Transworld Identity and Knowledge

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Where We are

Last Week

Modal Logic applied to metaphysics
(actualism vs. possibilism, necessities, *de re/de dicto*, essences)

This Week

Modal Logic applied to Epistemology

Today's Plan

Transworld Identity

Formalizing Knowledge in Epistemic Logic

Transmissibility

Problems and Paradoxes

J. Hintikka (1962), *Knowledge and Belief*, Cornell UP.

Before Getting Started Again

In this course, we are all encouraged to argue a lot, but . . .

> *There are three truths: my truth, your truth, and the truth. My truth, just like your truth, is no more than a fraction of the truth. Our truths are crescent moons situated on one side or another of the perfect circle of the full moon.* **Most of the time, when we argue . . . , our crescent moons turn their backs on one another. The more we argue the further they move apart.** *First we must turn them back towards one another, then our two crescent moons will be face to face, and they will gradually come closer and closer and perhaps in the end meet one another in the great circle of truth*
> *(Tienro Bokar, Master of the Tijaniyya in Mali.)*

Plan

Transworld identity

What is Identity?

Distinction:

- qualitative identity (=satisfying the same properties).
- numerical identity (=being the same).

two things may be qualitatively identical and yet not be numerically identical.
E.g., two electrons are qualitatively identical, but not numerically identical.

The **mystery** surrounding identity:

Saying of two things that they are the same is non-sense. Saying of one thing that is identical to itself is completely trivial. (Witty?)

Principles about Identity

RF $x = x$

LL $x = x \rightarrow [\varphi(x) \rightarrow \varphi(y)]$ (indiscernibility of identicals)

NI $x = y \rightarrow \Box x = y$

ND $x \neq y \rightarrow \Box x \neq y$

Paradox of change

At time t, the dog Oscar has the property being a puppy.

Later, when he grows up, at time t', he lacks that property.

So we have 'POscar' is true at t and '$\neg P$Oscar' is true at t'.

By principle **LL**, Oscar at t and Oscar at t' are not the same.

Lines of response:

1. We only say: Oscar-at-t has P, and Oscar-at-t' lacks P.

   So, we only talk about **temporal parts** of Oscar (things are four-dimensional worms).
2. Using essences? How?

Chrysippus' Paradox

Consider the dog Oscar at time t. Later, at time t', Oscar looses its tail.

Now, consider at t again Oscar but without its tail. Call this object at t Oscar-minus.

Clearly, we have Oscar$\neq$Oscar-minus. By principle **ND**, we have $\Box$Oscar$\neq$Oscar-minus.

By interpreting $\Box$ as a tense operator, we have that Oscar and Oscar-minus are different individual at time t', but they should be the same.

The Problem of Trans-world Identity

Fact 1 One can refer to the *same* individual in *different* possible world. E.g. Socrates sitting or Socrates standing; or Socrates philosopher and Socrates carpenter.

Fact 2 Recall **LL** or the principle of indiscernibility of the identicals:
$x = x \rightarrow [\varphi(x) \rightarrow \varphi(y)]$.

Problem Hence, Socrates sitting and Socrates standing are not the same individuals.

So, what is the *trans-world identity* between Socrates sitting and Socrates standing?

A Related Problem: Identity over Time

Fact 1 One can refer to the *same* individual in *different* instants in time
(what P.F. Strawson, *Individuals* (1959) calls *re-identification*).
E.g. Socrates at t and Socrates at $t + k$.

Fact 2 Individuals change (often radically) their properties over time.

Problem What is the basis for claiming that Socrates at t and Socrates at $t + k$ are the same individual?

Hume and Nietzsche have advanced powerful arguments against identity through time.

A Related Problem: Personal Identity

Formulation 1 Under what circumstances is a person at t identical (numerically identical) with a person at $t + k$?

Formulation 2 Under what circumstances is a person at t identical (numerically identical) with *something* at $t + k$?

Remark: Formulation 1 is the same as formulation 2, provided if someone is a person, it is a person essentially.

Two Views on Trans-world Identity

Lewis
: Each individual exists in only one world.
  This move dissolves the problem.
  Consequence: Each property is essential (why?).

Plantinga
: Essences can solve the problem of trans-world identity.

Lewis – Counterpart Theory

Fact: We make sense of sentences like
Socrates could have been an idiot.

Problem: How do we explain that, if Socrates exists only in one world?

Solution: Socrates exists in only one world, but some other world contain **counterparts** of Socrates
Socrates could have been an idiot is true iff there is a world in which Socrates-counterpart is an idiot.

Lewis – What is a Counterpart?

The counterpart relation C is a **resemblance relation**:

> *The counterpart relation is our substitute for identity between things in different worlds . . . Your counterparts resemble you closely in content and context in important respects. They resemble you more closely than do other things in their worlds. But they are not really you. For each of them is in his own world, and only you are here in the actual world.*

(Lewis (1968), 'Counterpart Theory and Quantified Modal Logic', *Journal of Philosophy*, 65.)

Conditions that the counterpart relation C should satisfy:

- Reflexive (every object is a counterpart to itself);
- Symmetric (if x is s counterpart to y, then y is a counterpart to x);
- Transitive (???).

Objection to Counterpart Theory (1)

Scenario Let α be the actual world. In a possible world w, Socrates-in-w resembles Xenophon-in-α more than Xenophon-in-w does, and viceversa.

Problem How do we account for this scenario in counterpart theory?
Either we cannot, or the truth conditions for modal statements about Socrates and Xenophon will turn out to be upside down.

This scenario is similar to Chisolm's switching role argument.

Objection to Counterpart Theory (2)

Consider:

(1) Socrates could have been different from Socrates (from the person who actually is Socrates).

Problem: In counterpart theory, (1) is true
(each Socrates-counterpart is different from Socrates).
But (1) is intuitively false.

Trans-world Identity and Essences

Consider the collection of all essential properties (=essence) of an individual 'a', and call it property 'E'.

The *different individuals* that in every world satisfy E are the *same individual* across different worlds.

So the problem of trans-world identity is solved by saying that Socrates-in-w_1 is the same as Socrates-in-w_2, provided they both satisfy the property E.

A Problem with Essentialist Approach: Fission

Scenario: In a possible world w, the same essence E is instantiated by two individuals having contradictory (accidental) properties.

Remedy: Introducing the notion of **hecceitas**, i.e., the essence of an individual that uniquely belongs to him.

Problem: The notion of **hecceitas** make sense for people, but what about artifacts or inanimate objects? So, it seems the essentialist approach run into difficulties in the second case.

Fission and Personal Identity

Assumption Psychological continuity is sometimes taken as a criterion for personal identity (e.g., memory, the sense of selfness).

Fission Suppose A's right hemisphere is transplanted in person R and A's left hemisphere is transplanted in person L.

Problem Supposedly A is psychologically continuous with both R and L. So A is (personally) identical with two people (or things). This is counterintuitive.

A Bigger Picture

- Actualism (A) goes with essentialism (E).
- Possibilism (P) goes with counterparts (C).

  (What about the theory of temporal parts, four-dimensionalism?)

Question: Is there a connection between theses A, E, P, C?

Remark: Possibilism and counterparts are expressions of the culture of suspicion in philosophy.
See P. Ricoeur, *Freud and Philosophy* (1970).
Marx, Freud and Nietzsche as masters of suspicion.

Plan

Formalizing Knowledge in Epistemic Logic

Preliminaries: Epistemic and Doxastic Logic

The operator $\Box$ will be written as:

$\mathbf{K}_i$ for knowing; $\mathbf{B}_i$ for believing.

The operator $\diamond$ will be written as:

$\mathbf{P}_i$ for being epistemically (=relative to knowledge) possible

$\mathbf{C}_i$ for being doxastically (=relative to belief) possible

Accessibility relations are relativized to:

Agents: R_i (or R_a, R_b, etc.);

Knowledge or belief: R^k or R^b.

Truth-conditions:

- $w \models \mathbf{K}_i\varphi$ iff for all w', if $wR_i^k w'$, then $w' \models \varphi$;
- $w \models \mathbf{B}_i\varphi$ iff for all w', if $wR_i^b w'$, then $w' \models \varphi$;
- $w \models \mathbf{P}_i\varphi$ iff for some w', $wR_i^k w'$ and $w' \models \varphi$;
- $w \models \mathbf{C}_i\varphi$ iff for some w', $wR_i^b w'$ and $w' \models \varphi$.

What is Knowledge? What is Belief?

Belief: What is held true (by the believing subject).

Knowledge (Plato's definition):

(i) Justified

(ii) true

(iii) belief

Warning: The **act** (of believing or knowing) is distinct from the **content** (believed or known).

Question: A logic of the act or a logic of the content?

Knowledge: Reflexivity of R_i

Valid: $\mathbf{K}_i\varphi \rightarrow \varphi$ (provided R_i is reflexive)
Argument: Knowledge is veridical.
Recall *Plato's definition:* Knowledge is justified **true** belief.

Knowledge: R_i is Transitive but not Symmetric

Valid:
$\mathbf{K}_i\varphi \rightarrow \mathbf{K}_i\mathbf{K}_i\varphi$ (provided R_i is transitive)
Argument: Knowledge cannot be given up.
(*Caveat*: $\mathbf{K}_i \rightarrow \mathbf{K}_j\mathbf{K}_i\varphi$ is obviously not valid, for $i \neq j$)

Not valid:
$\mathbf{P}_i\mathbf{K}_i\varphi \rightarrow \varphi$ (associated with R_i being symmetric)
Argument: Knowledge can increase in unforeseen ways.

If φ is not know, it might be known, or its opposite might be known, i.e., $\mathbf{P}_i\mathbf{K}_i\varphi$ and $\mathbf{P}_i\mathbf{K}_i\neg\varphi$. Accepting $\mathbf{P}_i\mathbf{K}_i\varphi \rightarrow \varphi$ would derive the contradiction $\varphi \wedge \neg\varphi$.

On the **KK** Principle

The argument given for the **KK** principle is that, once something is know, it cannot be retracted. Knowledge rests upon ultimate and unquestionable evidence.

Problem: However, the **KK** principle requires that knowledge meets very high standards, which need not be met in our ordinary understanding of knowledge, or even in scientific knowledge.

And there are further problems with the **KK** principle.

The Surprise Examination and the **KK** Principle

Next semester there will be a surprise exam, says the teacher:

- S1 It cannot be the last day, otherwise it won't be a surprise.
- S2 By the same token, it cannot be the second-to-last day, otherwise . . . .
- . . .
- Sk-1 By the same token, it cannot be the first day, otherwise . . .
- Sk So, there cannot be a surprise exam!

Claim: The above (unacceptable) reasoning is based on the **KK** principle.

S1 is known.

S2 is warranted, provided S1 <u>is known to be known</u>.

. . .

Belief: R_i is only Transitive

Valid:
$\mathbf{B}_i\varphi \rightarrow \mathbf{B}_i\mathbf{B}_i\varphi$ (provided R_i is transitive)
Argument: Beliefs cannot be given up (why not?).

Not valid:
$\mathbf{B}_i\varphi \rightarrow \varphi$ (associated with R_i being reflexive)
Argument:
Clearly, belief is not veridical.

Not valid:
$\mathbf{C}_i\mathbf{B}_i\varphi \rightarrow \varphi$ (associated with R_i being symmetric)
Argument: The same argument used for knowledge.

Knowledge and Belief Combined (1)

What one knows one also believes: $\mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\varphi$.

$\mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\varphi$ is valid, provided $R_i^b \subseteq R_i^k$.
(exercise)

Lemma: $\models \mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\mathbf{K}_i\varphi$.

Proof:

Assume $w \models \mathbf{K}_i\varphi$.

Then, $w \models \mathbf{K}_i\mathbf{K}_i\varphi$.

Then, $w \models \mathbf{B}_i\mathbf{K}_i\varphi$.

Thus, $\models \mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\mathbf{K}_i\varphi$.

Knowledge and Belief Combined (2)

What one beliefs one may not know: $\mathbf{B}_i\varphi \not\to \mathbf{K}_i\varphi$

$\mathbf{B}_i\varphi \to \mathbf{K}_i\varphi$ fails, since we may have $R_i^b \subset R_i^k$.
For one generally believes more than one knows.

Lemma: $\not\models \mathbf{B}_i\varphi \to \mathbf{K}_i\mathbf{B}_i\varphi$.

Counter-example:

Aside: Principle $\mathbf{B}_i\varphi \to \mathbf{K}_i\mathbf{B}_i\varphi$ and Introspection

Introspection: Believing that φ is a metal state. But mental states are transparent to the knowing subject (why?). Hence, I know that I believe that φ.

Thus, principle $\mathbf{B}_i\varphi \to \mathbf{K}_i\mathbf{B}_i\varphi$ should be accepted.

Counter objection: Introspection would support a principle like

$$\mathbf{P}_i\varphi \to \mathbf{K}_i\mathbf{P}_i\varphi,$$

which leads to the absurd (why?) conclusion that

$$\varphi \to \mathbf{K}_i\mathbf{P}_i\varphi$$

(recall $\varphi \to \mathbf{P}_i\varphi$).

How Do we Account for Epistemic Justification?

Plato's definition: Knowledge is justified true belief.

- That knowledge is **true** is taken care of by $\mathbf{B}_i\varphi \rightarrow \varphi$;
- That knowledge is a **belief** is taken care of by $\mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\varphi$;
- That knowledge is **justified** has no formal counterpart.

  What about principle $\mathbf{K}_i\varphi \rightarrow \mathbf{K}_i\mathbf{K}_i$? But it also holds for $\mathbf{B}_i$.

  We need a modal logic of justification
  (e.g., Artemov's provability logic).

Aside: Historical Remark

Knowledge is justified true belief is <u>not</u> Plato's definition of knowledge.

- It is historically wrong to say that Plato based knowledge (episteme) on belief (doxa).
- In Plato's view episteme and doxa are completely disjoint domains.

  Socrates: So, Theaetetus, neither perception, nor true belief, nor the addition of justification to true belief can be knowledge.
  (Plato, Theaetetus, 210a)

Plan

Transmissibility

Knowledge is Transmissible

Claim:
Given two agents a and b, the principle $\mathbf{K}_a\mathbf{K}_b\varphi \rightarrow \mathbf{K}_a\varphi$ is valid.

Proof:

Assume
(1) $w \models \mathbf{K}_a\mathbf{K}_b\varphi$, and
(2) $w \models \neg\mathbf{K}_a\varphi$ (absurd hypothesis).

From (2), $w \models \mathbf{P}_a\neg\varphi$, and so $w' \models \neg\varphi$, for some w' such that $wR_a^k w'$.

From (1), $w' \models \mathbf{K}_a\varphi$ for all w' such that $wR_a^k w'$, and so $w' \models \varphi$.

Contradiction: $w' \models \varphi$ and $w' \models \neg\varphi$.

Beliefs are not Transmissible

Claim:
Given two agents a and b, the principle $\mathbf{B}_a\mathbf{B}_b\varphi \rightarrow \mathbf{B}_a\varphi$ is <u>not</u> valid.

Explanation: The explanation must rely in the fact that the axiom $\mathbf{B}_i\varphi \rightarrow \varphi$ does not hold.

Plan

Problems

- Omniscience Problem
- Moore's Paradox
- Gettier's Problem
- Fitch's Paradox

Knowledge Spreads

Fact:
If $\vdash \mathbf{K}_i\varphi$, then $\vdash \mathbf{K}_i\psi$, provided $\vdash \varphi \rightarrow \psi$.

Proof:

Assume $\vdash \mathbf{K}_i\varphi$ and $\vdash \varphi \rightarrow \psi$.

By the rule of necessitation, we have $\vdash \mathbf{K}_i(\varphi \rightarrow \psi)$.

By distribution of $\mathbf{K}_i$ over implication, we have $\vdash \mathbf{K}_i\varphi \rightarrow \mathbf{K}_i\psi$.

By modus ponens, we have $\vdash \mathbf{K}_i\psi$.

This is called the **omniscience problem**.

Why is This a Problem?

Problem:
Epistemic logic requires that the knowing subject be able to draw **all** the (logical) consequences of what he knows.
Thus, the knowing subject is assumed to be **idealized**.

Solution:
Interpreting $\mathbf{K}_i\varphi$ as "it follows from what i knows that φ"

Skepticism Spreads

Fact:
If $\vdash \neg \mathbf{K}_i \psi$, then $\vdash \neg \mathbf{K}_i \varphi$, provided $\vdash \varphi \to \psi$.

Proof:

Assume $\vdash \neg \mathbf{K}_i \psi$ and $\vdash \varphi \to \psi$.

By the rule of necessitation, we have $\vdash \mathbf{K}_i(\varphi \to \psi)$.

By distribution of $\mathbf{K}_i$ over implication, we have $\vdash \mathbf{K}_i \varphi \to \mathbf{K}_i \psi$.

By contraposition, we have $\vdash \neg \mathbf{K}_i \psi \to \neg \mathbf{K}_i \varphi$.

By modus ponens, we have $\vdash \neg \mathbf{K}_i \varphi$.

This is the **reverse** of the **omniscience problem**.

Skepticism Spreads – Example

Premise 1:	I don't know I am not brain in a vat.
Premise 2:	If I have hands, then I am not a brain a vat.
Conclusion:	I don't know I have hands.

Plan

Some paradoxes

Moore's Paradox

Moore's sentence:

p but I do not believe that p

$p \wedge \neg \mathbf{B}_i p$

Moore's sentence is *not* logically inconsistent (why?), yet it is problematic (why?).

Hintikka's Explanation

Claim: $\mathbf{B}_i(p \wedge \neg \mathbf{B}_i p)$ is logically inconsistent or unsatisfiable.

Proof:

Suppose for contradiction that $w \models \mathbf{B}_i(p \wedge \neg \mathbf{B}_i p)$.

Then, $w \models \mathbf{B}_i \mathbf{B}_i p$ (why?) and $w \models \mathbf{B}_i \neg \mathbf{B}_i p$.

Then, $w' \models \mathbf{B}_i p$ and $w' \models \neg \mathbf{B}_i p$ for any $w' \in W$ such that $w R_i^b w'$.

Then, $w' \models \mathbf{B}_i p$ and $w' \models \mathbf{C}_i \neg p$.

Then, $w'' \models p$ and $w'' \models \neg p$, for $w'' \in W$ such that $w' R_i^b w''$.

Contradiction!

Upshot: Moore's sentence is **doxastically inconsistent**, but not logically inconsistent.

Compare Different Moore's Sentences

1. I believe this: That p is the case and that I do not believe that p (inconsistent).
2. a believes this: That p is the case and that a does not believe that p (inconsistent).
3. a believes this: That p is the case and b does not believe that p (consistent).

The gist of Hintikka's explanation is that $\mathbf{B}_i(p \wedge \neg \mathbf{B}_j p)$ is inconsistent only if $i = j$.

Moore's Explanation (and Block's)

'*p* but I don't believe that *p*' is odd whenever it is asserted.

Asserting a sentence presupposes believing that sentence (at least asserting it honestly)

The Two Accounts Compared

Hintikka If 'p but I don't believe that p' is **believed**, then it is inconsistent

Moore If 'p but I don't believe that p' is **asserted**, then it is inconsistent

Hintikka's explanation is less demanding than Moore's (why?).

Objection to Moore's explanation:
The sentence 'p but I cannot believe that p' is not odd.

Upshot: There are assertions whose content need not be believed by the speaker.

Epistemic Variant of Moore's Sentence

p but I don't know that p

$$p \wedge \neg \mathbf{K}_i p$$

Under Hintikka's account:

$\mathbf{K}_i(p \wedge \neg \mathbf{K}_i p)$ is inconsistent (exercise).

$\mathbf{B}_i(p \wedge \neg \mathbf{K}_i p)$ is consistent (exercise).

Under Moore's account:

$\mathbf{B}_i(p \wedge \neg \mathbf{K}_i p)$ is inconsistent (why?).

Gettier's Problem

Contra Knowledge as justified true belief.

Example 1 Suppose one is justified in holding φ true.

Thus, one is justified in holding $\varphi \vee \psi$ true.

(*assumption:* derivation rule preserves justification).

By chance, φ is fale, but ψ is true.

So, $\varphi \vee \psi$ is a justified true belief, but ...

Ficht's Knowability Paradox

P1: $\forall\varphi(\varphi \to \Diamond \mathbf{K}\varphi)$

P2: $p \wedge \neg \mathbf{K}p$

Put $\varphi := p \wedge \neg \mathbf{K}p$

Thus, $(p \wedge \neg \mathbf{K}p) \to \Diamond \mathbf{K}(p \wedge \neg \mathbf{K}p)$.

Thus, $\Diamond \mathbf{K}(p \wedge \neg \mathbf{K}p)$.

However

Assume $\mathbf{K}(p \wedge \neg \mathbf{K}p)$ for contradiction.

$\mathbf{K}p \wedge \mathbf{K}\neg \mathbf{K}p$, by distributivity of $\mathbf{K}$.

$\mathbf{K}p \wedge \neg \mathbf{K}p$, by veridicality of $\mathbf{K}$.

$\neg \mathbf{K}(p \wedge \neg \mathbf{K}p)$, by reductio rule.

$\Box \neg \mathbf{K}(p \wedge \neg \mathbf{K}p)$, by necessitation rule.

$\neg \Diamond \mathbf{K}(p \wedge \neg \mathbf{K}p)$.

Some Philosophical Claims in Epistemic Logic

$\varphi \wedge \Diamond\neg\mathbf{K}_i\varphi$ (realism)
$\varphi \rightarrow \Box\mathbf{K}_i\varphi$ (idealism)
$\varphi \rightarrow \Diamond\mathbf{K}_i\varphi$ (*ens et bonum convertuntur*)
$\varphi \rightarrow \neg\Diamond\mathbf{K}_i\varphi$ (epistemic nihilism, e.g., Gorgias)

Plan

Quantifying into Epistemic Contexts

Enriching the Language

- We shall allow one to combine epistemic ($\mathbf{K}_i, \mathbf{P}_i$) or doxastic ($\mathbf{B}_i, \mathbf{C}_i$) operators with qualifiers ($\forall x, \exists x$).
- Such a combination is not trivial, if we want to keep the logic close to our intuitions.

Troubles with Substitutivity

Substitutivity $a = b \to (\varphi(a) \leftrightarrow \varphi(a)[b/a])$ is a valid principle.

Problem $a = b \to (K_i Pa \leftrightarrow K_i Pb)$ is not an intuitive principle, but it should be valid (assuming substitutivity).

Espistemic subsititutivity: $\mathbf{K}_i(a = b) \to (\varphi(a) \leftrightarrow \varphi(a)[a/b])$.

Troubles with Existential Introduction

Fact If $\mathbf{K}(a = b)$, then $\exists x \mathbf{K}_i(x = b)$, by existential introduction.

Problem If I know that a is (called the same as) b, it does not follow that there there is an individual x of which I know that he/she is (called) b.

What does the problem consist of exactly?

P1 $\exists x \mathbf{K}_i(x = b)$ implies that agent i can identify (across possible worlds) the individual x who is b.

P2 $\exists x \mathbf{K}_i(x = b)$ implies an ontological commitment to the existence of b.

Hintikka's Solution (for this special case)

Fact If $\mathbf{K}(a = b)$, then $\exists x \mathbf{K}_i(x = b)$, by existential introduction.

Problem If I know that a is (called the same as) b, it <u>does not follow</u> that there there is an individual x of which I know that he/she is (called) b.

Solution:
Existential introduction within epistemic contexts can be performed provided $\exists y \mathbf{K}_i(y = a)$, where a is the constant that is going to be replaced by an existentially quantified variable.

Hintikka's Solution (2)

$\exists \mathbf{K}_i(x = b)$ means 'agent i knows who b is'.

So, Hintikka solution amounts to saying:

> If agent i knows who b is, and i also knows that b is a, then agent i also knows who a is.

But it is not true that:

> If agent i knows that a is b, then he knows who b is and he knows who a is.

Rigidity and Epistemic Contexts

Rigidity: $I_w(a) = I_{w'}(a)$ for any constant a and any $w, w' \in W$.

Rigidity justifies:

i) $a = b \rightarrow (\mathbf{K}_i Pa \leftrightarrow \mathbf{K}_i Pb)$
For rigidity makes valid $a = b \rightarrow \mathbf{K}_i(a = b)$.

ii) $\mathbf{K}_i(a = b) \rightarrow \exists x \mathbf{K}_i(x = b)$

Rigidity Fails in Epistemic Contexts

Rigidity: $I_w(a) = I_{w'}(a)$ for any constant a and any $w, w' \in W$.

Fact $a = b \rightarrow \mathbf{K}_i(a = b)$ (which follows from rigidity) is not epistemically intuitive.

Fact $\mathbf{K}_i(a = b) \rightarrow \exists x \mathbf{K}_i(x = b)$ (which follows from rigidity) is not epistemically intuitive.

Conclusion Rigidity fails in epistemic contexts.

Upshot: Epistemic contexts are **opaque contexts** (e.g., contexts where subsitutivity fails).

Plan

Information.

Three Notions of Information

- Being informative (as opposed to trivial).
- Becoming informed.
- Being informed (=holding the information that)

A Modal Logic of Being Informed

- Agent i is informed that (holds the information that) φ.
  $\mathbf{I}_i\varphi$
- φ is consistent with what i is informed of.
  $\mathbf{U}_i\varphi$
  (the information that i holds can be consistently updated with φ)

A Modal Logic of Being Informed

Satisfies:

- Distributivity Axiom: $\mathbf{I}_i(\varphi \to \psi) \to (\mathbf{I}_i\varphi \to \mathbf{I}_i\psi)$.
- Veridicality: $\mathbf{I}_i\varphi \to \varphi$ (reflexivity).
- Consistency: $\mathbf{U}_i\varphi \to \mathbf{I}_i\varphi$ (seriality).
- Brower's axiom: $\varphi \to \mathbf{I}_i\mathbf{U}_i\varphi$ (symmetry).
- Trasmissibility: $\mathbf{I}_i\mathbf{I}_j\varphi \to \mathbf{I}_i\varphi$ (theorem).

Does not satisfy:

- $\mathbf{I}_i\varphi \to \mathbf{I}_i\mathbf{I}_i\varphi$ (transitivity).
- $\mathbf{U}_i\varphi \to \mathbf{I}_i\mathbf{U}_i\varphi$ (euclidicity).