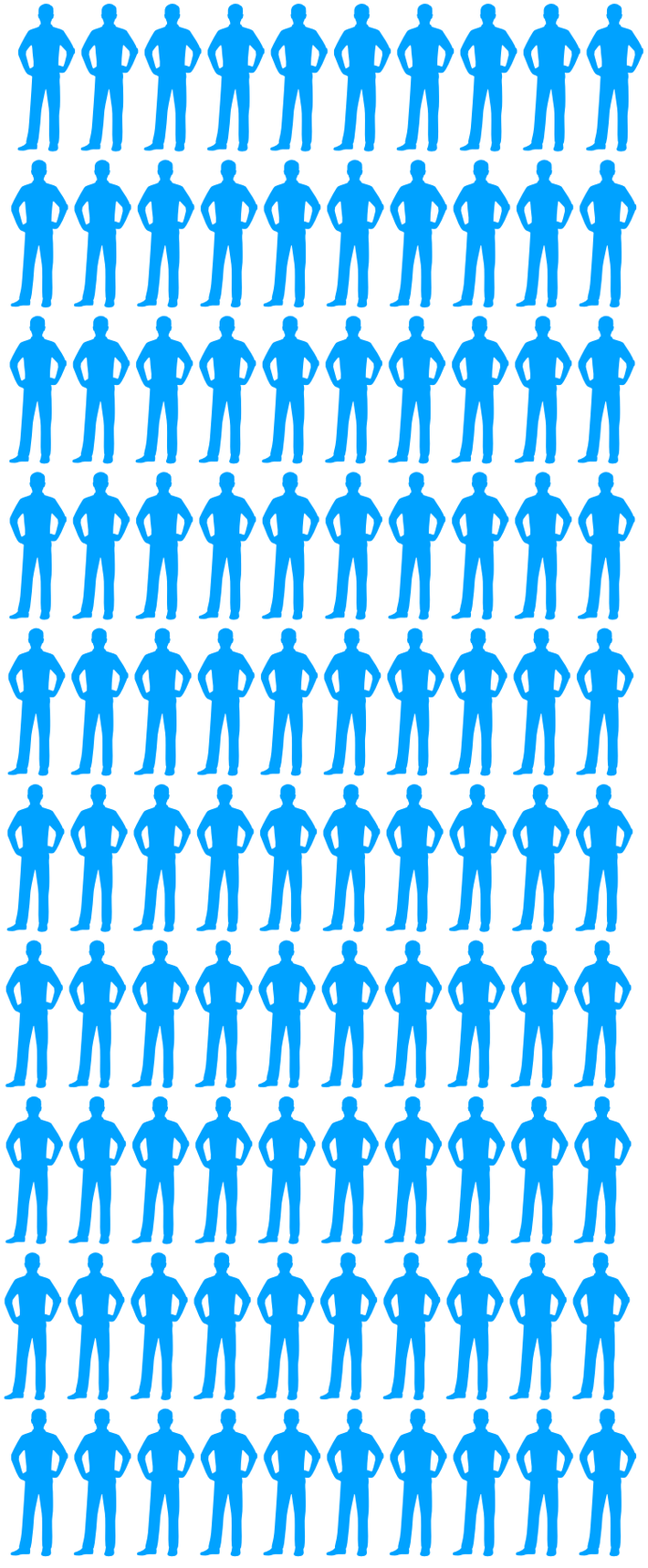
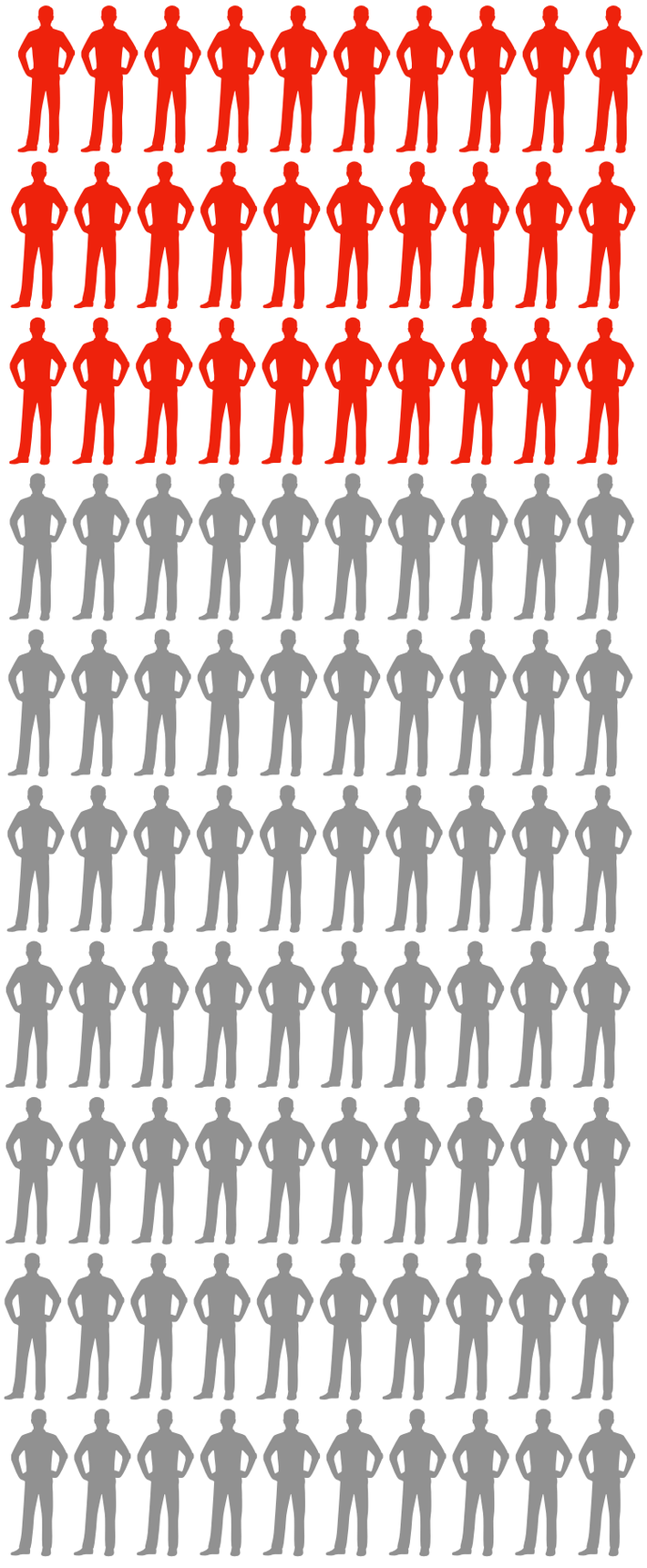
Chouldechova's Impossibility *In Pictures*

One Group

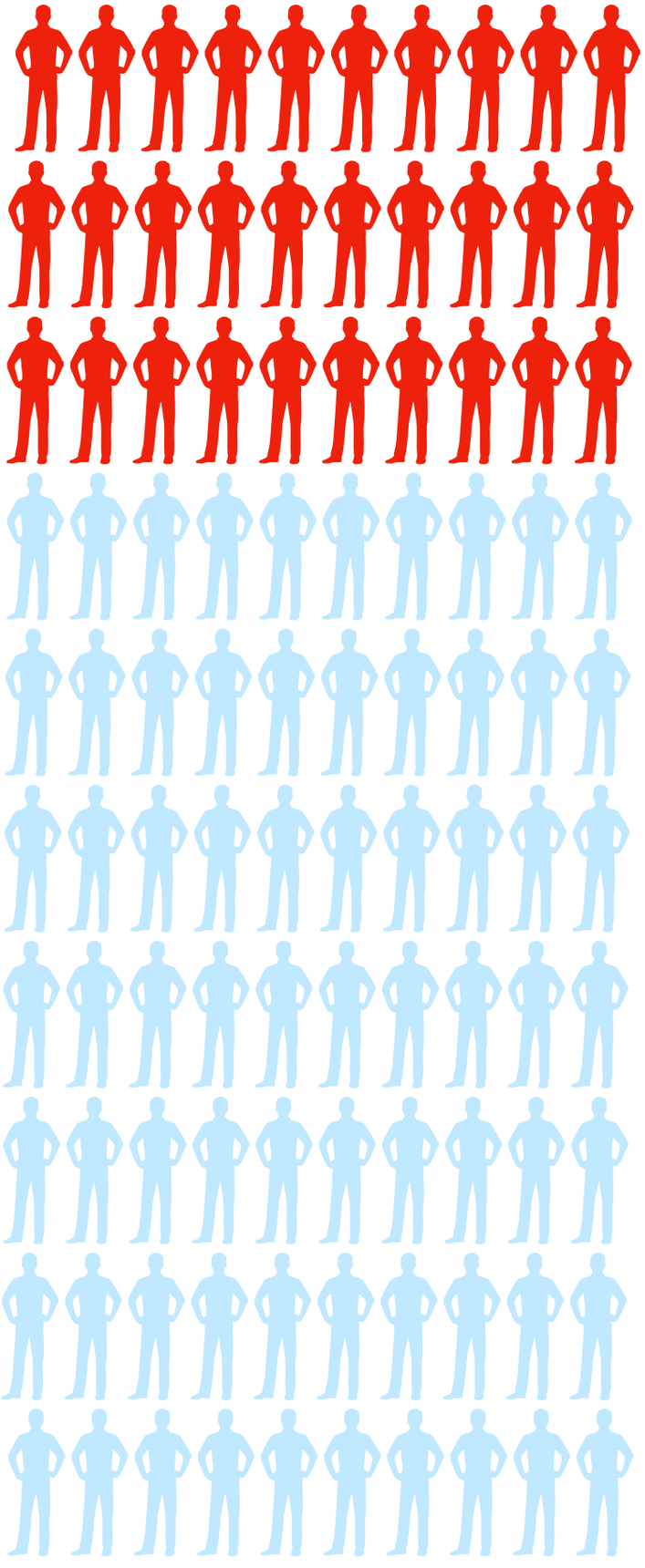
Base rate, FPR, FNR, PPV



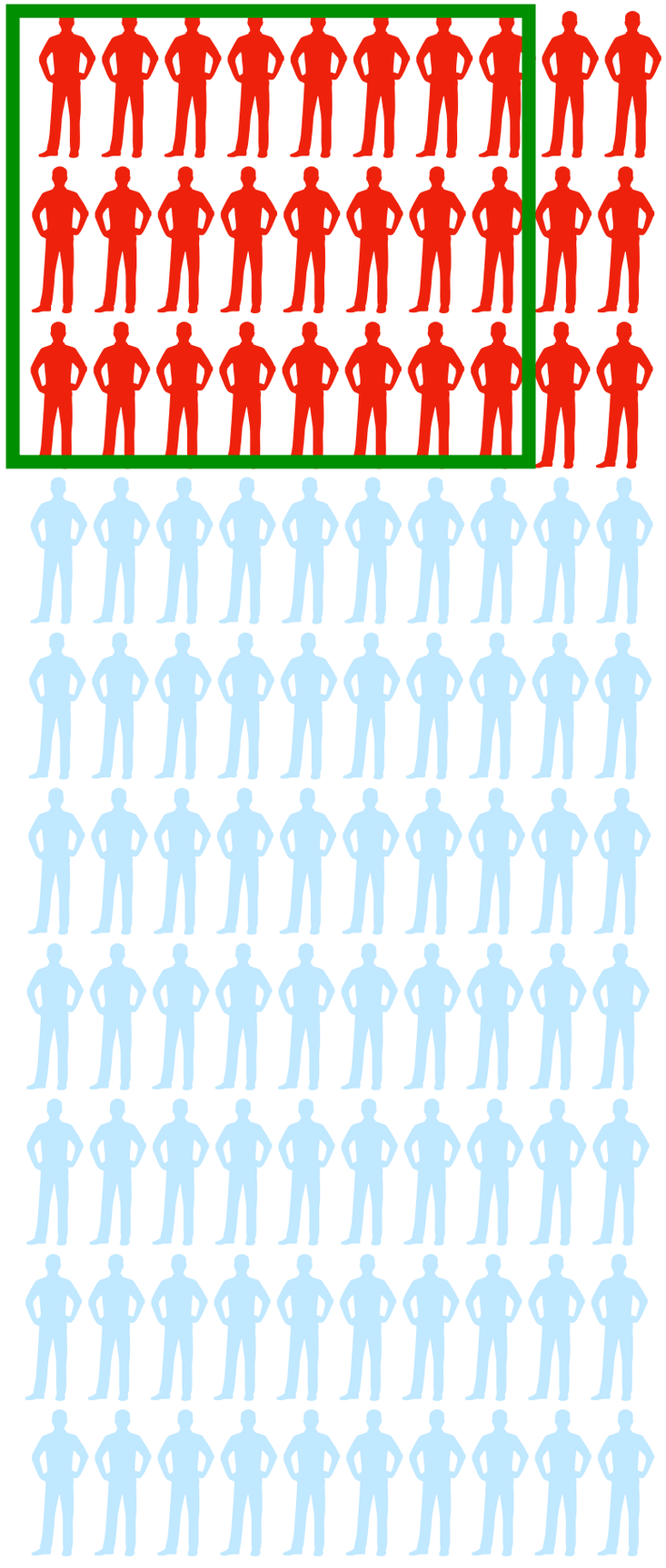


br = 30%

1-br = 70%

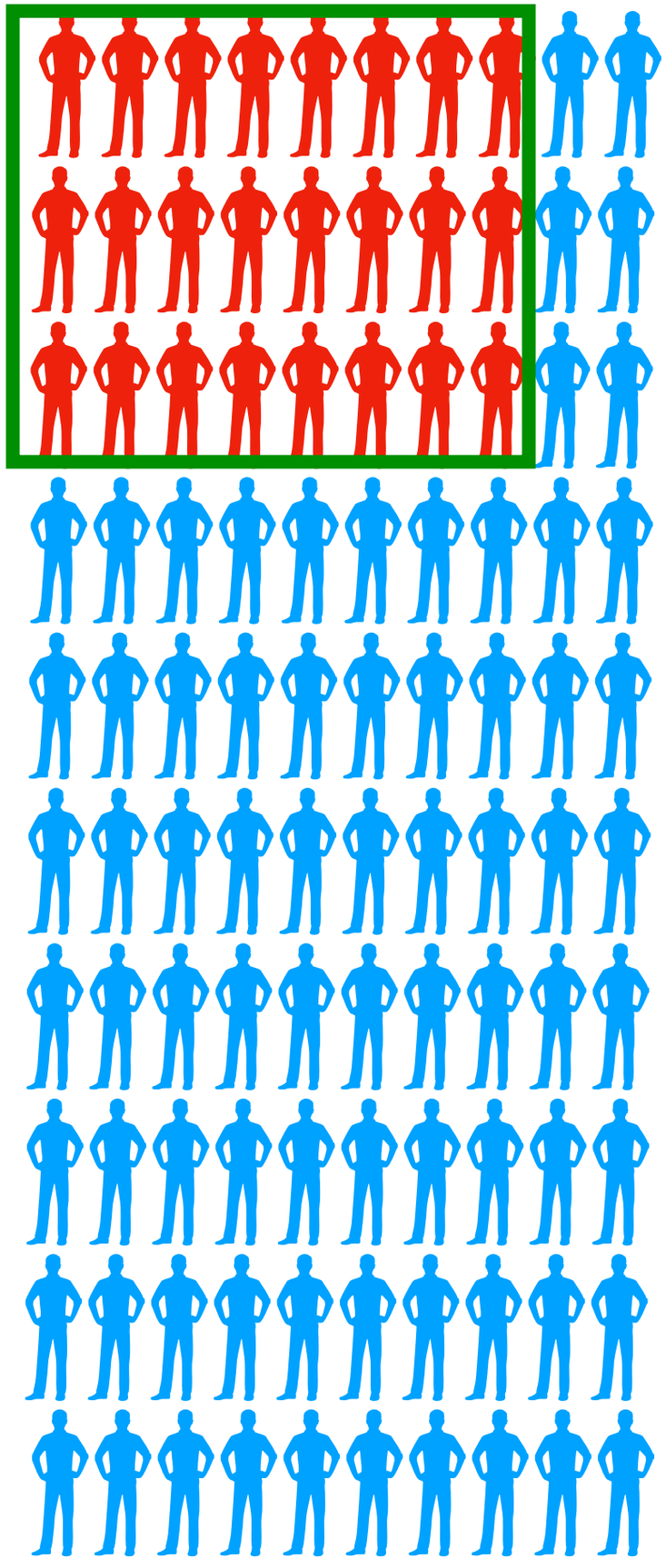


Br = 30%



Br = 30%

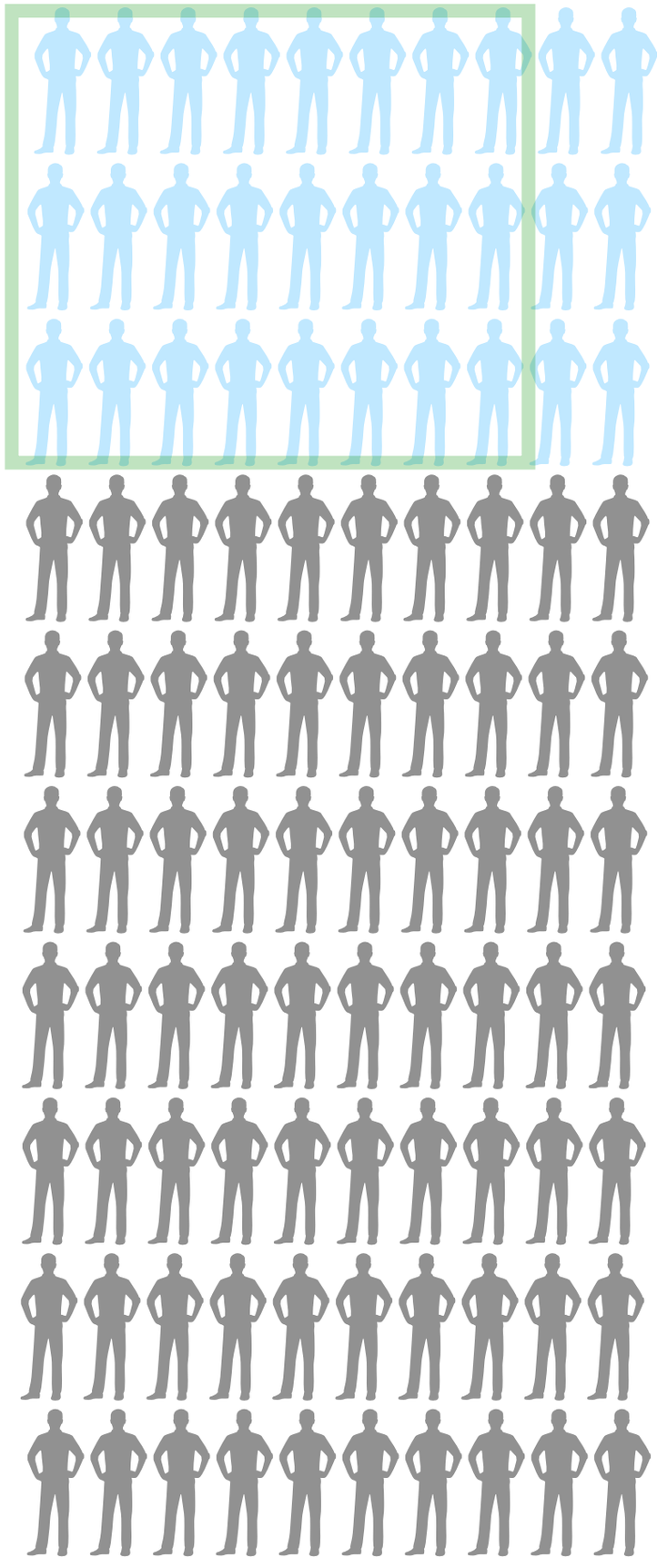
TPR = 80% = 1- FNR



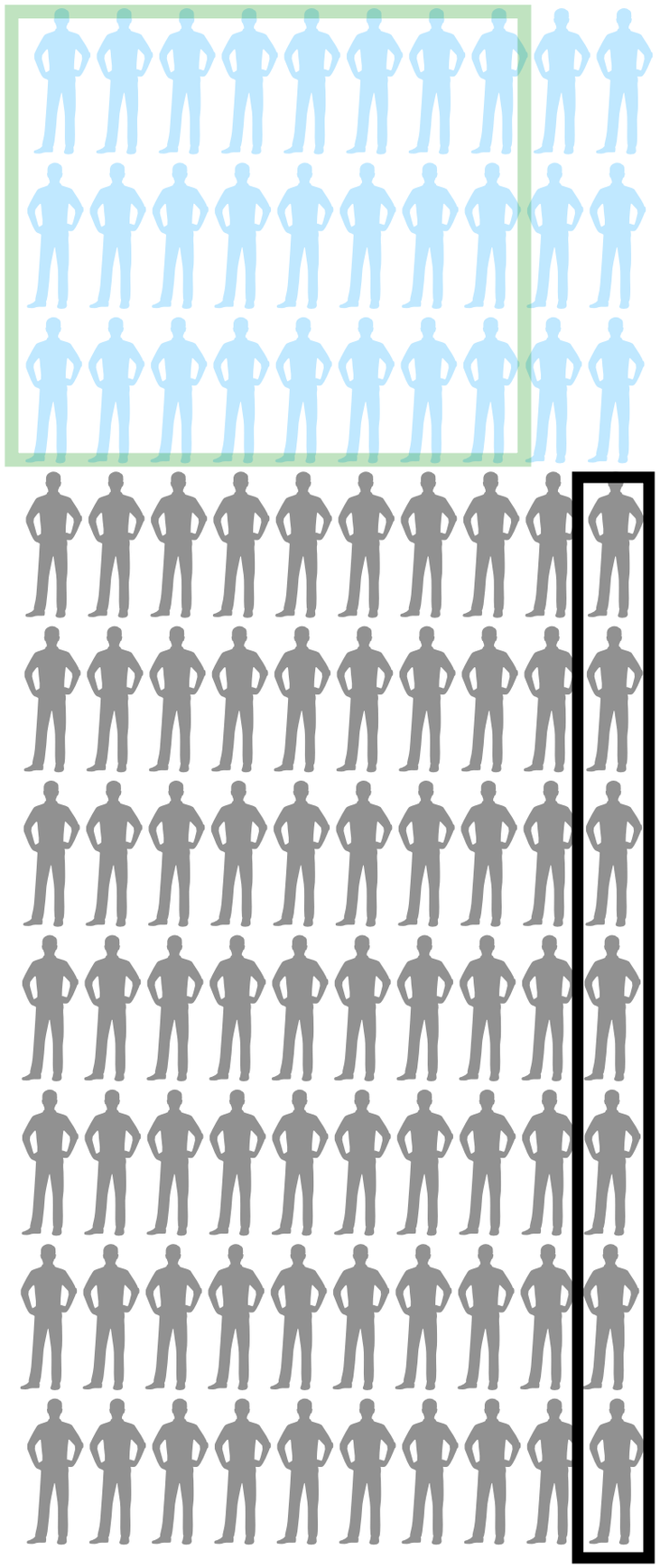
Br = 30%

TPR = 80% = 1- FNR

TP = br x TPR = 30% x 80%= 24%

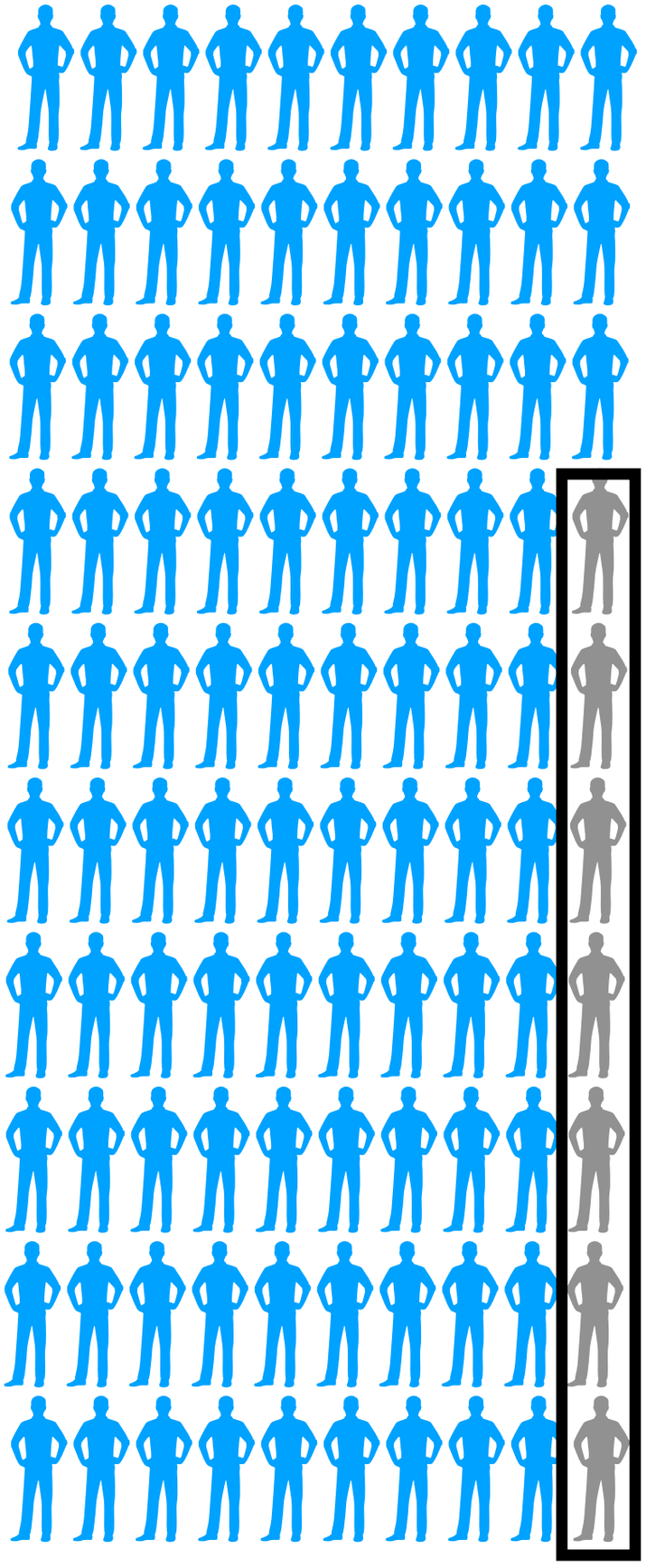


1-br = 70%



1-br = 70%

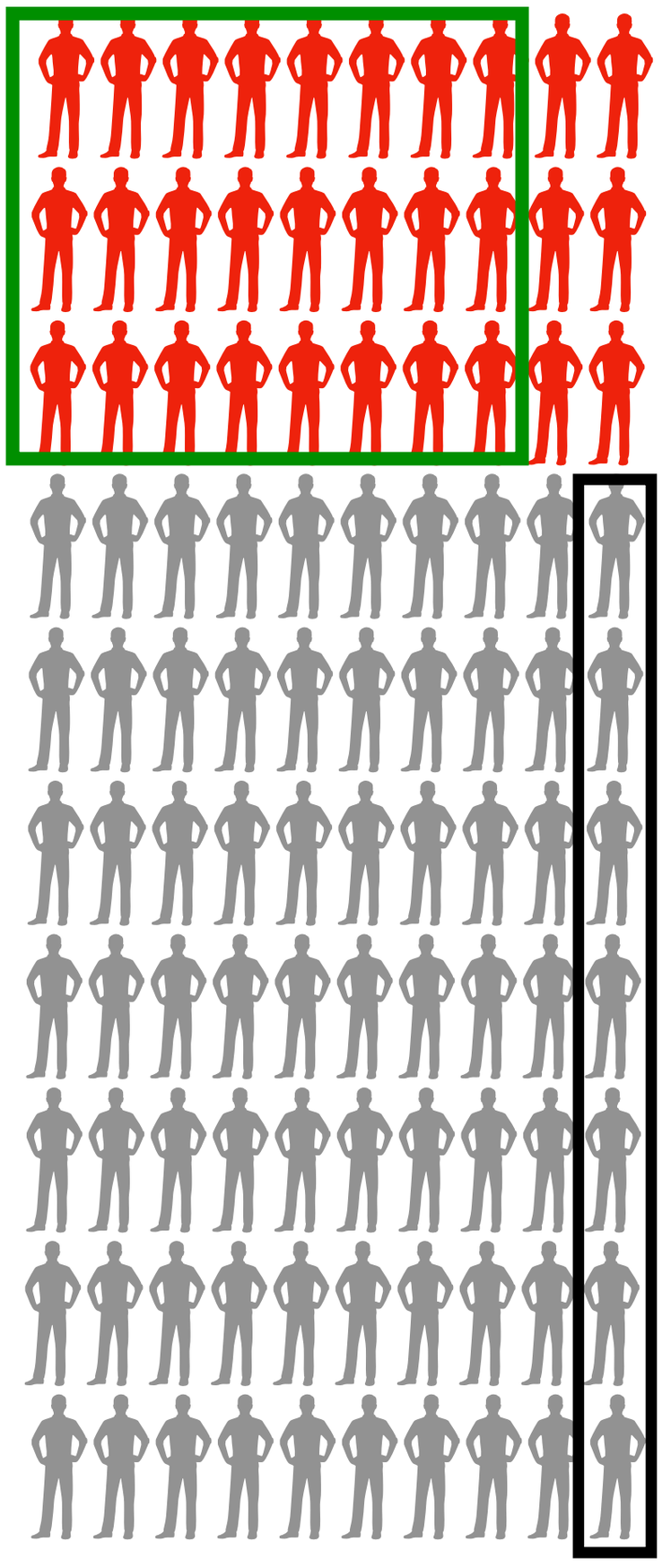
FPR = 10% = 1- TNR



1-br = 70%

FPR = 10% = 1- TNR

FP = (1-br) x FPR = 70% x 10% = 7%



Br = 30%

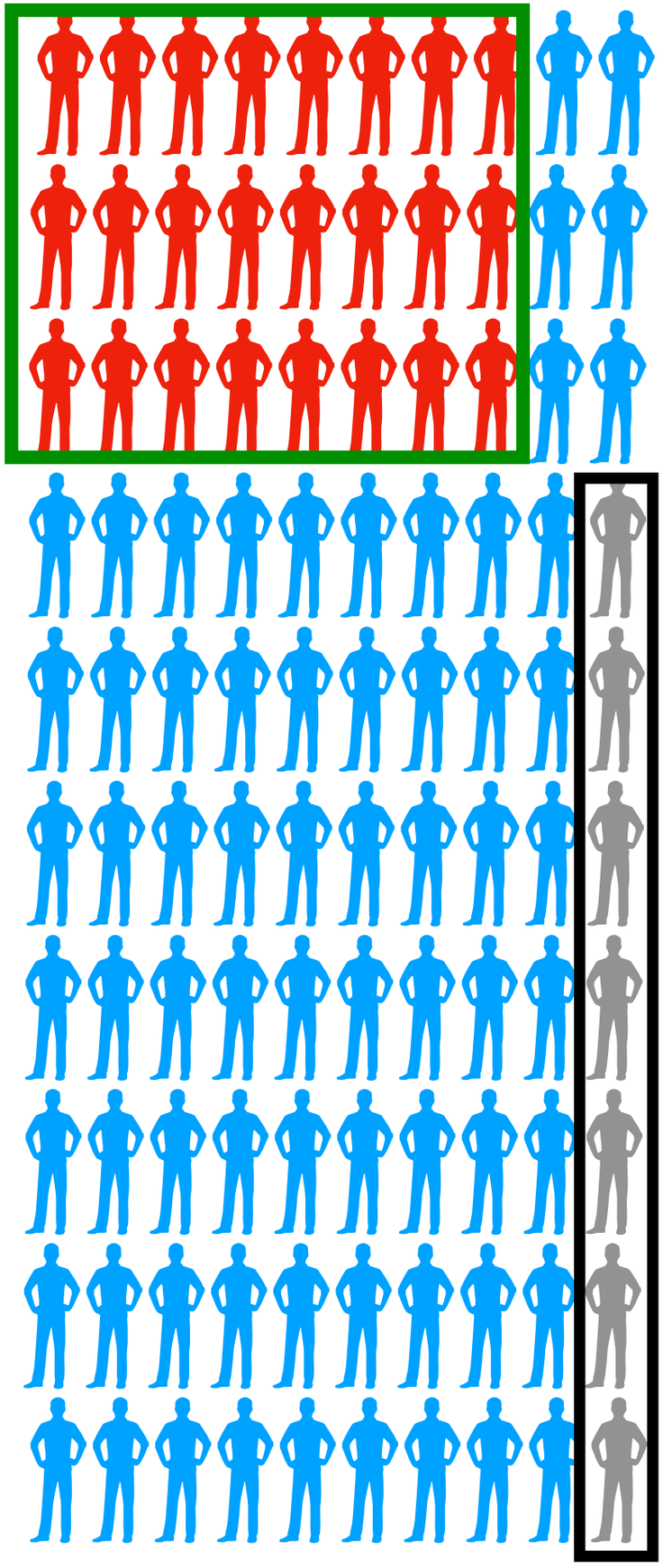
1-br = 70%

TPR = 80% = 1- FNR

TP = br x TPR = 30% x 80%= 24%

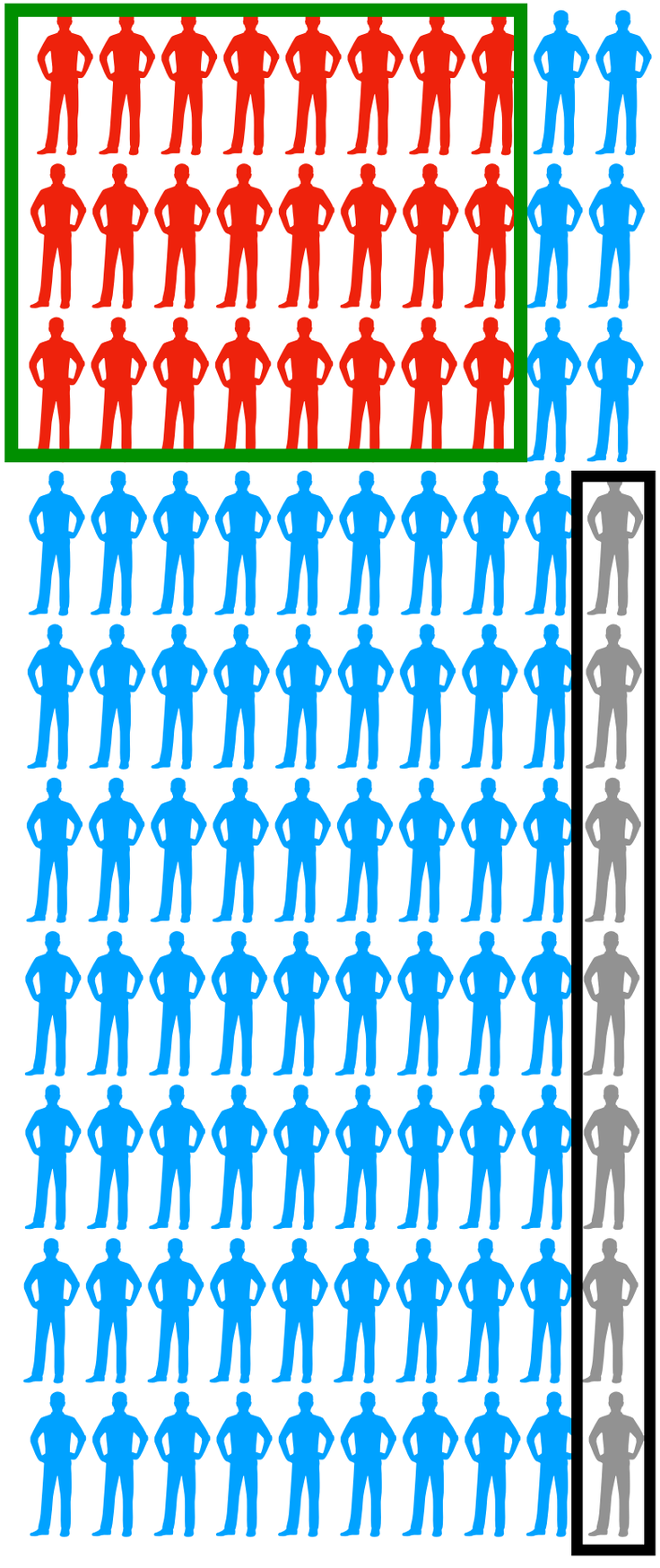
FPR = 10% = 1- TNR

FP = (1-br) x FPR = 70% x 10% = 7%



TP = br x TPR = 24%

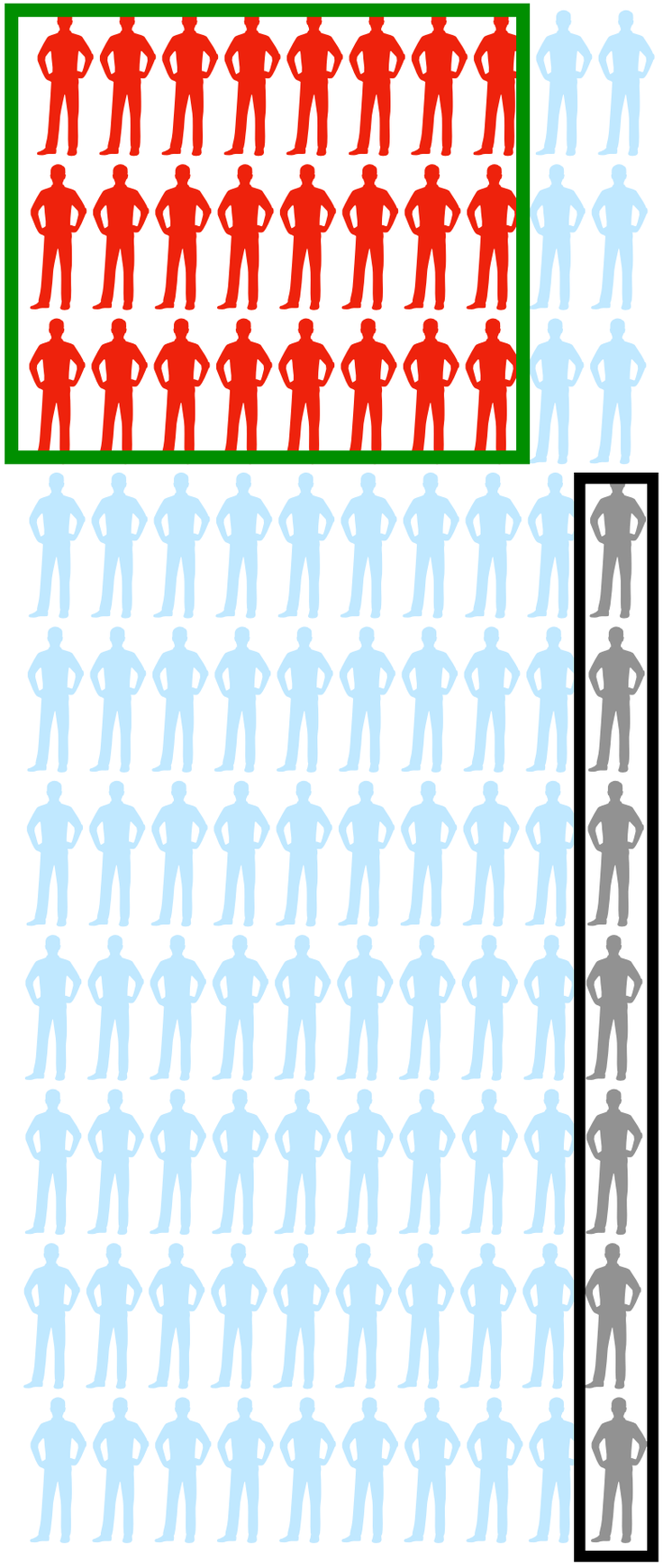
FP = (1-br) x FPR = 7%



TP = br x TPR = 24%

FP = (1-br) x FPR = 7%

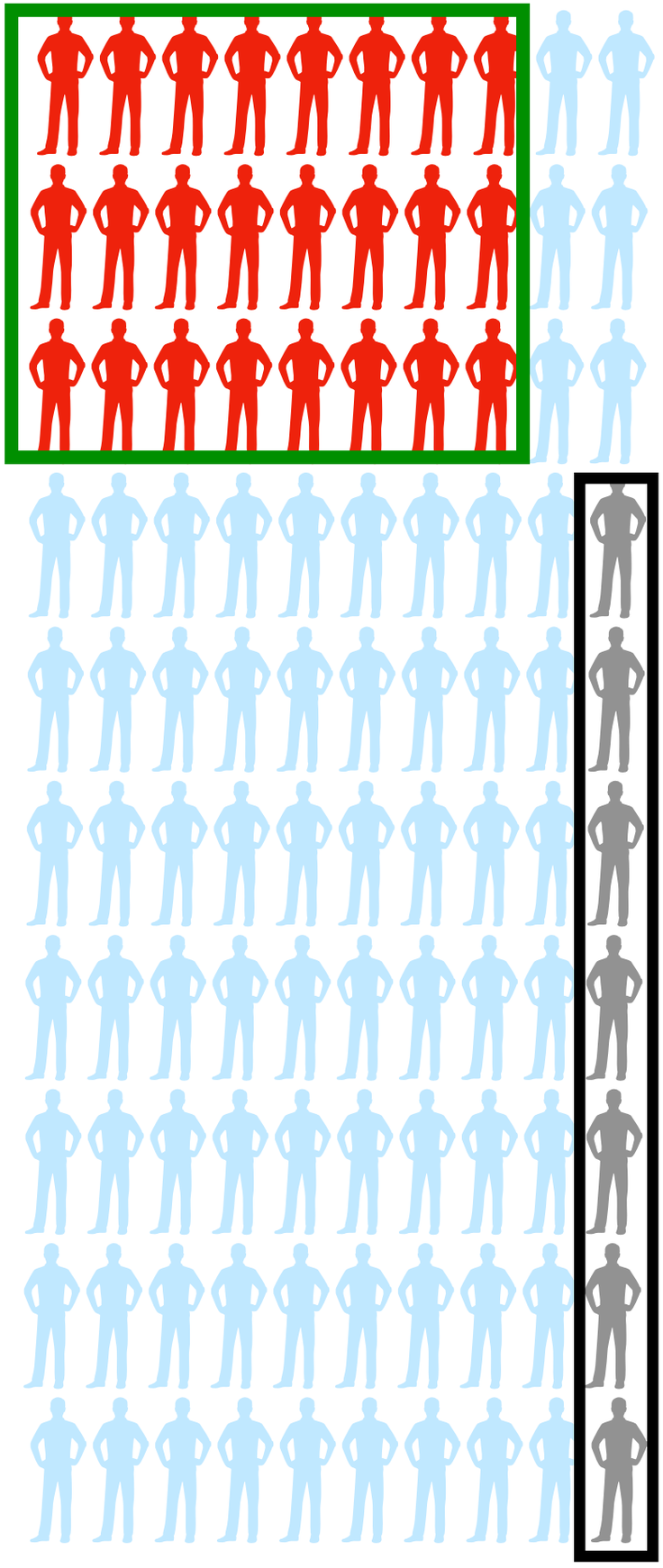
$$PP = TP + FP = 31\,\%$$



TP = br x TPR = 24%

FP = (1-br) x FPR = 7%

$$PP = TP + FP = 31\,\%$$

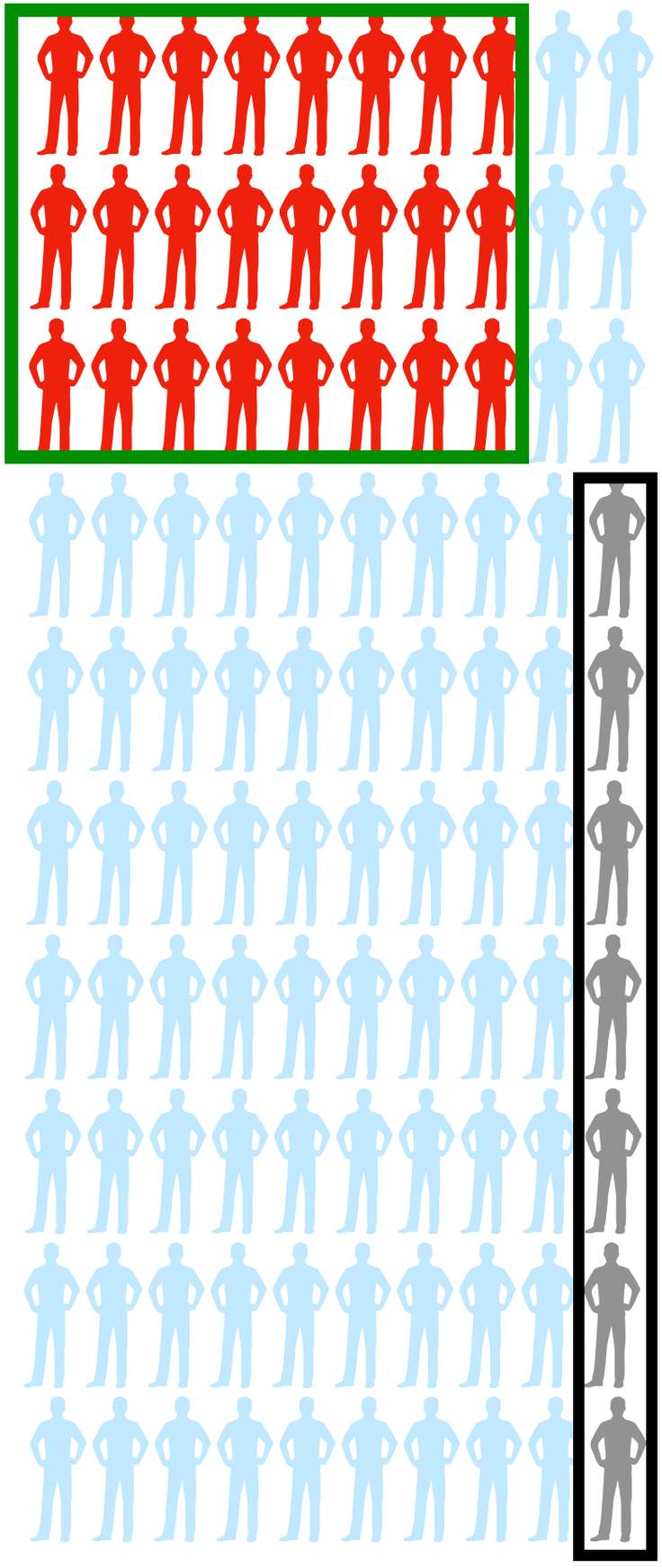


TP = br x TPR = 24%

FP = (1-br) x FPR = 7%

$$PP = TP + FP = 31\,\%$$

$$PPV = \frac{TP}{TP + FP} = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



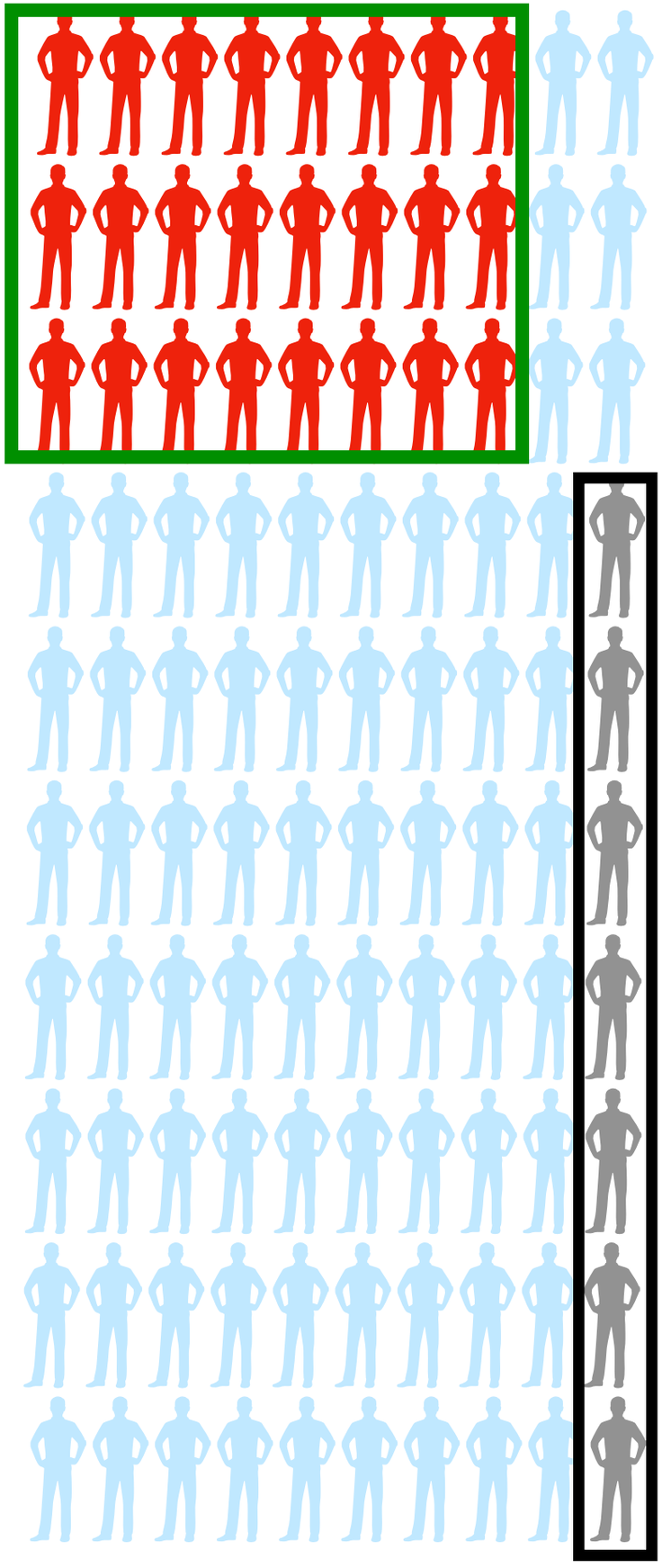
TP = br x TPR = 24%

FP = (1-br) x FPR = 7%

$$PP = TP + FP = 31\,\%$$

$$PPV = \frac{TP}{TP + FP} = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

$$PPV = \frac{br \times TPR}{br \times TPR + (1 - br) \times FPR}$$



TP = br x TPR = 24%

FP = (1-br) x FPR = 7%

$$PP = TP + FP = 31\,\%$$

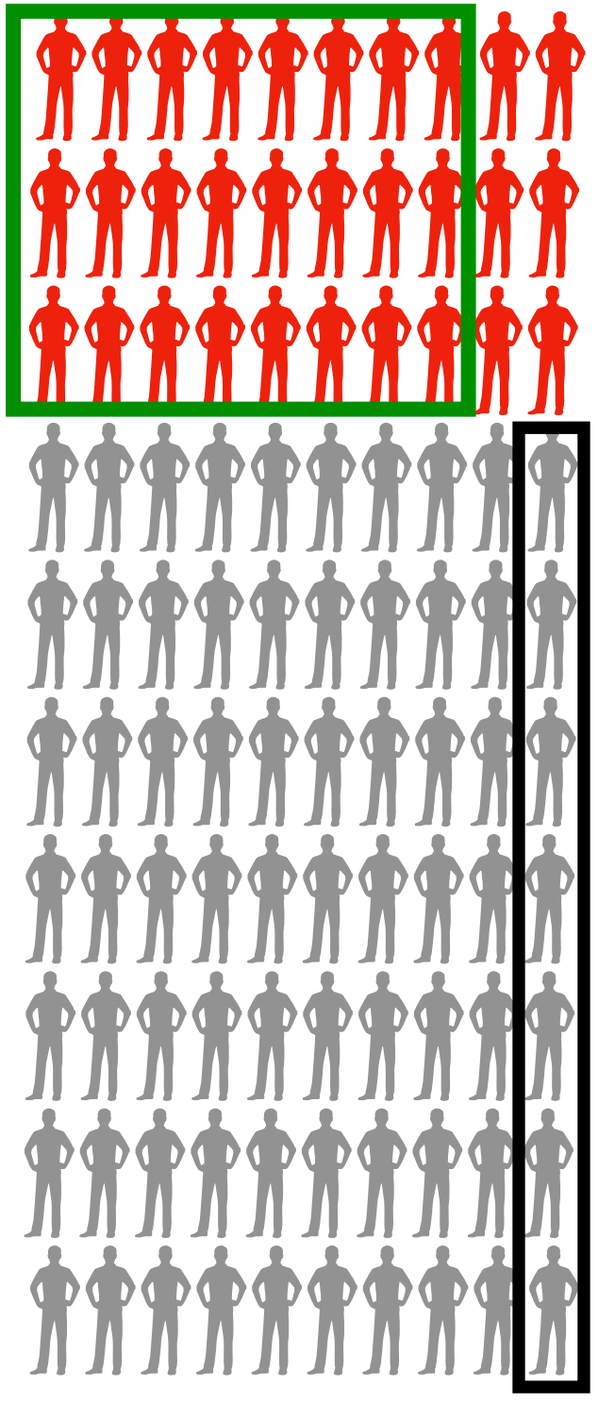
$$PPV = \frac{TP}{TP + FP} = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

$$PPV = \frac{br \times TPR}{br \times TPR + (1 - br) \times FPR}$$

Two Groups

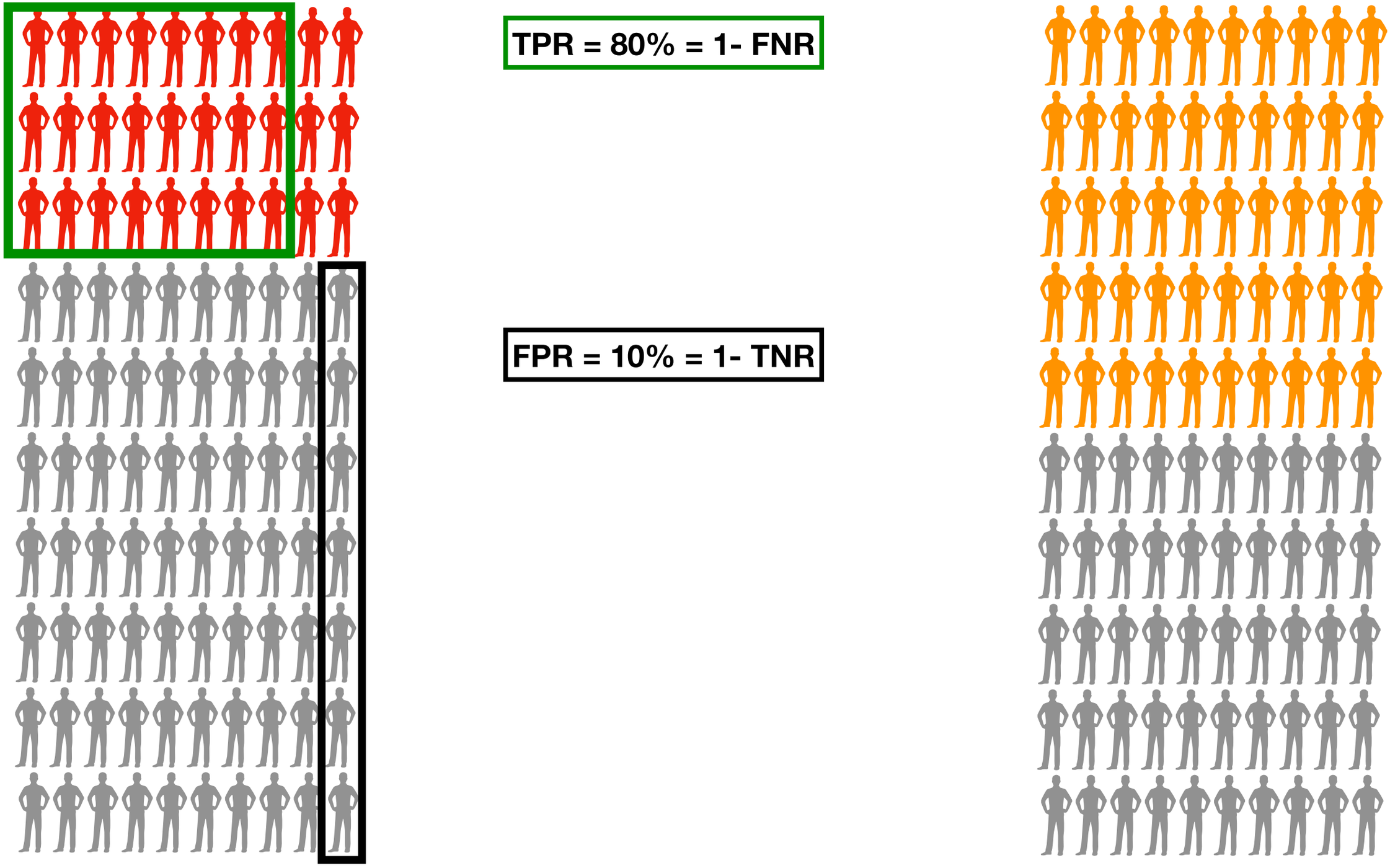
Base rate, FPR, FNR, PPV

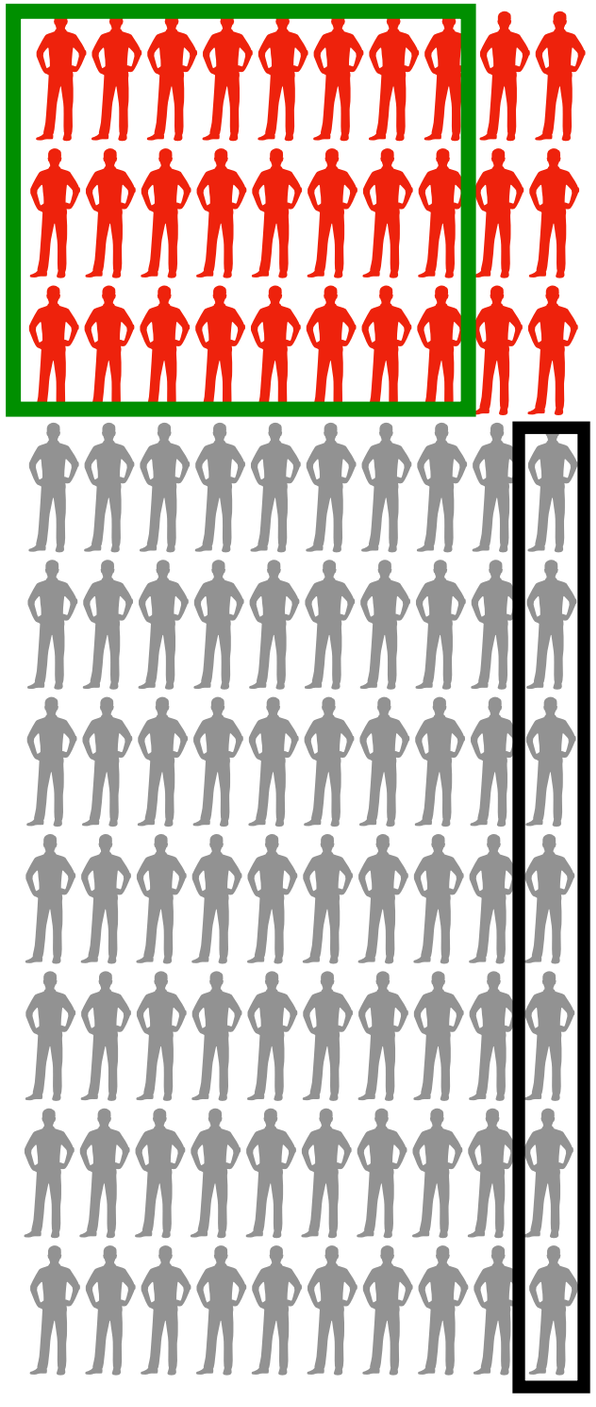
Suppose FPR and FNR Are the Same Across Groups



TPR = 80% = 1- FNR

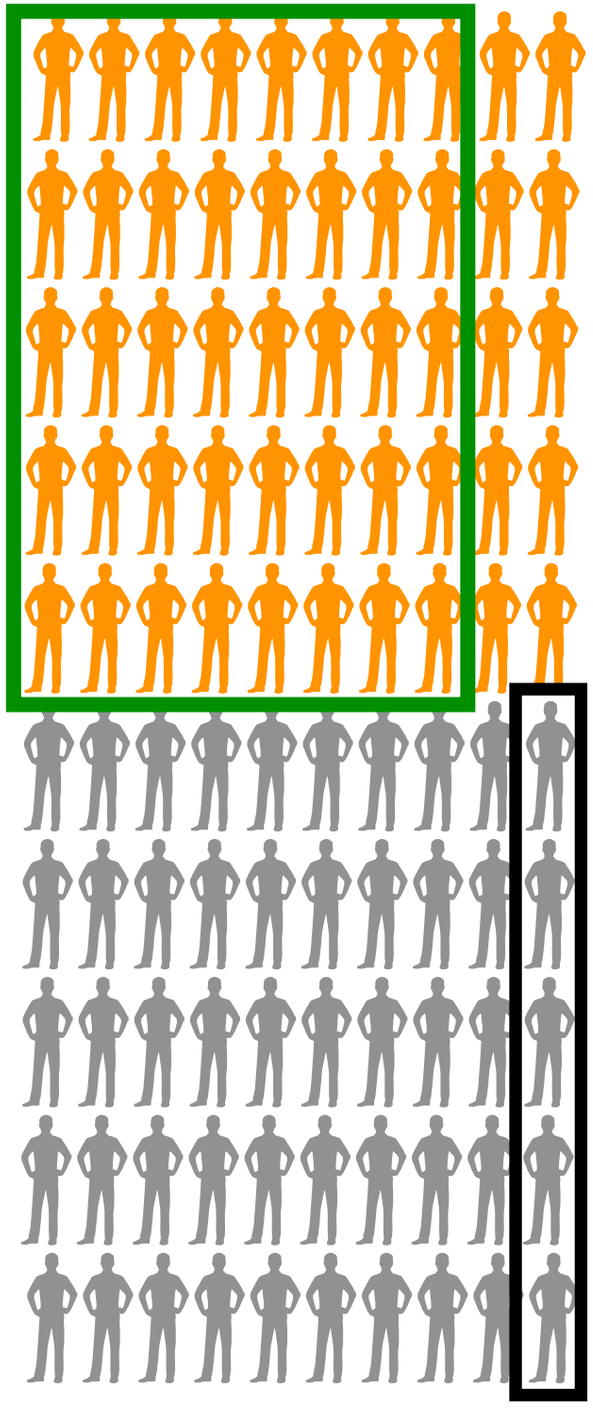
FPR = 10% = 1- TNR

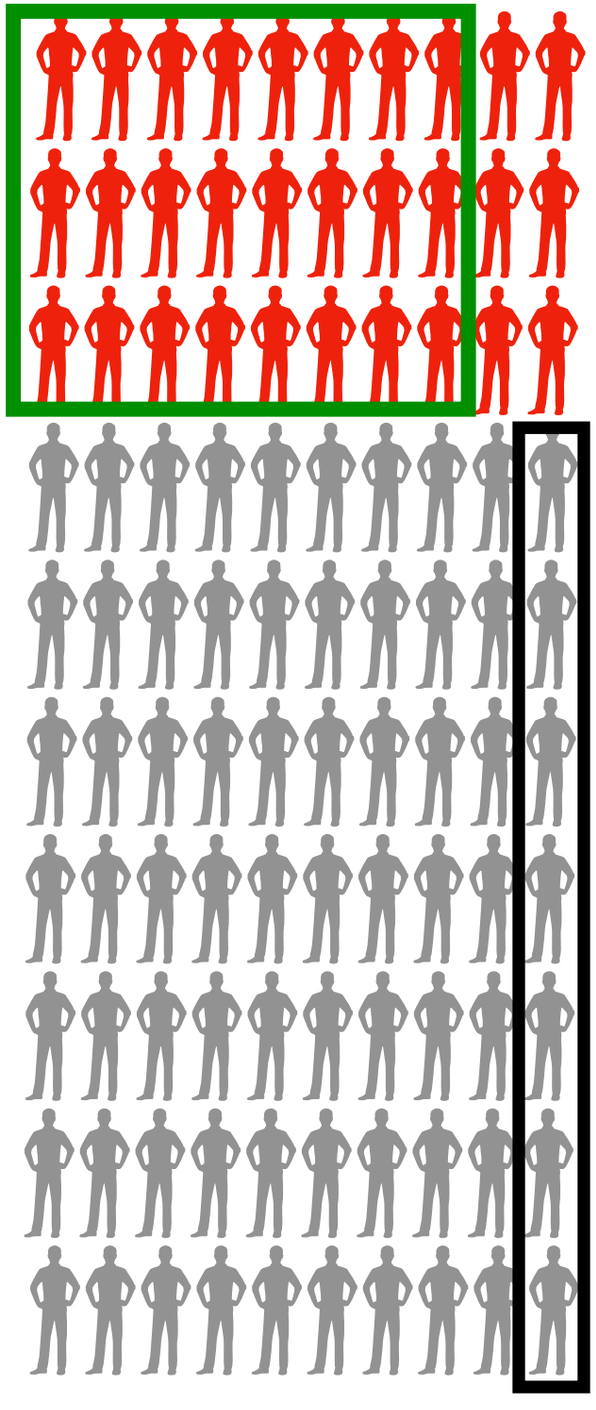

TPR = 80% = 1- FNR
FPR = 10% = 1- TNR



TPR = 80% = 1- FNR

FPR = 10% = 1- TNR

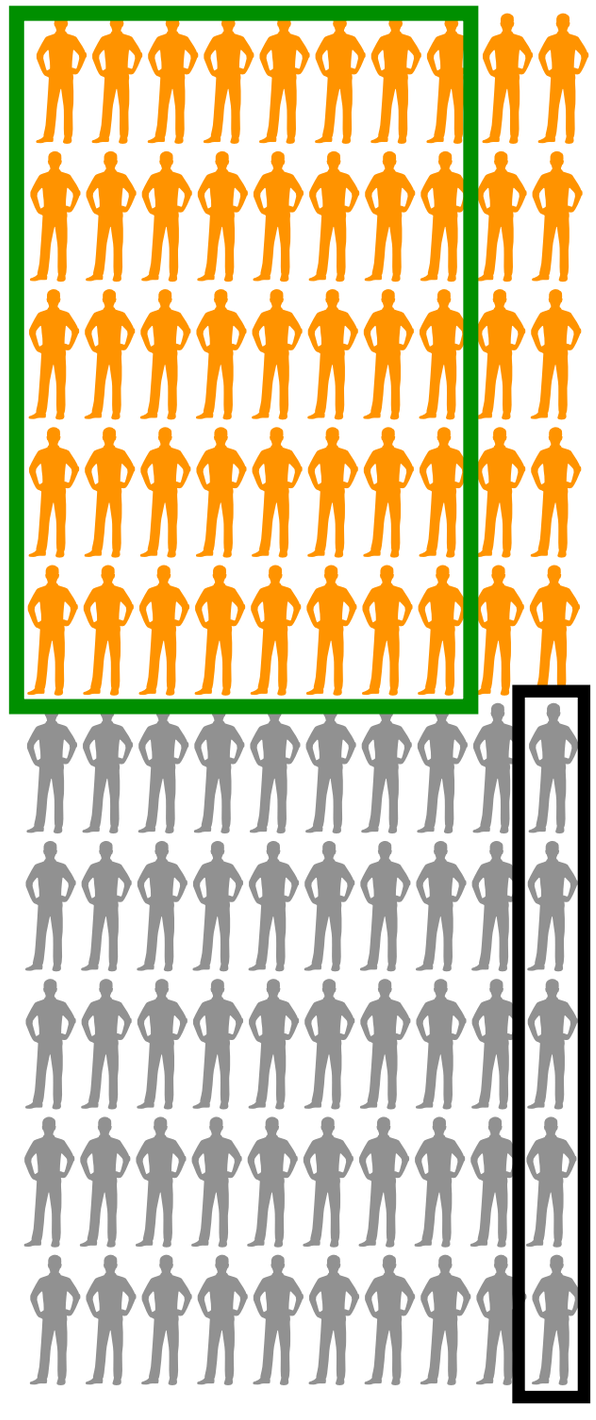


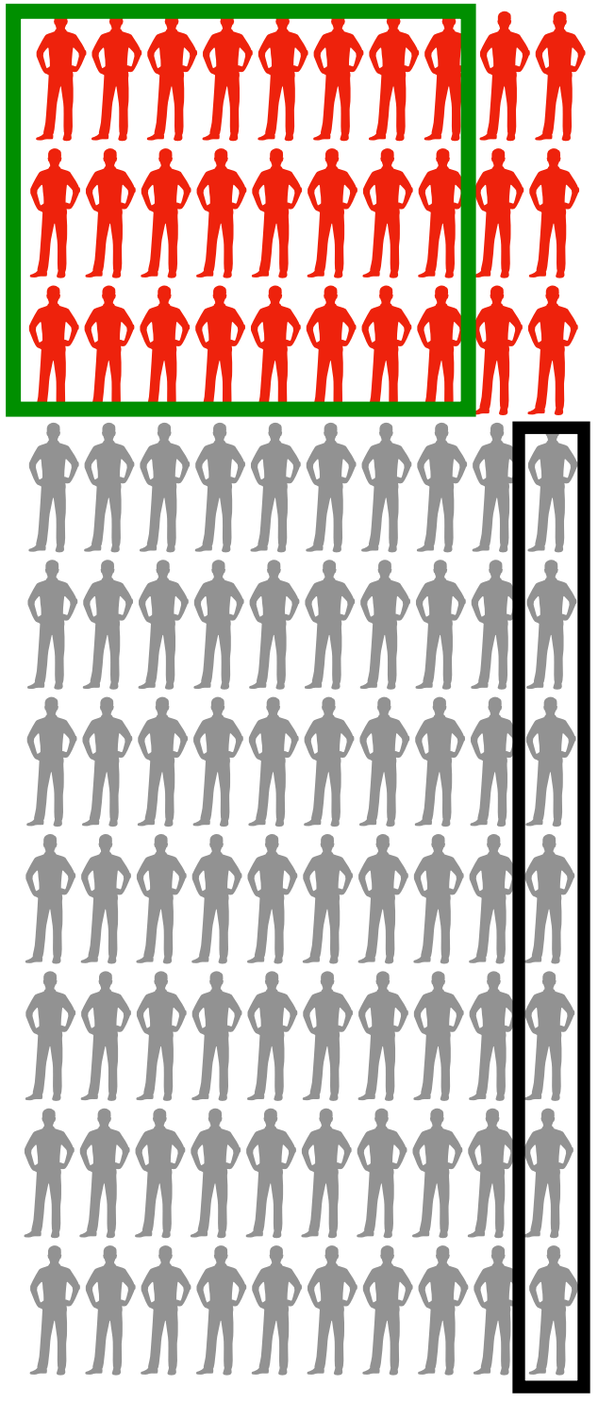


TPR = 80% = 1- FNR

FPR = 10% = 1- TNR

$$PPV = \frac{TP}{TP + FP}$$

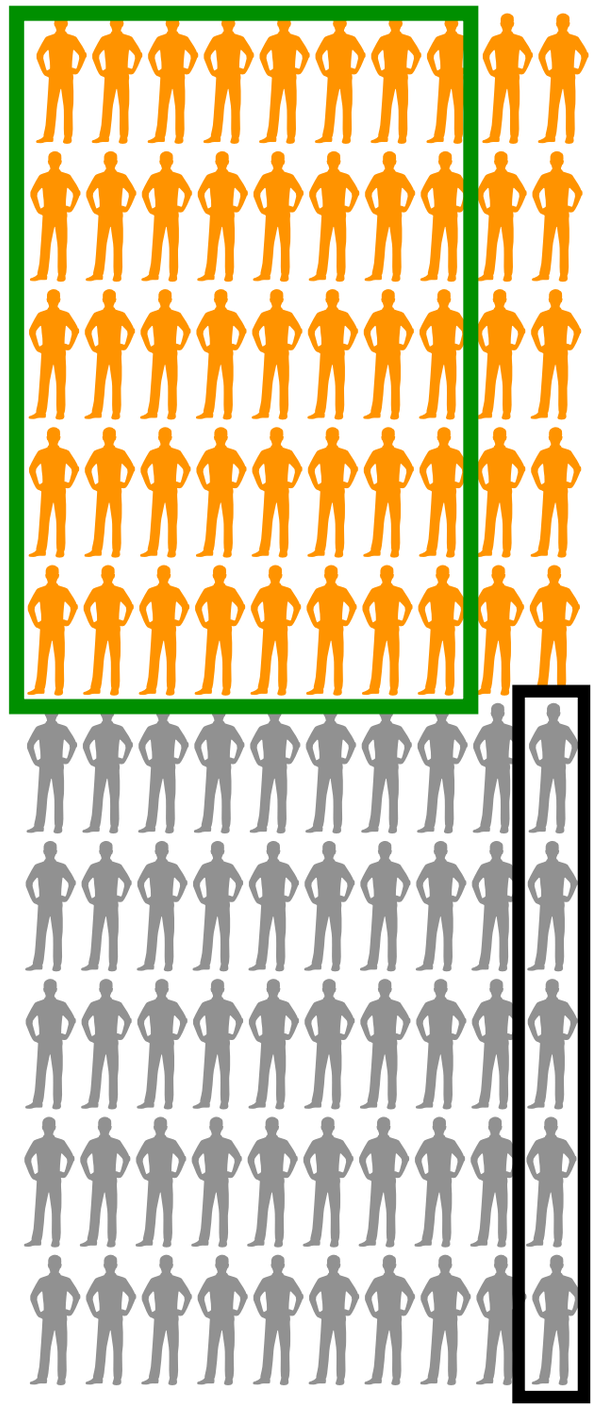




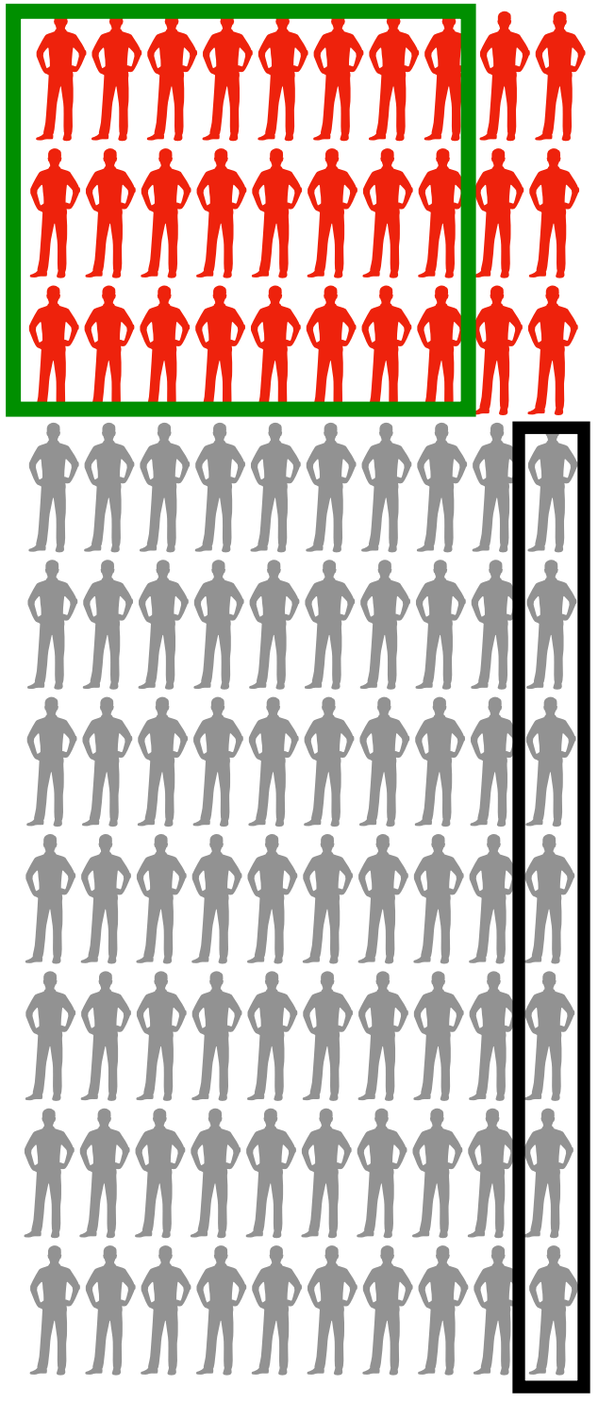
TPR = 80% = 1- FNR

FPR = 10% = 1- TNR

$$PPV = \frac{TP}{TP + FP}$$



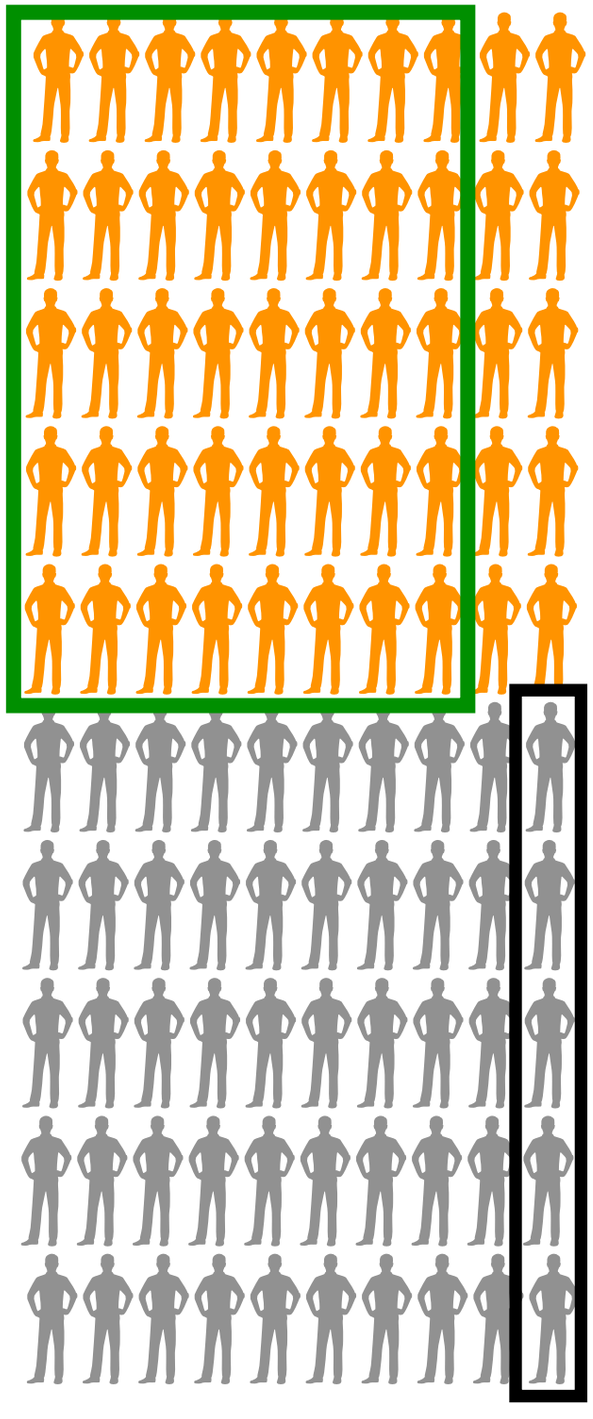
$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



TPR = 80% = 1- FNR

FPR = 10% = 1- TNR

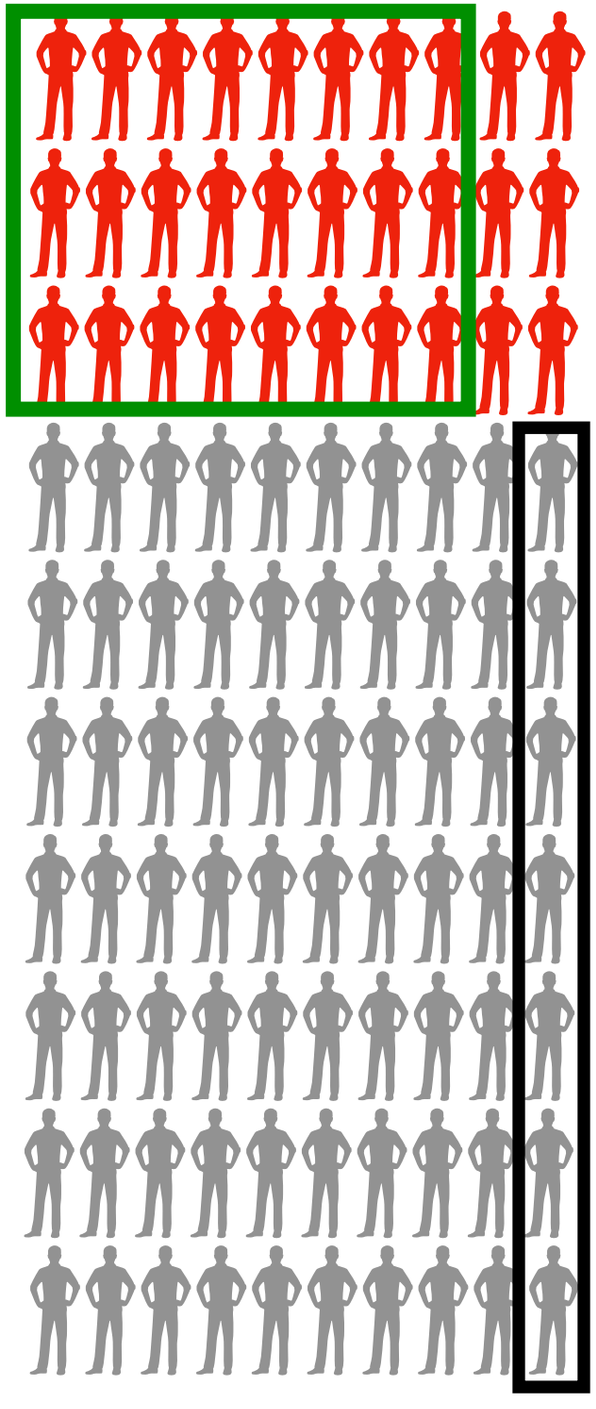
$$PPV = \frac{TP}{TP + FP}$$

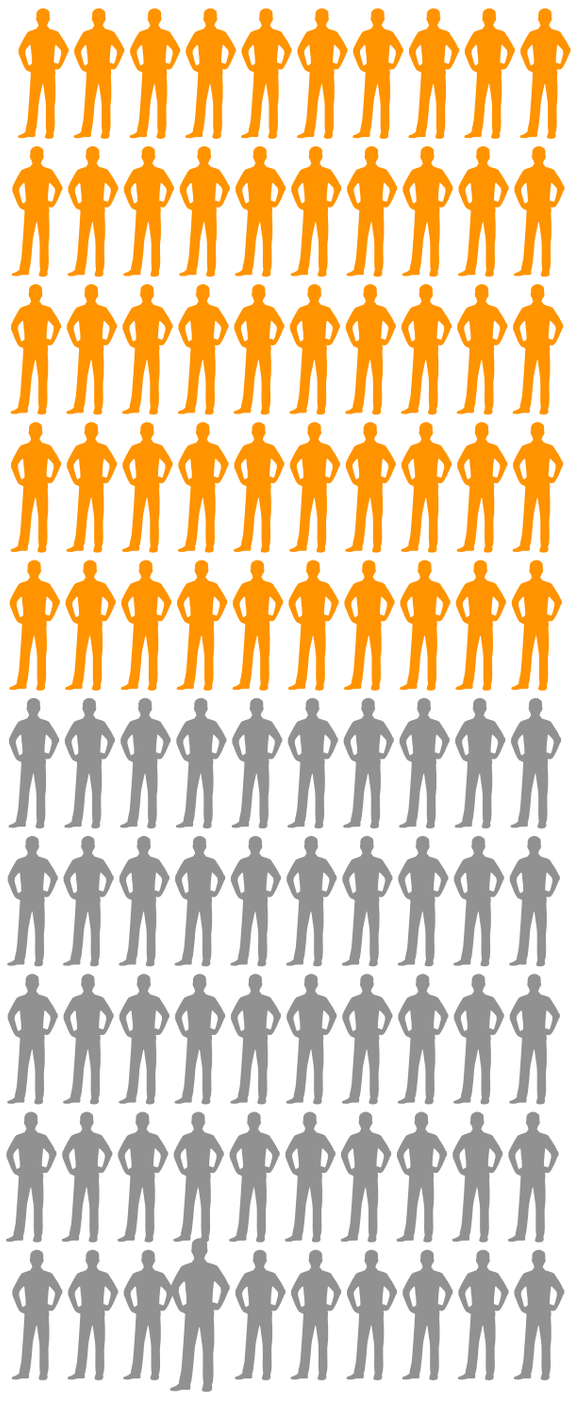


$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

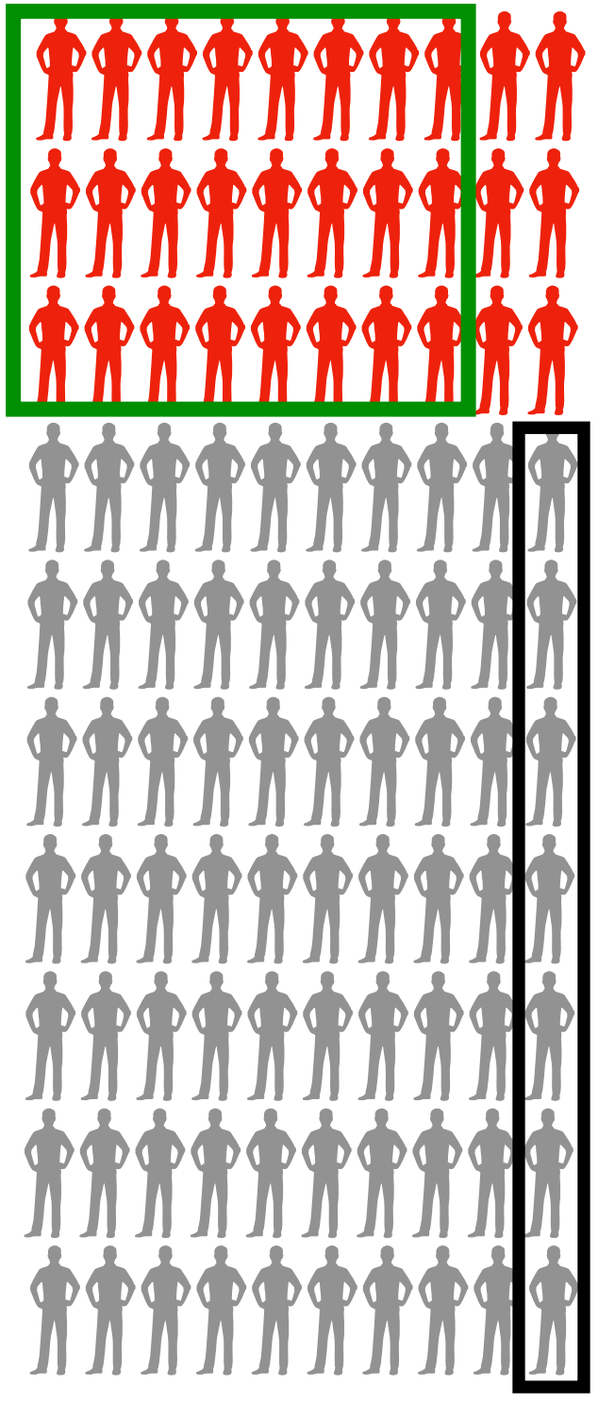
$$PPV_2 = \frac{40\,\%}{40\% + 5\,\%} \approx 88\,\%$$

What if we Hold PPV the Same Across Groups?



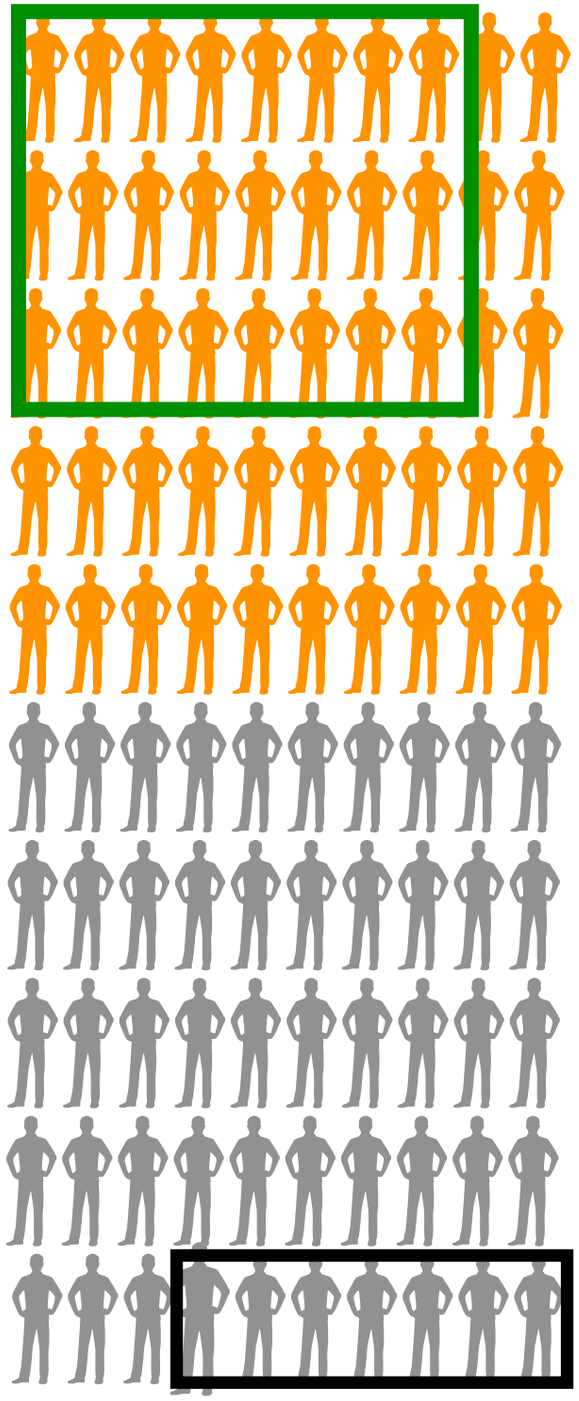


$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

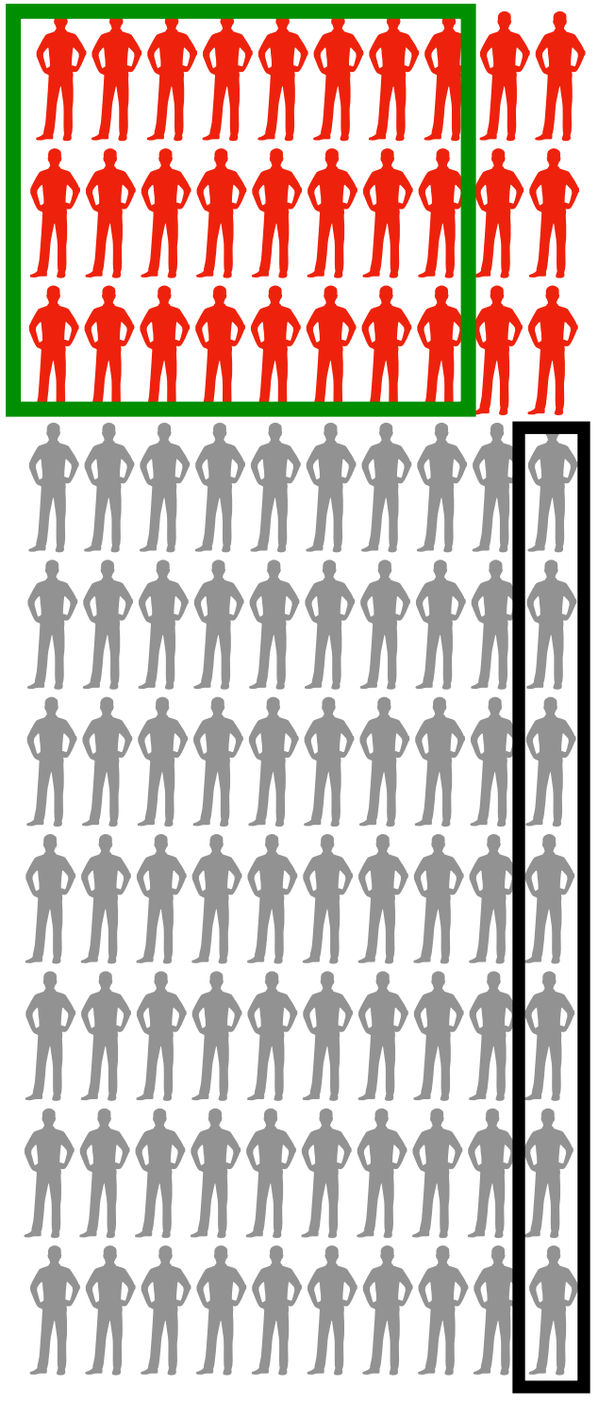


$$PPV = \frac{TP}{TP + FP}$$

$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



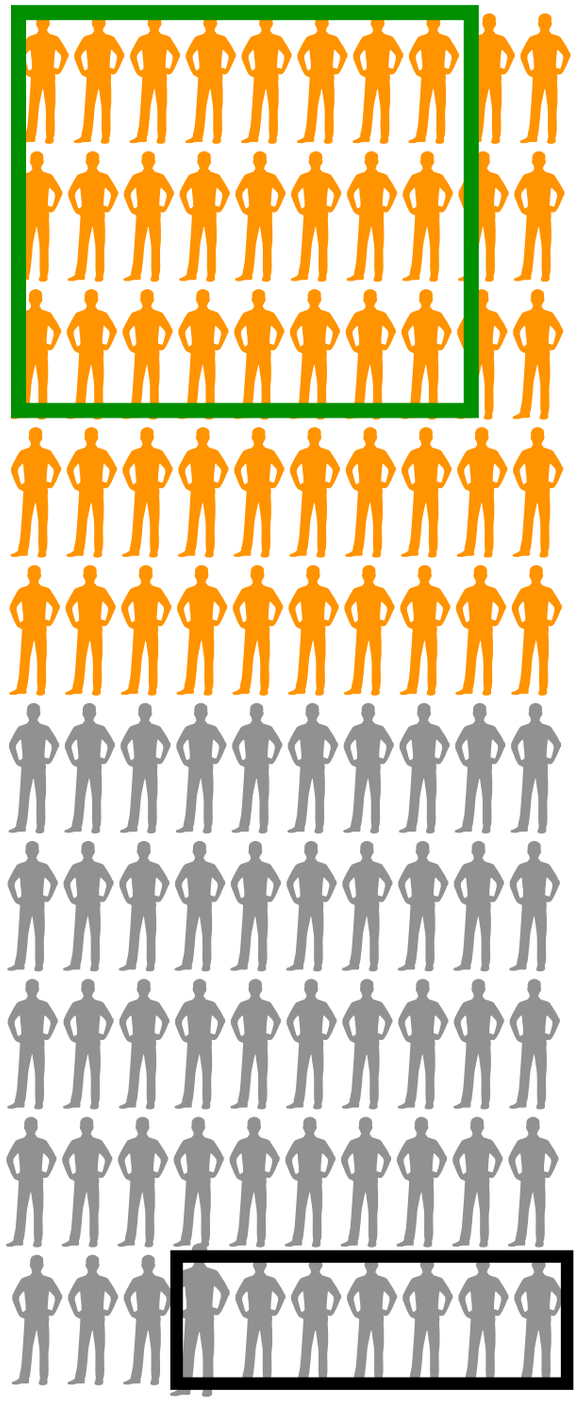
$$PPV_2 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



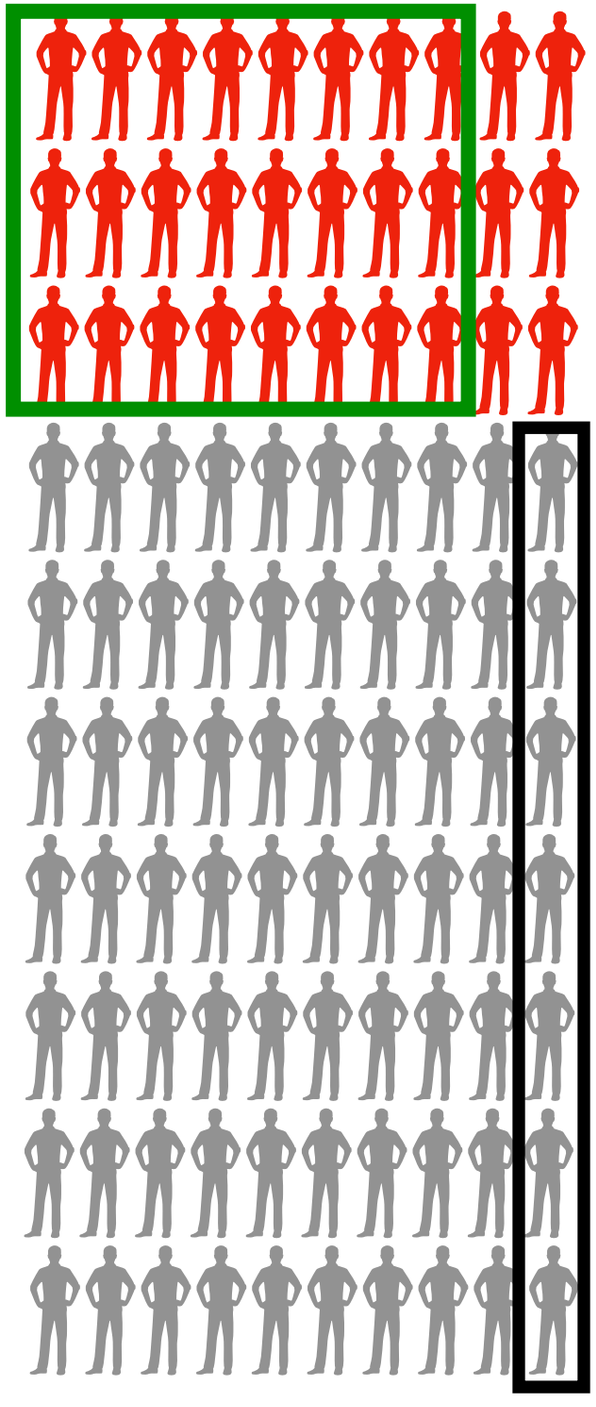
TPR_1 = 80%

FPR_1 = 10%

$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



$$PPV_2 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



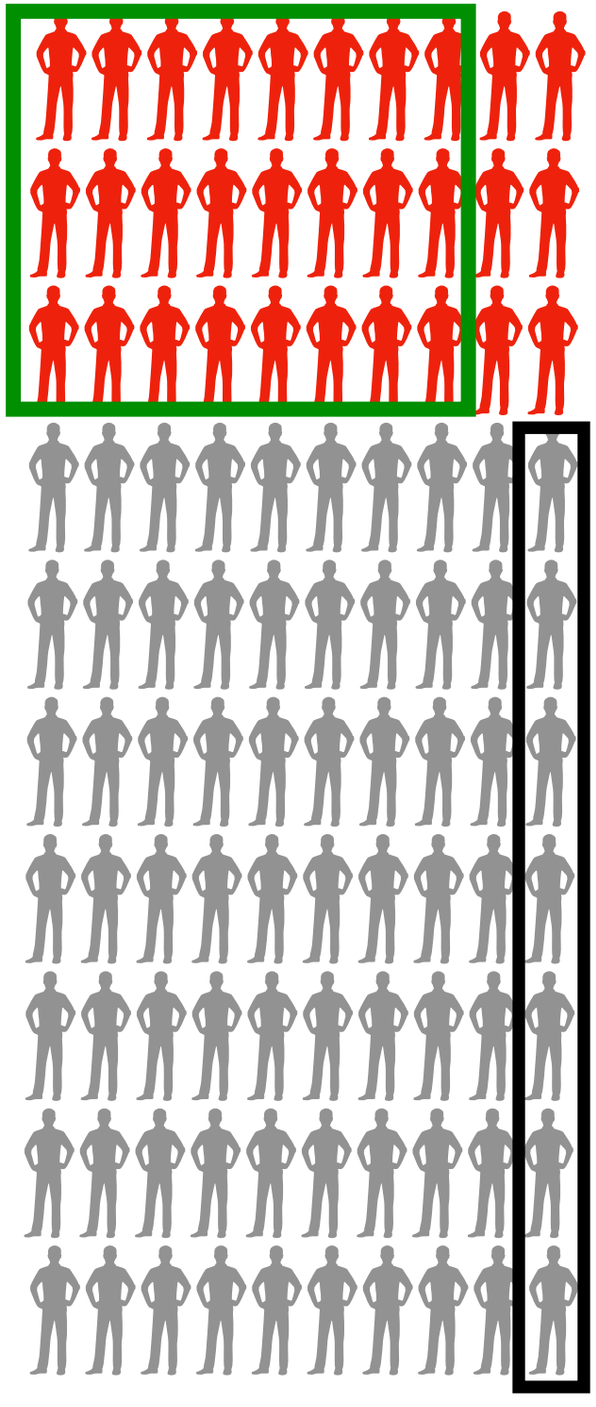
TPR_1 = 80%

FPR_1 = 10%

$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

TPR_2 = 24/50=48%

$$PPV_2 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$



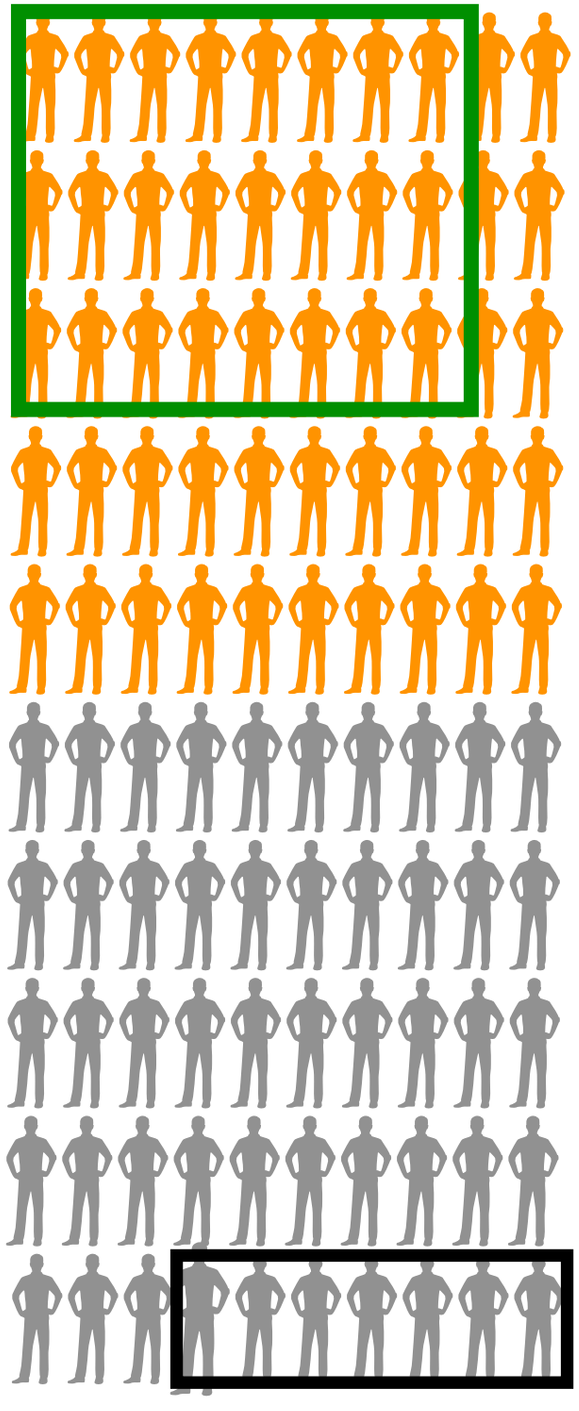
TPR_1 = 80%

FPR_1 = 10%

$$PPV_1 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

TPR_2 = 24/50=48%

FPR_2 = 7/50 =14%



$$PPV_2 = \frac{24\,\%}{24\% + 7\,\%} \approx 77\,\%$$

If base rates are different, it is impossible to have the same PPV and same TPR and FPR across groups