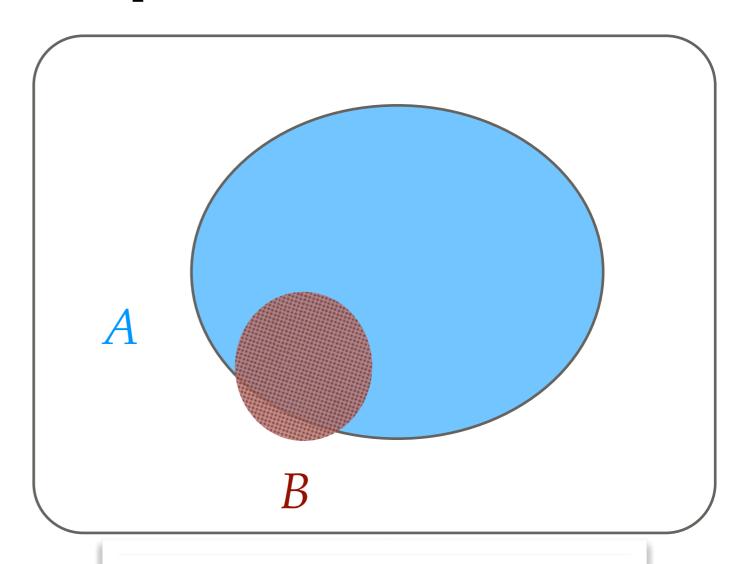
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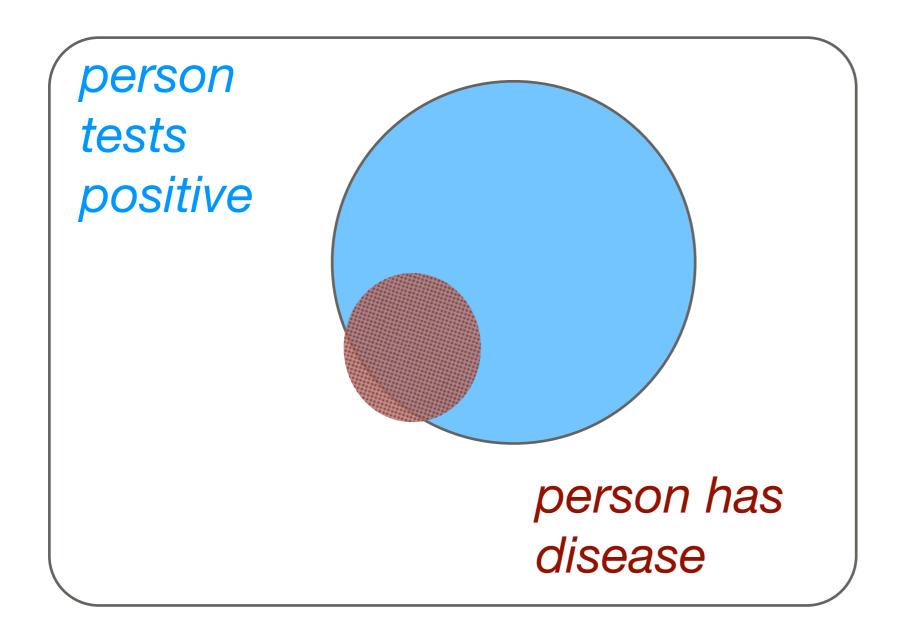
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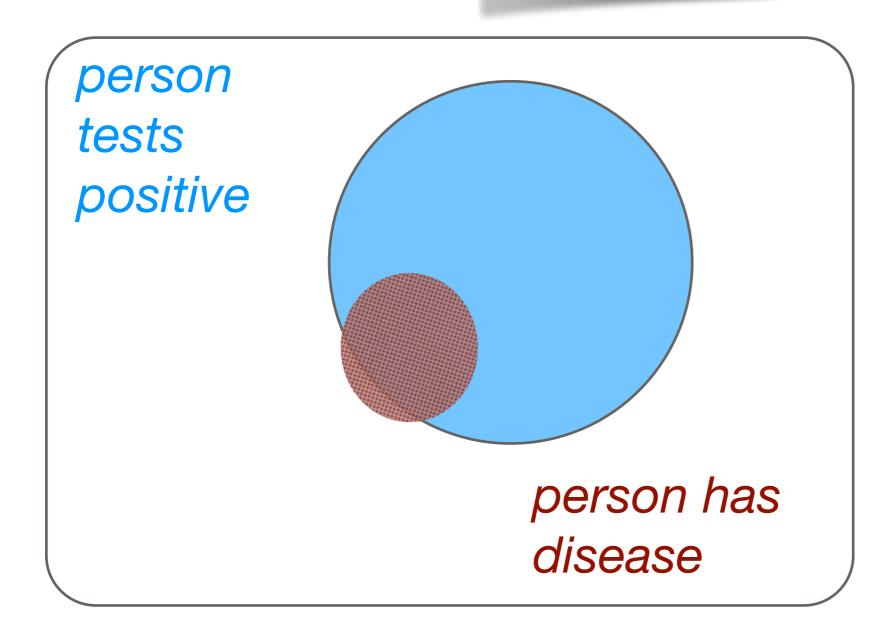
Pr(A|B) can be **HIGH** Pr(B|A) while is **LOW** 

# Example



# Example

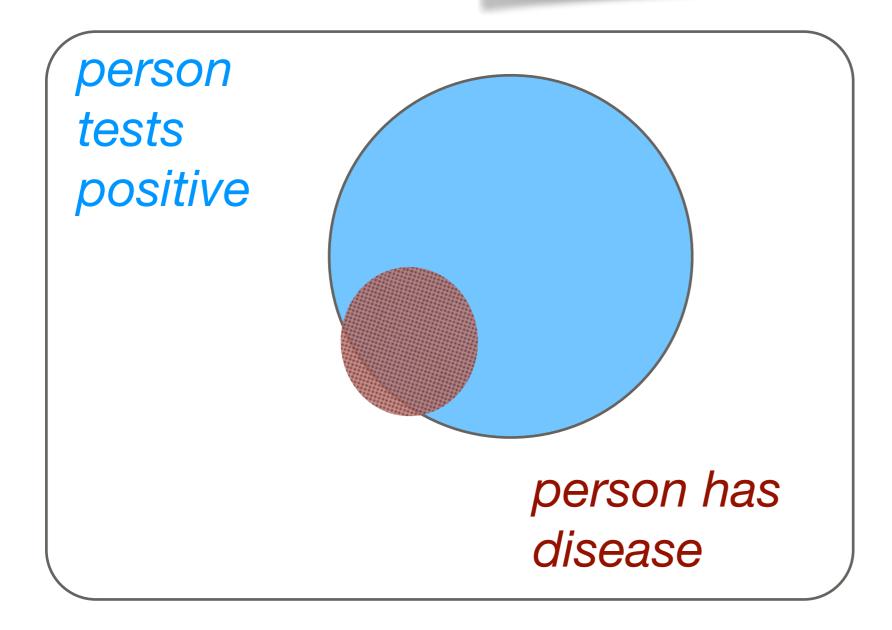
Pr(test positive disease) is HIGH

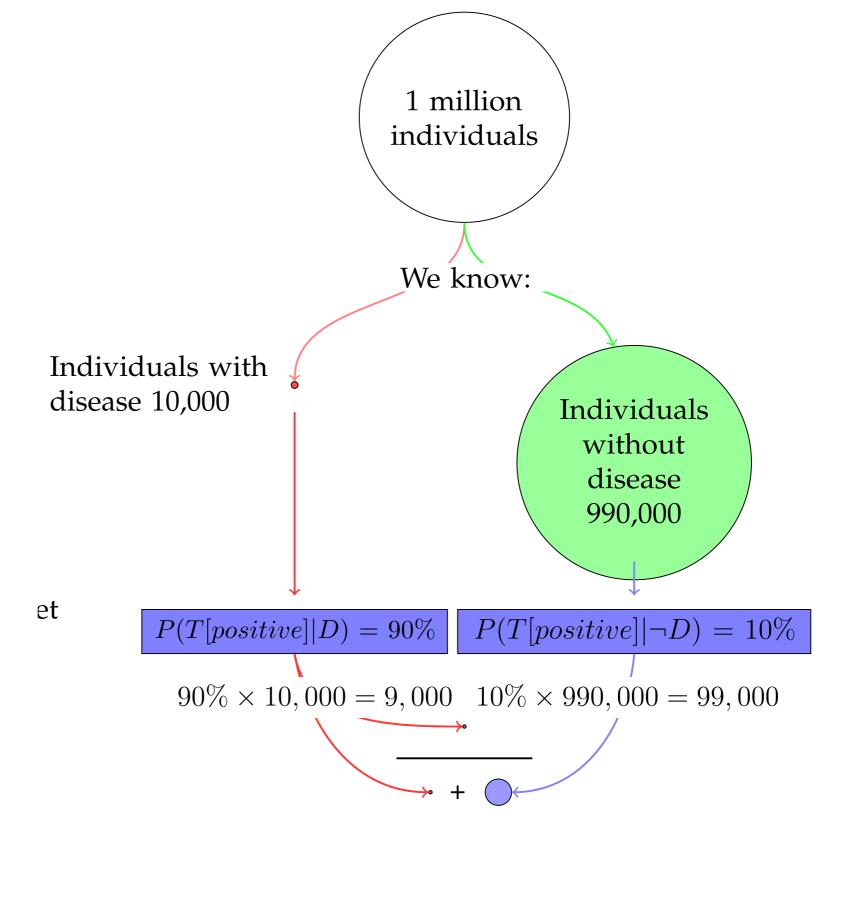


## Example

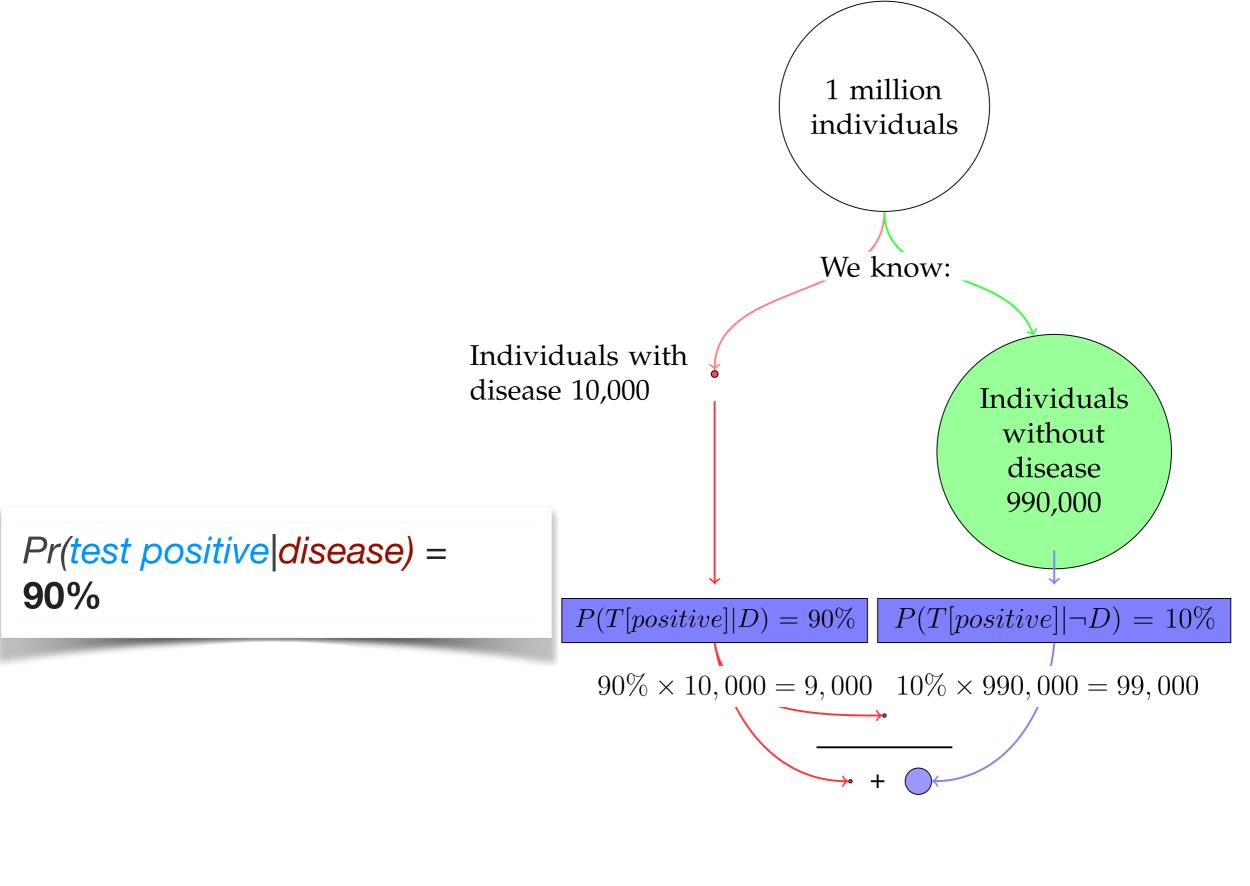
Pr(test positive disease) is HIGH

Pr(disease test positive) is LOW

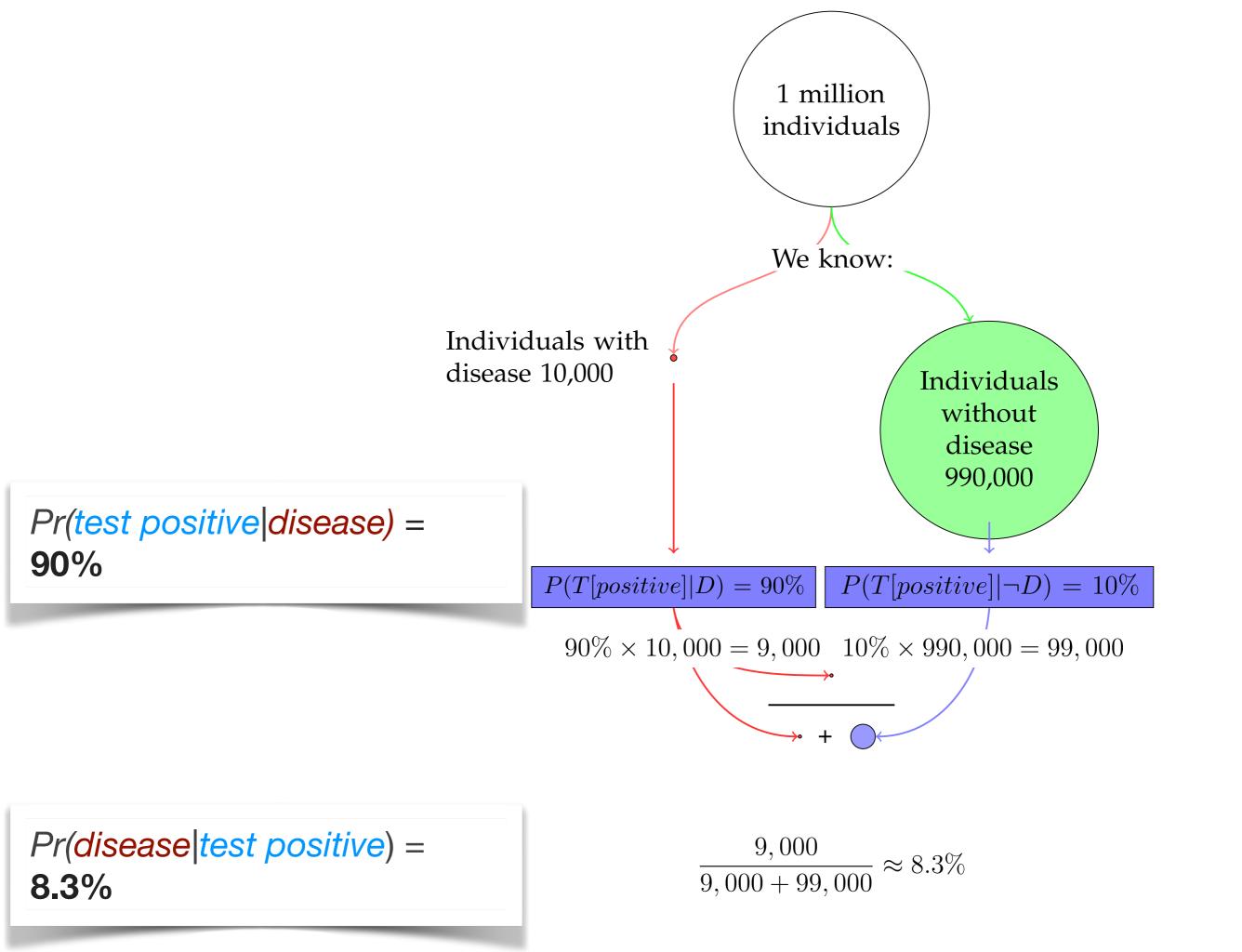




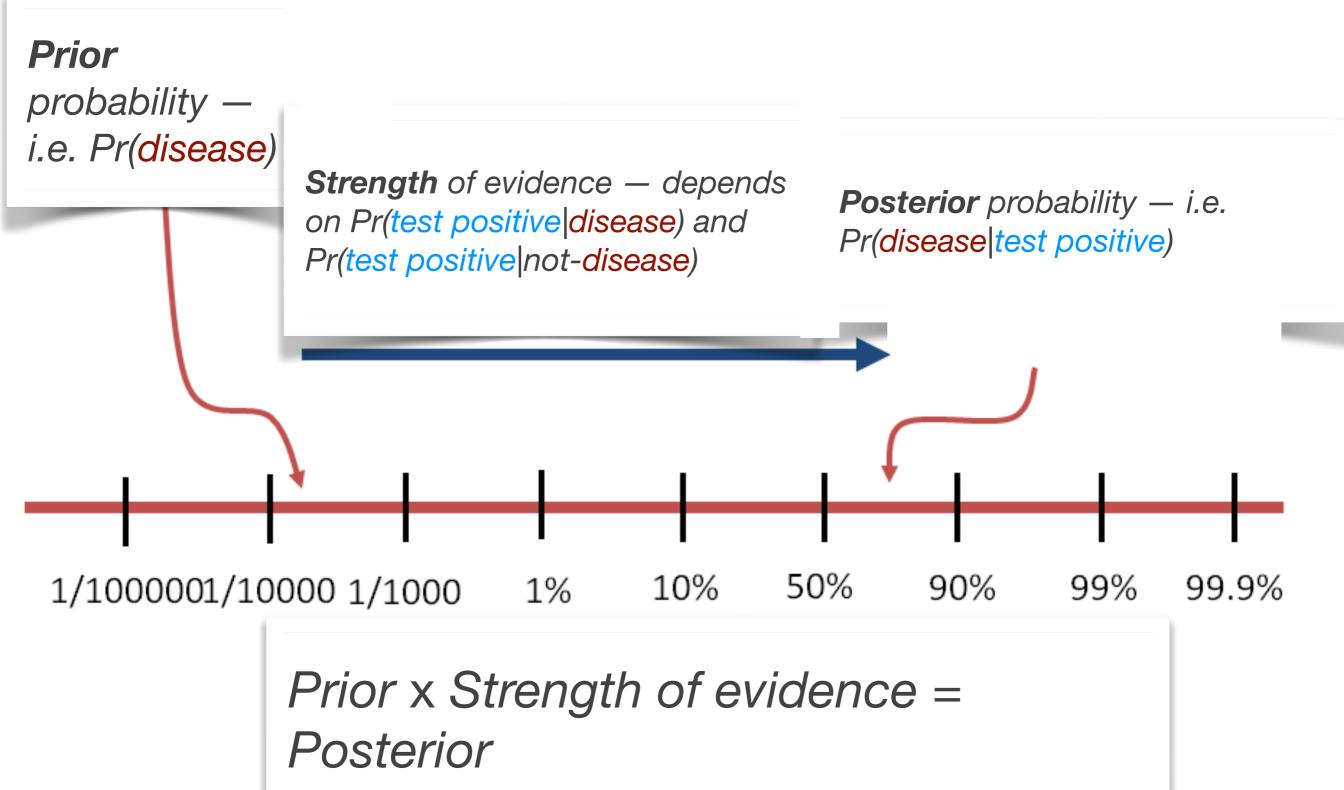
$$\frac{9,000}{9,000 + 99,000} \approx 8.3\%$$



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#### Bayes' Theorem (graphical representation)



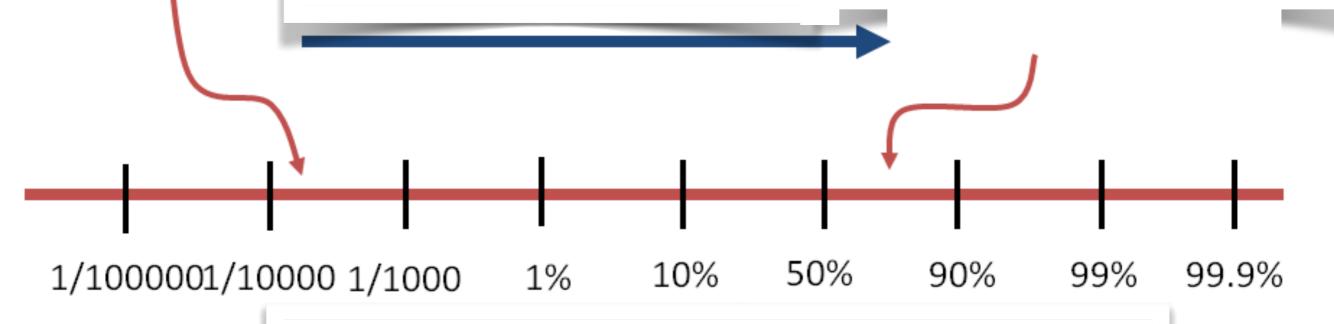
#### Bayes' Theorem

Upshot: even if the test is good, the posterior probability of having the disease given a positive test result, could still be low if the priors are low

**Prior**probability —
i.e. Pr(disease)

**Strength** of evidence — depends on Pr(test positive|disease) and Pr(test positive|not-disease)

**Posterior** probability — i.e. Pr(disease|test positive)



Prior x Strength of evidence = Posterior

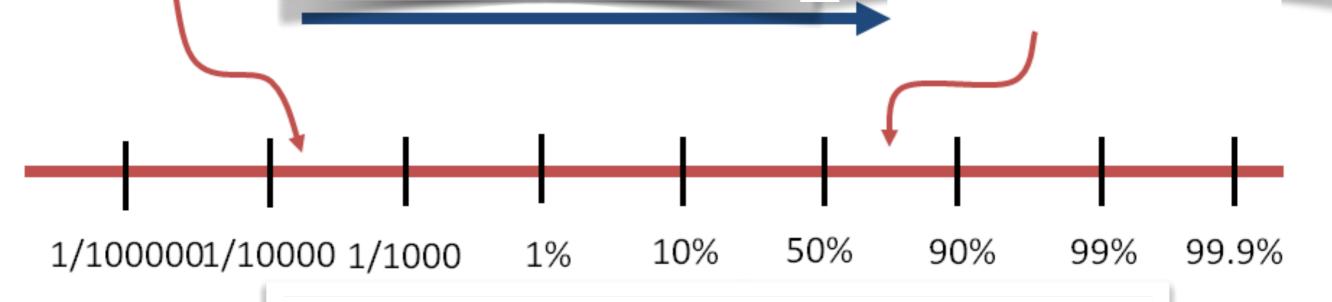
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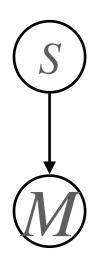


Prior x Strength of evidence = Posterior

# Back to Bayesian Networks

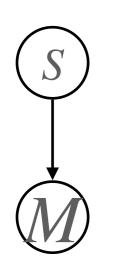
# Examples of Bayesian Networks for Assessing DNA Evidence and Eyewitness Evidence

Graph



#### Graph

#### Probabilities



$$P(S = yes) = prior$$

$$P_0(M = yes | S = yes) = 1$$
  
 $P(M = yes | S = no) = RMP$   
Random Match Probability

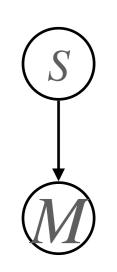
S=yes	Prior
S=no	1-prior

	S=yes	S=no
M=yes	100%	RMP
M=no	0%	1-RMP

#### Graph

#### Probabilities

#### **Probability Tables**



$$P(S = yes) = prior$$

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 $P(M = yes | S = no) = RMP$   
Random Match Probability

	S=yes	S=no
M=yes	100%	RMP
M=no	0%	1-RMP

Bayes' theorem needed to calculate P(S = yes | M = yes), as follows:

$$P_0(S = yes \mid M = yes) = \frac{P(M = yes \mid S = yes)}{P(M = yes)}P(S = yes)$$

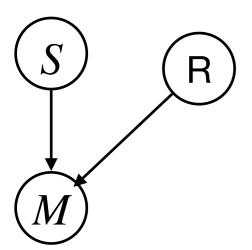
$$= \frac{P(M = yes \mid S = yes)}{P(M = yes \mid S = yes)P(S = yes) + P(M = yes \mid S = no)P(S = no)}P(S = yes)$$

# Aside How Are Random Match Probabilities Calculated?

See Charles H. Brenner's "Forensic mathematics of DNA matching" available at <a href="https://dna-view.com/profile.htm">https://dna-view.com/profile.htm</a>

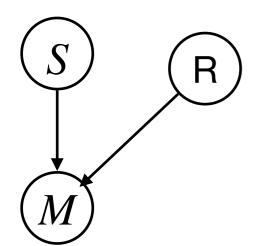
DNA P	rofile	Allele frequency from database		Genotype frequency for loc				
Locus	Alleles	times allele observed	size of database	size of database Frequency		formula	number	
CSF1PO	10	109	432	p=	0.25	2na	0.16	
CSF1FO	11	134	432	q=	0.31	2pq	0.16	
TPOX	8	229	432	n=	0.53	2	0.28	
ITOX	8	229	432	p=	0.55	$p^2$	0.26	
THO1	6	102	428	128	p=	0.24	2na	0.07
	7	64		q=	0.15	2 <i>pq</i>	0.07	
vWA	16	91	428	n=	0.21	2	0.05	
VVVA	16	71		p=	0.21	$p^2$	0.03	
	profile frequency=			0.00014				

Graph



Graph

#### Probabilities



$$P(S = yes) = prior for S$$

$$P(R = yes) = prior for R$$

Graph

Probabilities

S R

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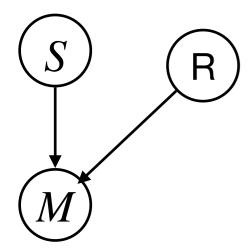
$$P(R = yes) = prior for R$$

S=yes	Prior (low?)
S=no	1-prior

R=yes	Prior (high?)
R=no	1-prior

Graph

#### Probabilities



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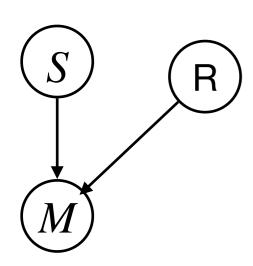
$$P_0(M = yes | S = yes \& R = yes) = 1$$
 $P(M = yes | S = no \& R = yes) = RMP$ 
Random Match Probability
 $P_0(M = yes | S = yes \& R = no) = 0.5$ 
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S=yes	Prior (low?)
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Graph

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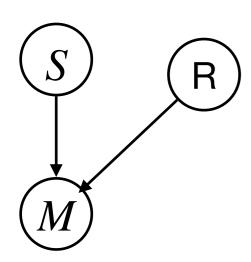
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	S=yes & R=yes	S=no & R=yes	S=yes & R=no	S=no & R=no
M=yes	100%	RMP	50%	50%
M=no	0%	1-RMP	50%	50%

Graph

#### Probabilities

Probability Tables



$$P(S = yes) = prior for S$$

$$P(R = yes) = prior for R$$

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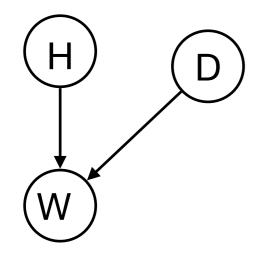
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M=yes	100%	RMP	50%	50%
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Bayes' theorem needed to calculate P(S = yes | M = yes). **But manual calculations quickly become unmanageable!** 

### Example 3: Eyewitness and Distance

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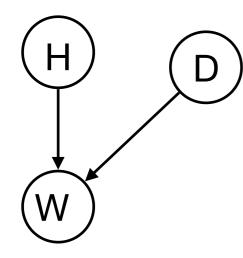
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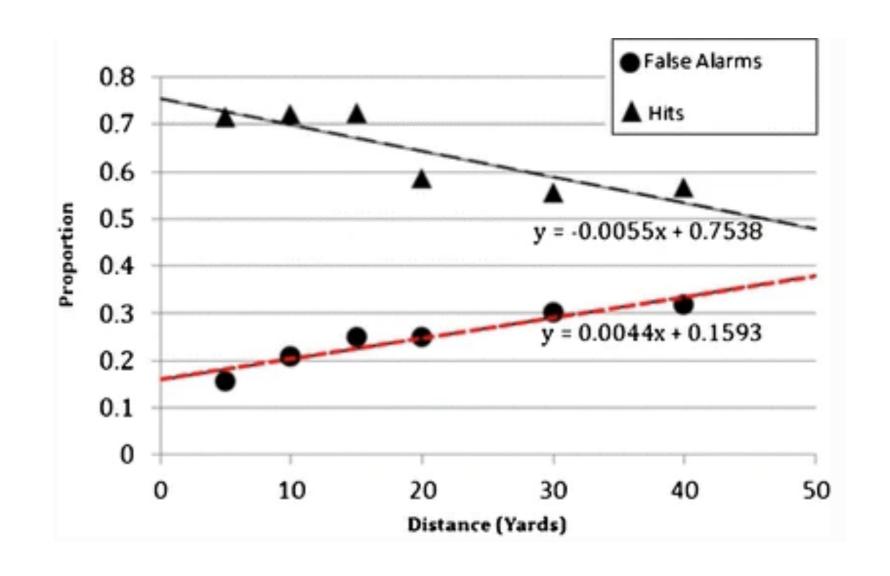
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Lampinen, James Michael, Erickson, William Blake, Moore, Kara N., & Hittson, Aaron (2014), "Effects of distance on face recognition: implications for eyewitness identification", *Psychonomic Bulletin & Review*, 21.

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(2) **Numerical**: The strength of these relationships is expressed numerically with probabilities tables

(3) **Reasoning**: Able to calculate probabilities of hypotheses based on evidence using Bayes' theorem (or dedicated software)

# PART II Group Exercise and Discussion

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Fred goes to the shooting range 4 days a week.

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Daniela's accuracy in correctly identifying and remembering Fred is 99%. In other words, if Fred was at the shooting range that day, there is a 1% chance that she will incorrectly report that he was not there, and if he was not, there is a 99% chance that she will correctly report that he was not there.

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What is the probability that Fred shot Chris?

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- Sketch how a graph of a Bayesian network (nodes and arrows) could look like. Is there only one possible graph or multiple graphs seem appropriate here?

- Start with an informal analysis of the case: what are the main pieces of evidence? How would a judge or a lawyer analyze this case? How strong is the evidence against Fred? Is there a reasonable doubt about Fred's guilt?
- Sketch how a graph of a Bayesian network (nodes and arrows) could look like. Is there only one possible graph or multiple graphs seem appropriate here?
- Fill in the **probability tables** with the right numbers. Do you have all the numbers you need or are some numbers missing?

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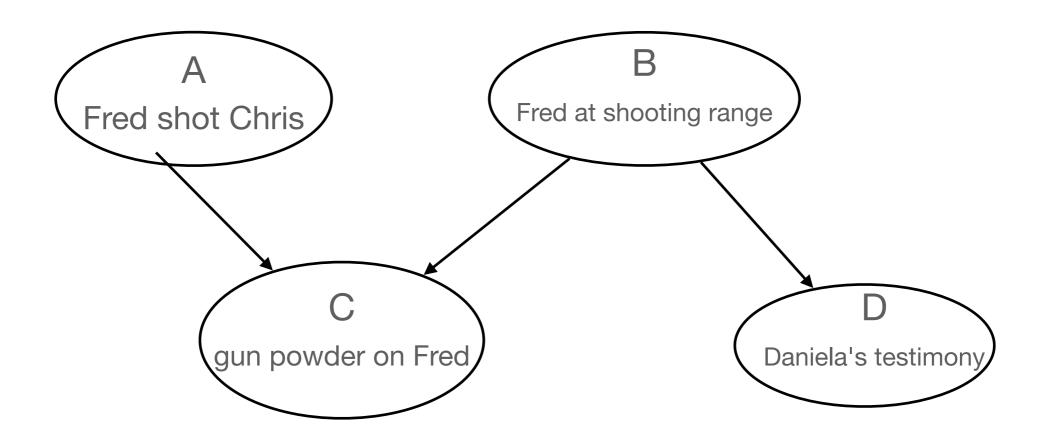
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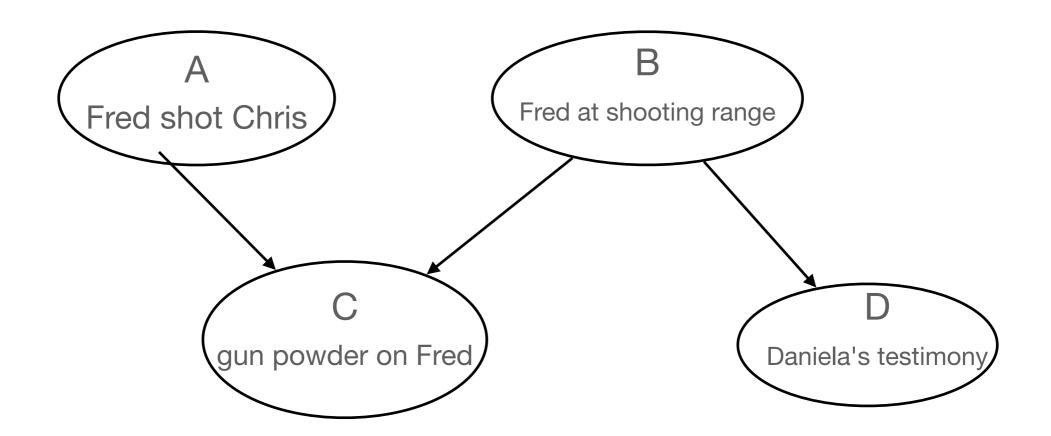
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**Daniela's testimony** changes things. She is highly reliable (99%). If the hypothesis that Fred was at the shooting range that day is ruled out, the most likely explanation is that Fred did indeed shot Chris.

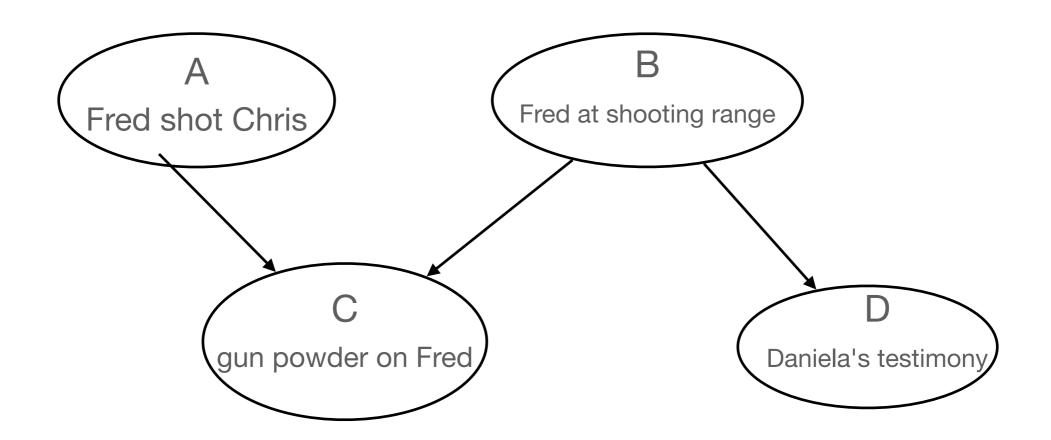
## Bayesian Network Approach

#### Bayesian Network - Graph

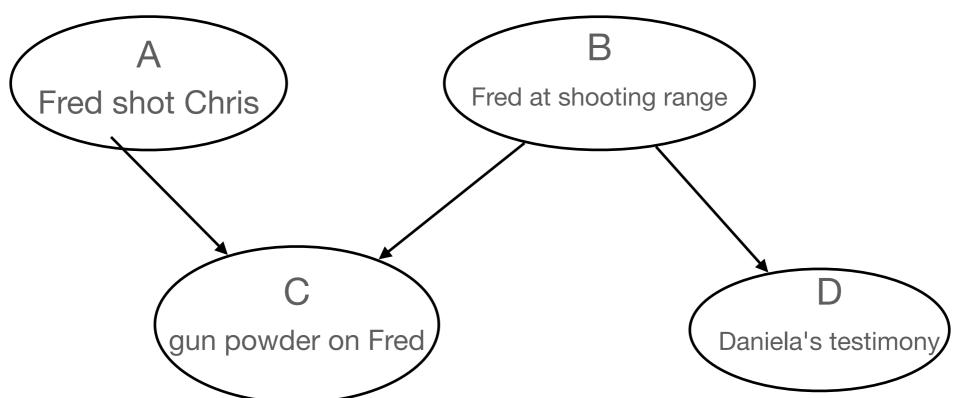




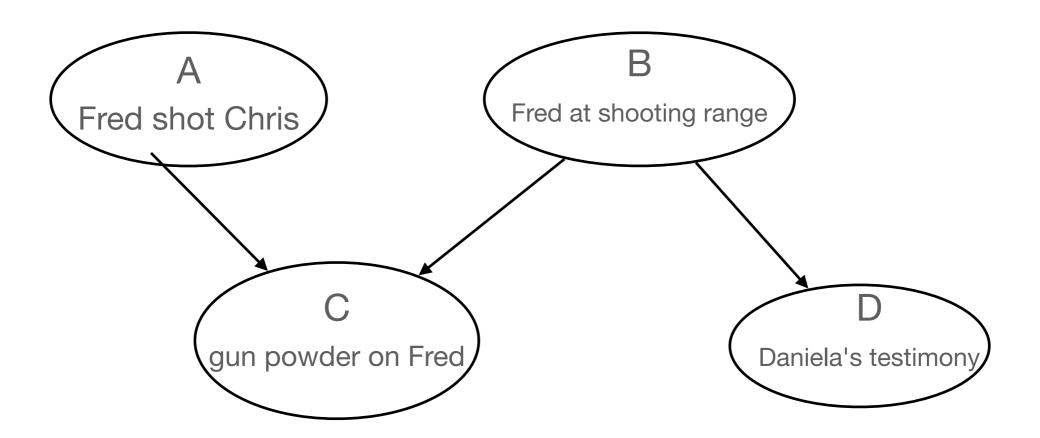
A=yes	1/100=1%	
A=no	99%	





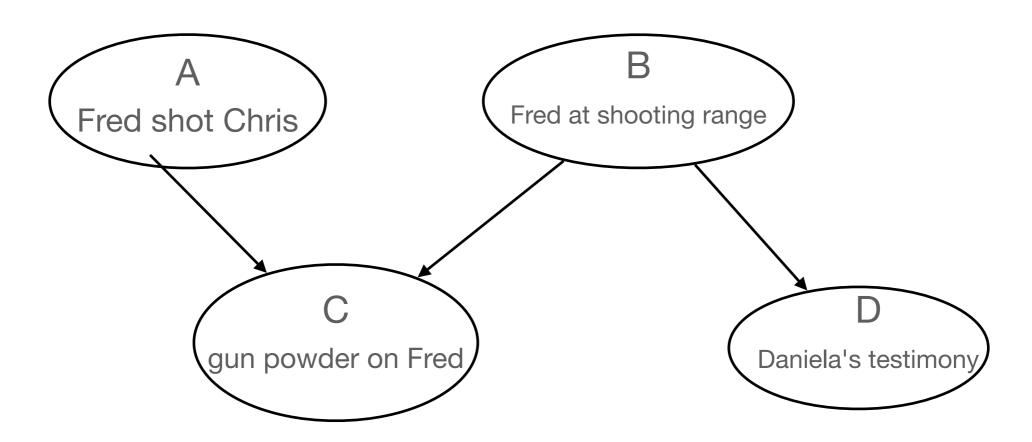






	B=yes	B=no
D=yes	99%	1%
D=no	1%	99%

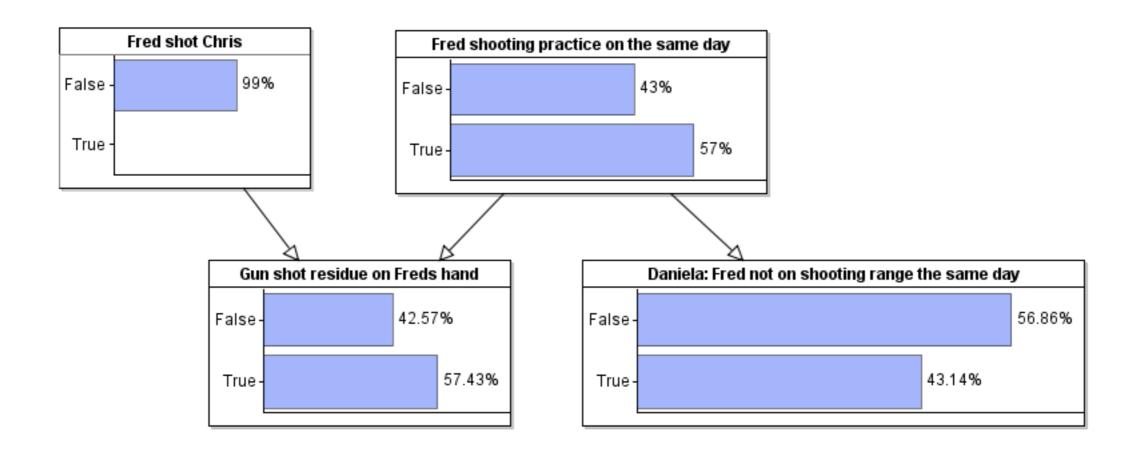




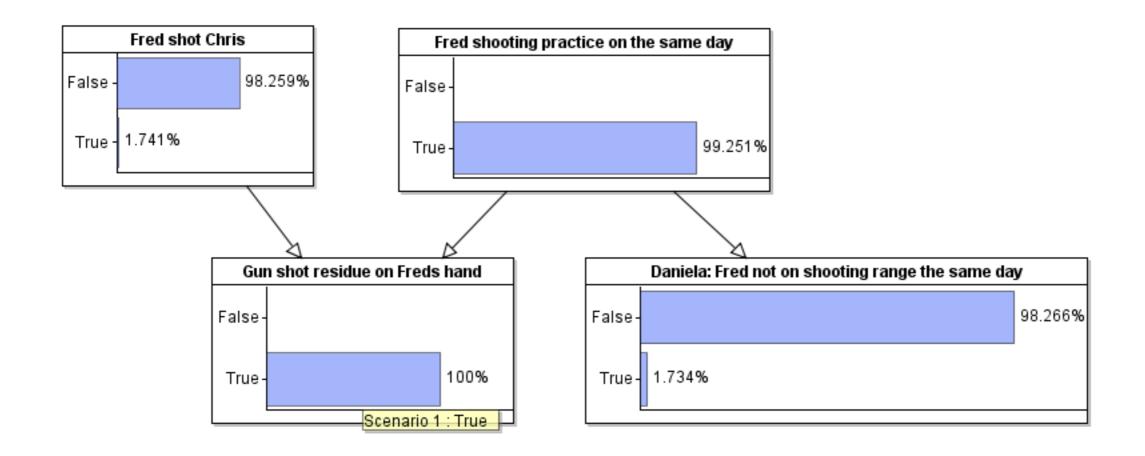
	A=yes & B=yes	A=no & B=yes	A=yes & B=no	A=no & B=no
C=yes	100%	100%	100%	0%
C=no	0%	0%	0%	100%

	B=yes	B=no
D=yes	99%	1%
D=no	1%	99%

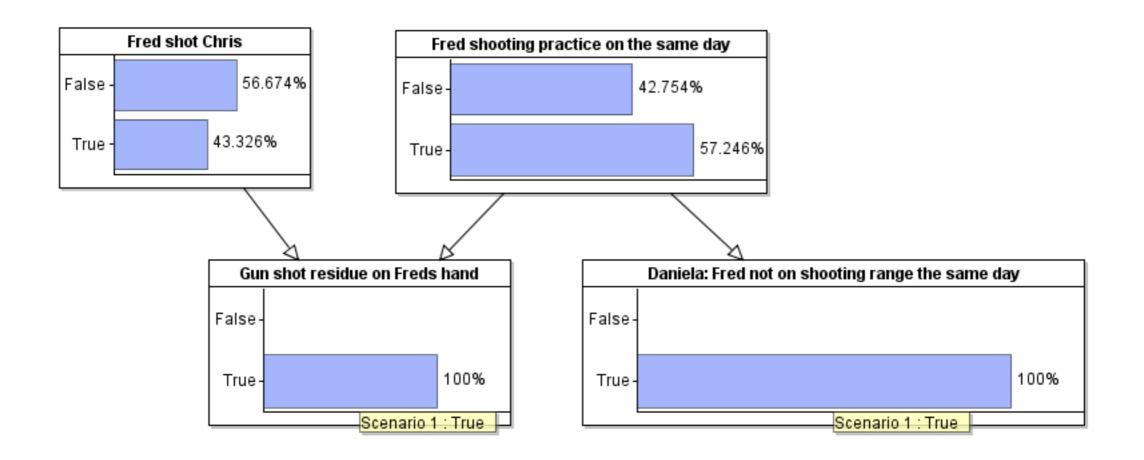
#### No Evidence: Unlikely Fred Shot Chris



#### Gun Powder on Fred: Still *Unlikely Fred Shot Chris*



### Gun Powder on Fred *plus* Daniela's Testimony: Still *Unlikely Fred Shot Chris*



#### **Questions for Discussion**

Feel Free to Add Your Own!

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#### **Questions for Discussion**

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If yes, wouldn't such subjectivity be a problem?

3. Where do the numbers needed to fill the probability tables come from?

# PART III Analysis of a Legal Case Using Bayesian Networks

#### Tasks of a Judge

- (1) Gatekeeping: apply exclusionary rules about relevance, hearsay, character evidence, privileges, etc.
- (2) Assess the evidence for and against the defendant, and then finally decide

- (2) Seek
  evidence and
  asks questions
- (4) Write down a written opinion that lays down in detail the reasoning that supports to the decision

#### Simonshaven case

## If You Were a Judge Writing the Opinion, How Would You Organize Your Analysis?

(NB: Matters of fact only)

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(1) Identify factual propositions (=hypotheses) under dispute.

These can be ultimate probanda or intermediate propositions.

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- (2) Identify key pieces of evidence which favor or oppose the factual propositions under dispute
- (3) Make an assessment of the case as a whole, all things considered.

This can can require an assessment of the balance of the evidence for/against the accused or an assessment of whether a reasonable doubt about guilt exists.

## The Analysis That Follows Is Taken From this Paper

#### **Analyzing the Simonshaven Case using Bayesian Networks**

Norman Fenton\*, School of Electronic Engineering and Computer Science, Queen Mary

University of London

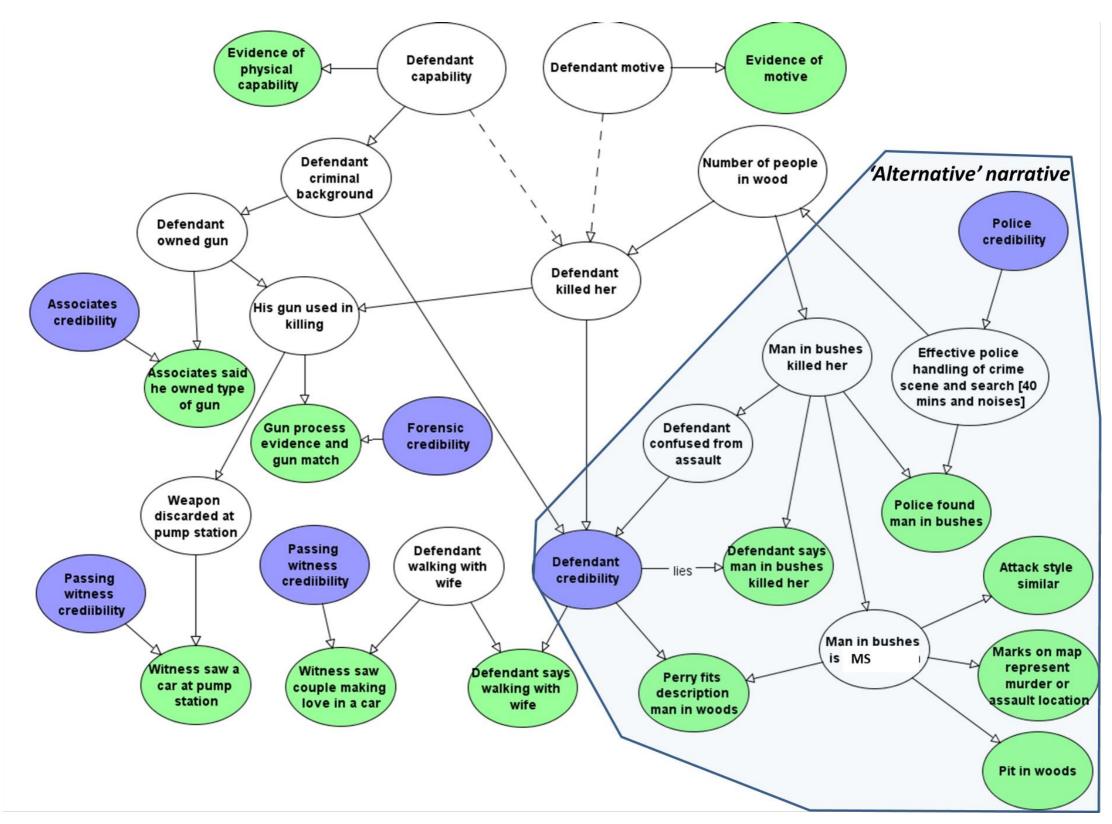
Martin Neil, School of Electronic Engineering and Computer Science, Queen Mary

University of London

Barbaros Yet, Department of Industrial Engineering, Hacettepe Universitesi, Turkey

David Lagnado, Department of Experimental Psychology, University College London

#### Full Bayesian Network



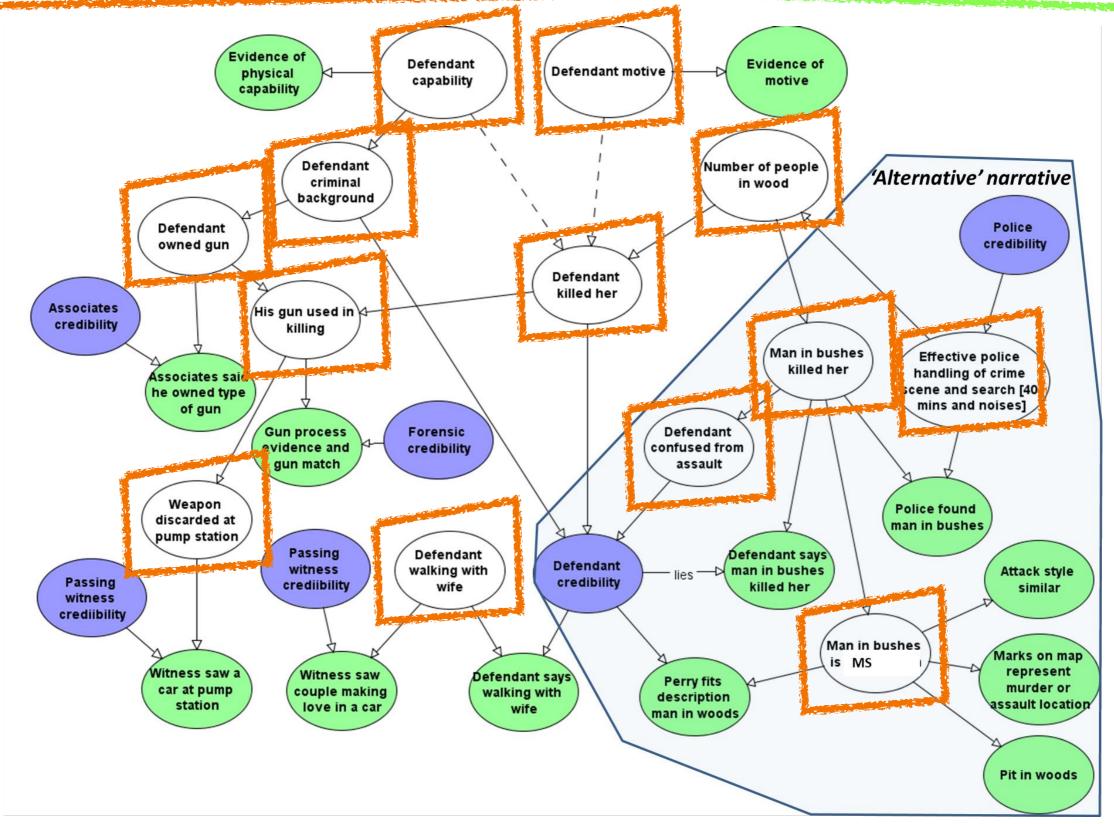


Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

#### Examples: Evidence/Hypothesis Idioms

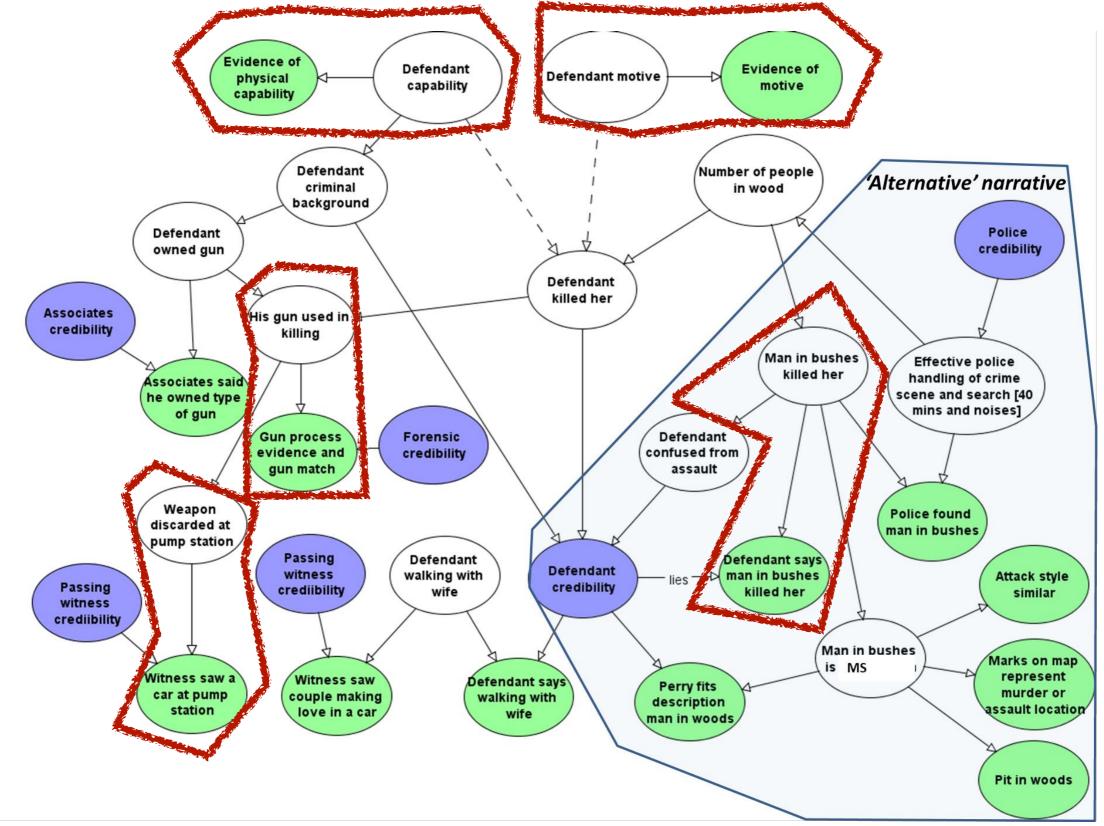


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### Examples: Evidence Credibility Idiom

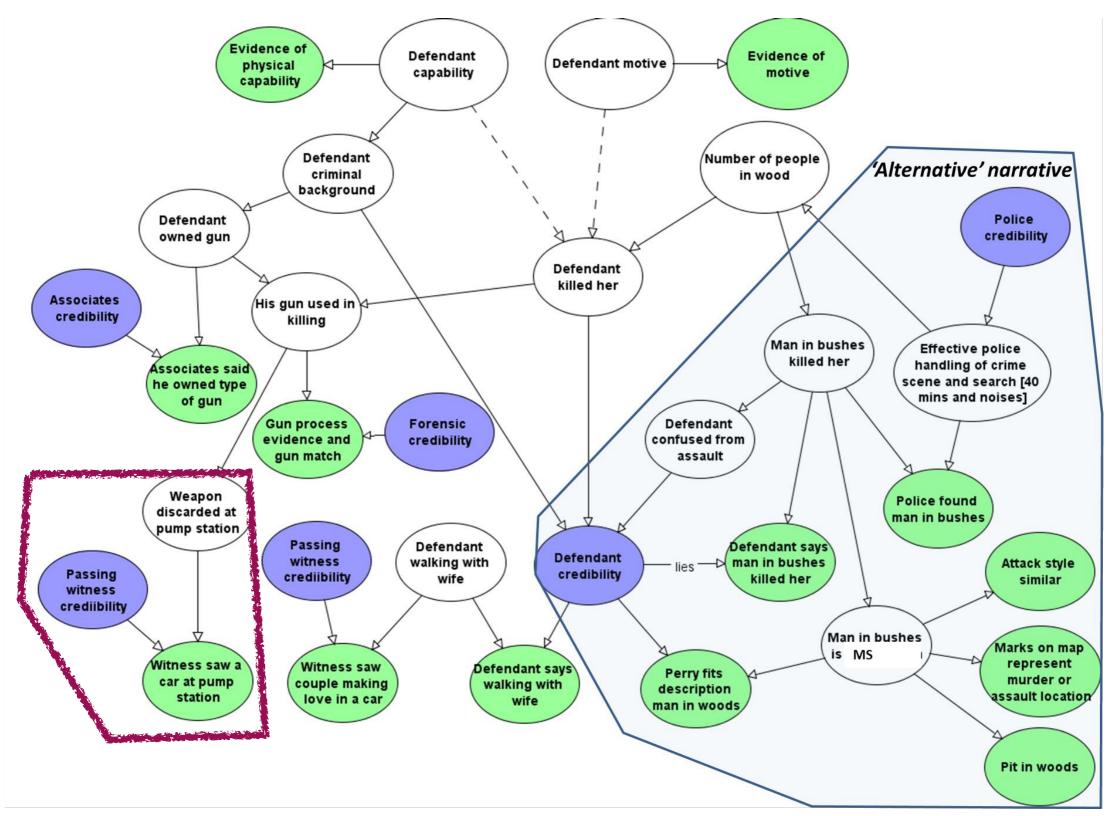
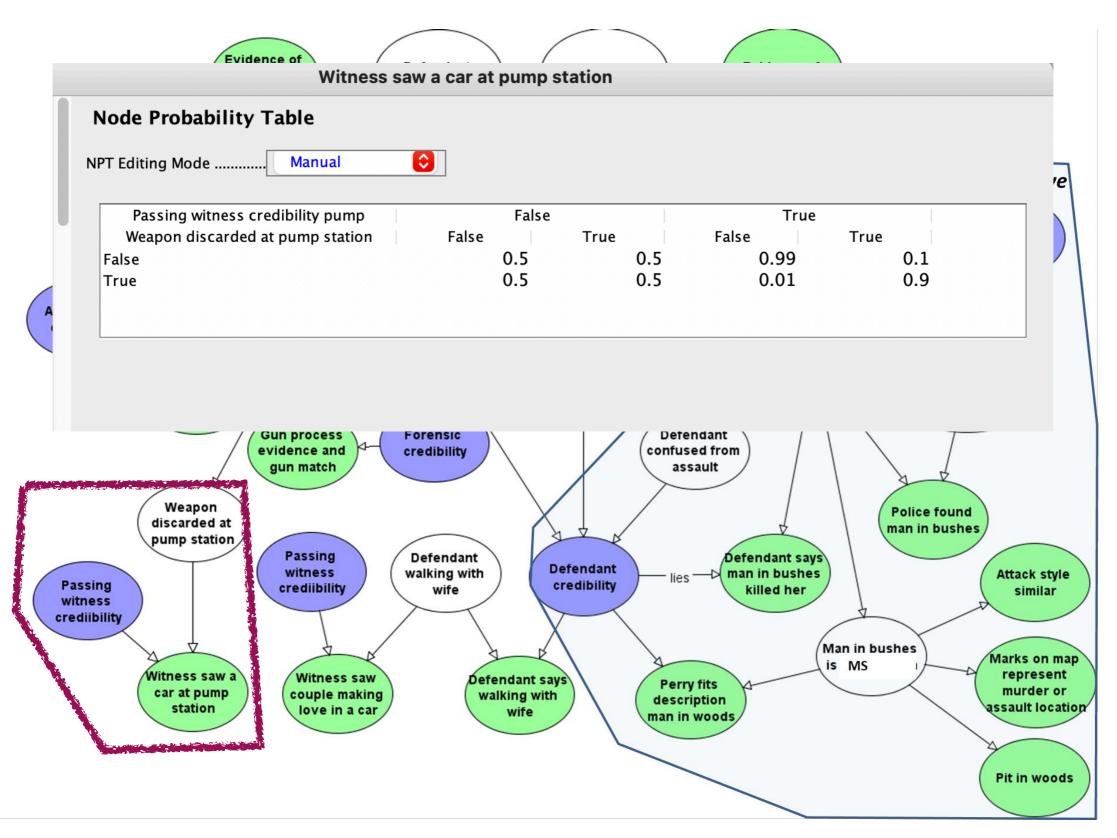


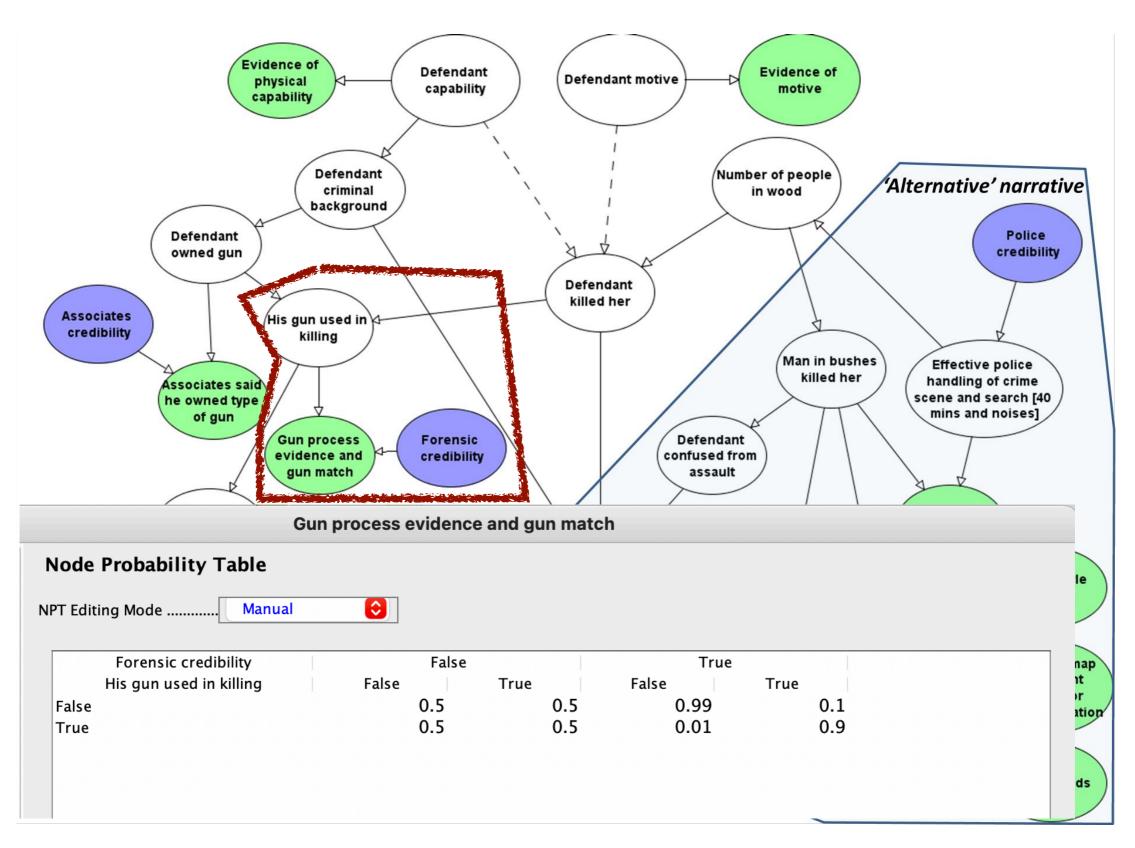
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# Examples of Probability Tables

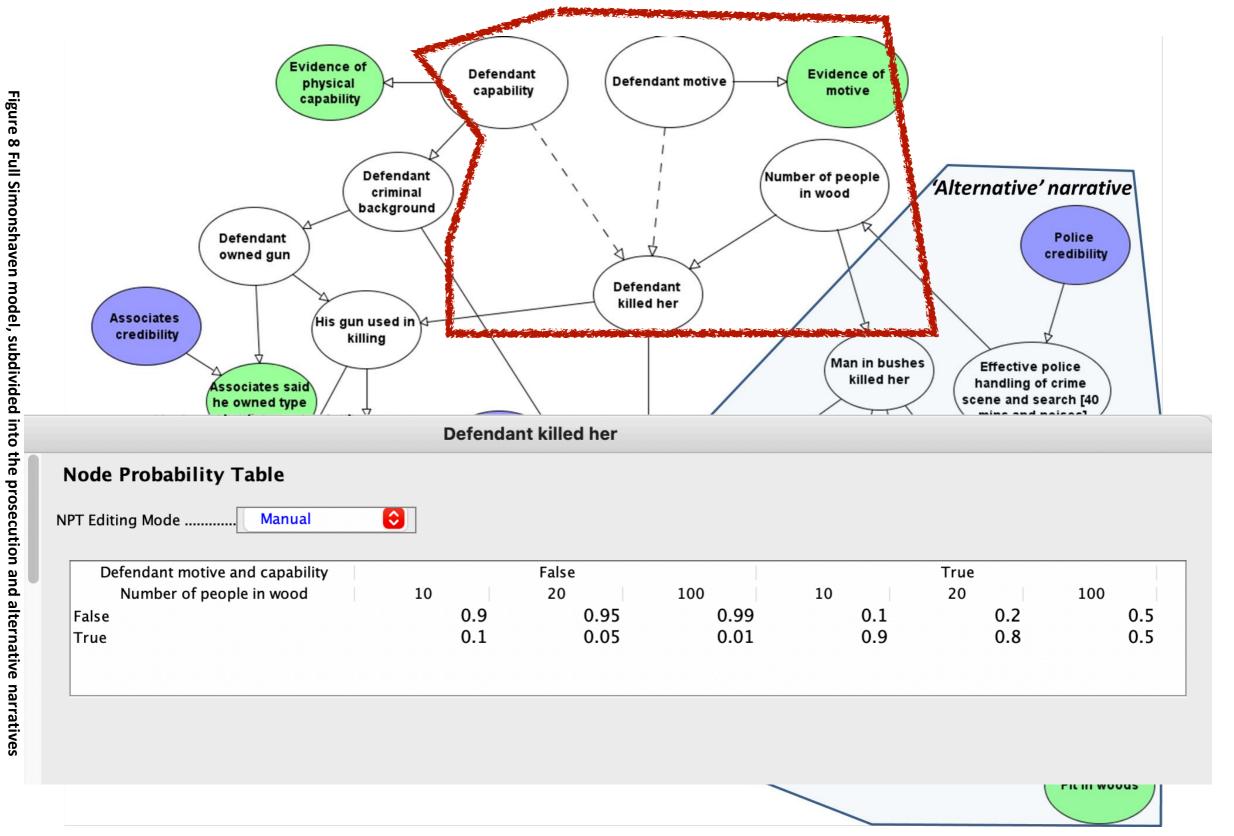
#### Weapon Discarded at Pump Station?



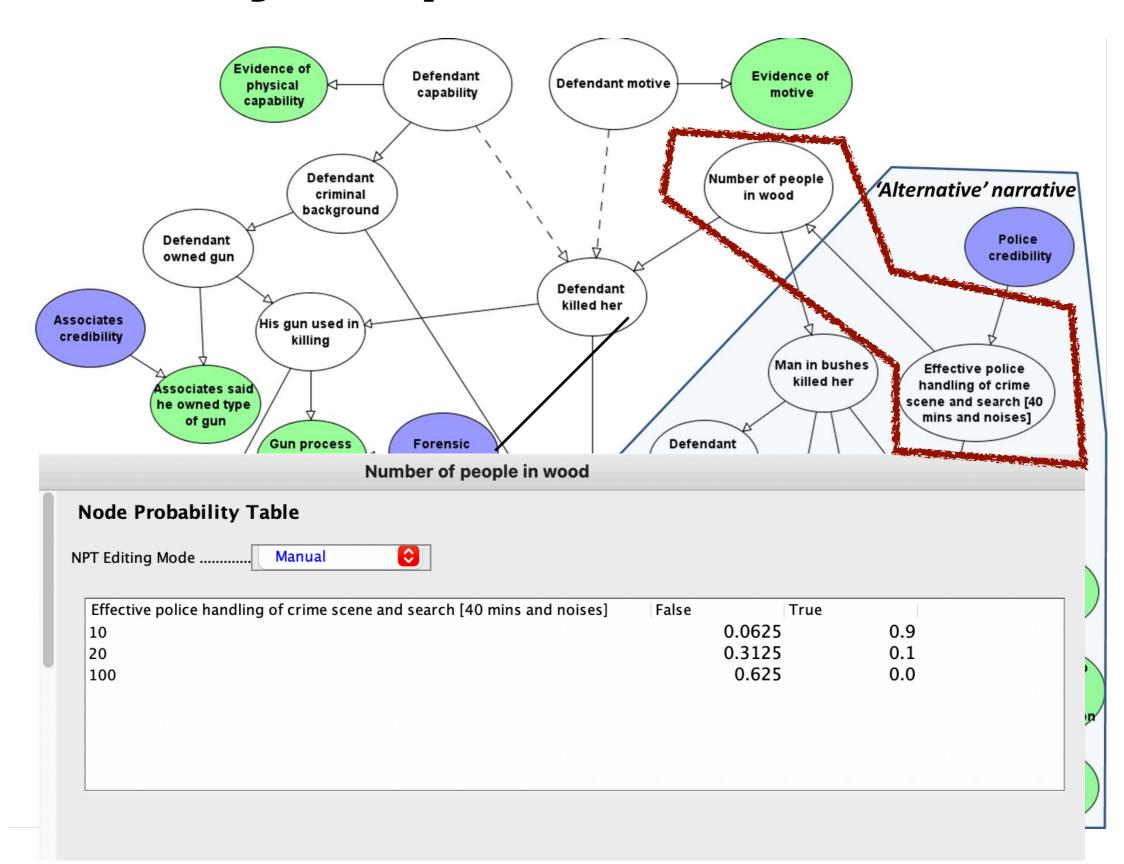
#### Gun Match Evidence



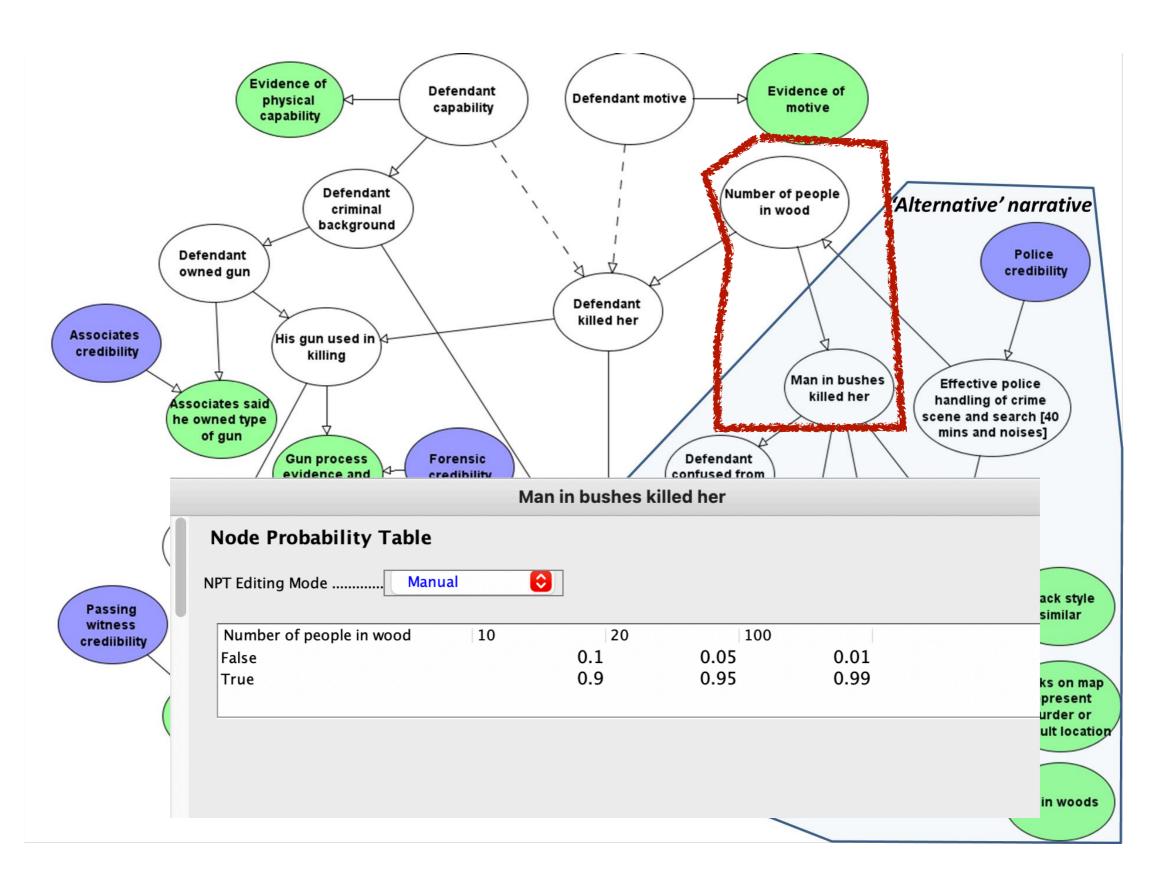
#### Did the Defendant Kill the Victim?



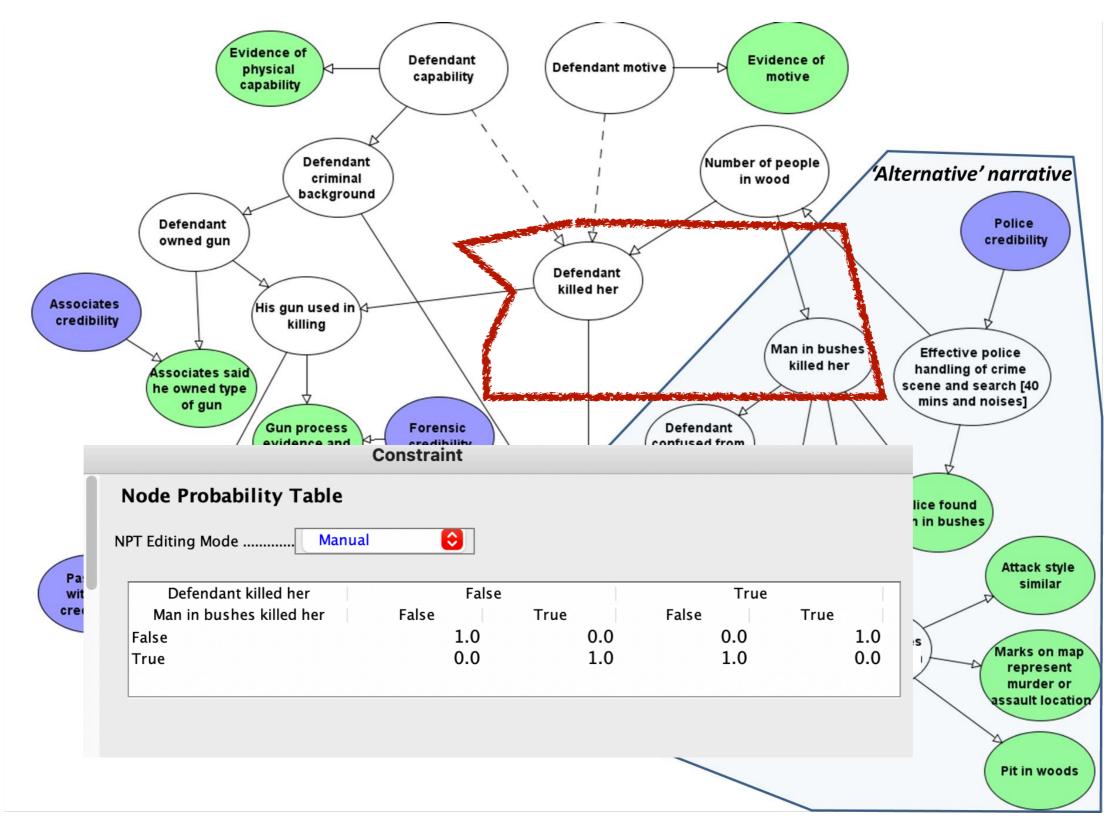
#### How Many People Were in the Woods?



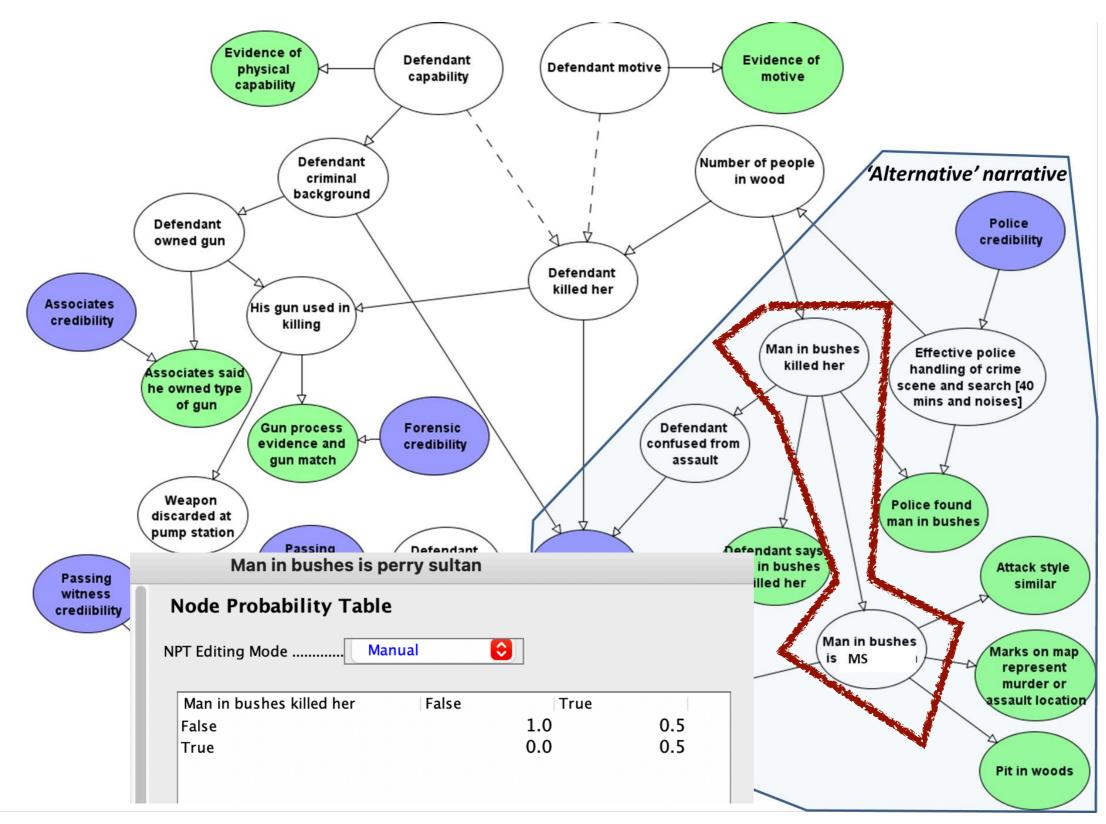
#### Did The Man in the Bushes Kill the Victim?



#### Incompatible Hypotheses

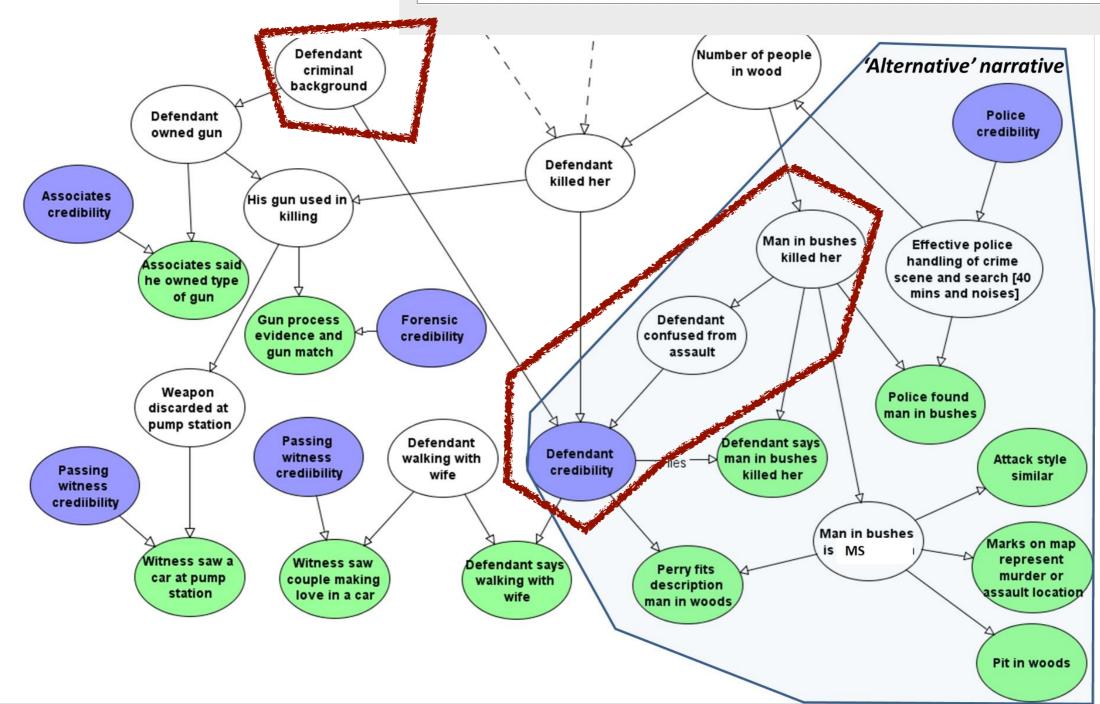


#### Was Perry Sultan in the Woods?



## Defendant's credibility

#### **Defendant credibility Node Probability Table** NPT Editing Mode ...... Defendant killed her False True Defendant criminal background False True False True Defendant confused from assault False False True False True False True True 0.1 0.1 0.99 0.5 0.6 0.9 0.9 0.99 False 0.9 0.5 0.9 0.4 0.1 0.1 0.01 0.01 True



ull Simonshaven model, subdivided into the prosecution and alternative narratives

Defendant capability

Defendant motive

Number of people in wood

Police credibility

Police credibility



ull Simonshaven model, subdivided into the prosecution and alternative narratives

Table 2 Probability assignments for credibility nodes

Credibility node	Probability credible (%)
Police credibility	90
Forensic credibility	90
Defendant credibility (in absence of any evidence, except existence of	53
crime). Note that the figure here is determined automatically by the	
priors for this node's parent nodes.	
Associates credibility (perhaps criminal?)	30
Passing witness credibility (pump station)	90
Passing witness credibility (car)	90

Pass witne credii

Pit in woods

ap

#### Changes in Probability as Evidence is Added

Table 3 Changes to probability of guilt, and defendant credibility, as evidence is entered in model (P refers

to prosecution evidence and D to defence evidence)

Evidence (cumulative)	Probability	Probability
	defendant	defendant
	guilty (%)	credible
	[rounded down]	[rounded down]
None	1	55
Evidence physical capability and Evidence of motive (P)	21	41
Associates said he owned type of gun + witness saw car at pump station	53	25
(P)		
Gun process evidence and gun match (P)	93	5
Witness saw couple making love on car (P) but defendant says walking	96	< 1
with wife at time (D)		
Police failed to find man in bushes and poor handling of crime scene (D)	80	2
Various bits of MS evidence {attack style, marks on map, pit in woods}	46	6
and fact that defendant says man in bushes killed her (D)		
MS does not fit suspect's description of the man in the woods (P)	74	4

### Sensitivity Analysis: What If We Had Assigned Different Numbers?