Knowledge and Information

Marcello Di Bello
University of Amsterdam

August 15, 2007

Today's Plan

- Transmissibility of knowledge
- Problems and paradoxes in epistemic logic
- Formalizing information using modal logic

J. Hintikka (1962), *Knowledge and Belief*, Cornell UP.
L. Floridi. (2006), 'The Logic of Being Informed', *L & A*, n. 196.

Chrysippus' Paradox (clarification)

Consider the dog Oscar at time t. Later, at time t', Oscar looses its tail.

Now, consider at t again Oscar but without its tail. That is: consider the **proper part** of Oscar at t which is Oscar lacking its tail. Call this object at t Oscar-minus.

Clearly, we have Oscar$\neq$Oscar-minus. By principle **ND**, we have $\Box$Oscar$\neq$Oscar-minus.

By interpreting $\Box$ as a tense operator, we have that Oscar and Oscar-minus are different individual at time t', but they should be the same.

Williamson's Argument Against the **KK** Principle (1)

Scenario:

Mr. M is looking at a tree from a long distance. The tree is actually 665 meter tall. But Mr. M does not know it. Clearly, he knows that the tree is not 0 meter tall.

Premise:

I $\mathbf{K}(t_{i+1} \rightarrow \neg\mathbf{K}\neg t_i)$

Argument for **I**:

- Mr. M only has approximate estimate of how tall the tree is.
- Clearly, he knows that the tree is not 0 meters tall, and that it is not 2 million meters tall.
- Anyway, Mr. M does not precisely know how tall the tree is.
- So, if the tree is n meters tall, he cannot distinguish if it is $n+1$ or $n-1$ meters tall.
- So, if the tree is n meters tall, he does not know if it is not $n+1$ meters tall.
- Further, Mr. M knows the above implication.

Williamson's Argument Against the **KK** Principle (2)

Premisses:

I	$\mathbf{K}(t_{i+1} \rightarrow \neg\mathbf{K}\neg t_i)$
K	$\mathbf{K}(\varphi \rightarrow \psi) \rightarrow (\mathbf{K}\varphi \rightarrow \mathbf{K}\psi)$
T	$\mathbf{K}\varphi \rightarrow \varphi$
KK	$\mathbf{K}\varphi \rightarrow \mathbf{K}\mathbf{K}\varphi$

Claim 1: $\mathbf{K}\neg t_i \rightarrow \mathbf{K}\neg t_{i+1}$

Assume $\mathbf{K}\neg t_i$.

$\mathbf{K}\mathbf{K}\neg t_i$, by **KK**.

$\mathbf{K}(\mathbf{K}\neg t_i \rightarrow \neg t_{i+1})$, by contraposition from **I**.

$\mathbf{K}\mathbf{K}\neg t_i \rightarrow \mathbf{K}\neg t_{i+1}$, by **K**.

$\mathbf{K}\neg t_{i+1}$, by modus ponens.

Williamson's Argument Against the **KK** Principle (3)

Scenario:

Someone is looking at a tree from a long distance. The tree is actually 665 meter heigh. But this person does not know it. Clearly, he knows that the tree is not 0 meter heigh.

Claim 1: $\mathbf{K}\neg t_i \rightarrow \mathbf{K}\neg t_{i+1}$

Claim 2: $\mathbf{K}\neg t_0 \rightarrow \ldots \mathbf{K}\neg t_n$, for any $n \in \mathbb{N}$.

It follows from **Claim 1** by substitution and iteration.

Claim 3: $\neg t_{665} \wedge t_{665}$.

t_{665} by scenario.

Clearly, $\mathbf{K}\neg t_0$.

$\mathbf{K}\neg t_{665}$, by **Claim 2**.

$\neg t_{665}$, by **T**.

Plan

Transmissibility

Knowledge is Transmissible

Claim:
Given two agents a and b, the principle $\mathbf{K}_a\mathbf{K}_b\varphi \rightarrow \mathbf{K}_a\varphi$ is valid.

Proof:

Assume
(1) $w \models \mathbf{K}_a\mathbf{K}_b\varphi$, and
(2) $w \models \neg\mathbf{K}_a\varphi$ (absurd hypothesis).

From (2), $w \models \mathbf{P}_a\neg\varphi$, and so $w' \models \neg\varphi$, for some w' such that $wR_a^k w'$.

From (1), $w' \models \mathbf{K}_a\varphi$ for all w' such that $wR_a^k w'$, and so $w' \models \varphi$.

Contradiction: $w' \models \varphi$ and $w' \models \neg\varphi$.

Beliefs are not Transmissible

Claim:
Given two agents a and b, the principle $\mathbf{B}_a\mathbf{B}_b\varphi \rightarrow \mathbf{B}_a\varphi$ is <u>not</u> valid.

Explanation: The explanation must rely in the fact that the axiom $\mathbf{B}_i\varphi \rightarrow \varphi$ does not hold.

Plan

Knowledge Spreads

Fact:
If $\vdash \mathbf{K}_i\varphi$, then $\vdash \mathbf{K}_i\psi$, provided $\vdash \varphi \rightarrow \psi$.

Proof:

Assume $\vdash \mathbf{K}_i\varphi$ and $\vdash \varphi \rightarrow \psi$.

By the rule of necessitation, we have $\vdash \mathbf{K}_i(\varphi \rightarrow \psi)$.

By distribution of $\mathbf{K}_i$ over implication, we have $\vdash \mathbf{K}_i\varphi \rightarrow \mathbf{K}_i\psi$.

By modus ponens, we have $\vdash \mathbf{K}_i\psi$.

This is called the **omniscience problem**.

Why is This a Problem?

Problem:
Epistemic logic requires that the knowing subject be able to draw **all** the (logical) consequences of what he knows.
Thus, the knowing subject is assumed to be **idealized**.

Solution:
Interpreting $\mathbf{K}_i\varphi$ as "it follows from what i knows that φ"

Skepticism Spreads

Fact:
If $\vdash \neg \mathbf{K}_i \psi$, then $\vdash \neg \mathbf{K}_i \varphi$, provided $\vdash \varphi \rightarrow \psi$.

Proof:

Assume $\vdash \neg \mathbf{K}_i \psi$ and $\vdash \varphi \rightarrow \psi$.

By the rule of necessitation, we have $\vdash \mathbf{K}_i(\varphi \rightarrow \psi)$.

By distribution of $\mathbf{K}_i$ over implication, we have $\vdash \mathbf{K}_i \varphi \rightarrow \mathbf{K}_i \psi$.

By contraposition, we have $\vdash \neg \mathbf{K}_i \psi \rightarrow \neg \mathbf{K}_i \varphi$.

By modus ponens, we have $\vdash \neg \mathbf{K}_i \varphi$.

This is the **reverse** of the **omniscience problem**.

Skepticism Spreads – Example

Premise 1: I don't know I am not brain in a vat.

Premise 2: If I have hands, then I am not a brain a vat.

Conclusion: I don't know I have hands.

I Know that Everything Is False

I don't know the Löb formula $((p \to q) \leftrightarrow p) \to p$.

$\mathbf{K}\neg\top$

$\varphi \to \top$, for any φ

$\neg\top \to \neg\varphi$, for any φ

$\mathbf{K}(\neg\top \to \neg\varphi)$

$\mathbf{K}\neg\top \to \mathbf{K}\neg\varphi$

$\mathbf{K}\neg\varphi$, for any φ

Moore's Paradox

Moore's sentence:

p but I do not believe that p

$p \wedge \neg \mathbf{B}_i p$

Moore's sentence is *not* logically inconsistent (why?), yet it is problematic (why?).

Hintikka's Explanation

Claim: $\mathbf{B}_i(p \wedge \neg \mathbf{B}_i p)$ is logically inconsistent or unsatisfiable.

Proof:

Suppose for contradiction that $w \models \mathbf{B}_i(p \wedge \neg \mathbf{B}_i p)$.

Then, $w \models \mathbf{B}_i p \wedge \mathbf{B}_i \neg \mathbf{B}_i p$.

Then, $w \models \mathbf{B}_i \mathbf{B}_i p$ (why?) and $w \models \mathbf{B}_i \neg \mathbf{B}_i p$.

Then, $w' \models \mathbf{B}_i p$ and $w' \models \neg \mathbf{B}_i p$ for any $w' \in W$ such that $w R_i^b w'$.

Contradiction!

Upshot: Moore's sentence is **doxastically inconsistent**, but not logically inconsistent.

Compare Different Moore's Sentences

1. I believe this: That p is the case and that I do not believe that p (inconsistent).
2. a believes this: That p is the case and that a does not believe that p (inconsistent).
3. a believes this: That p is the case and b does not believe that p (consistent).

The gist of Hintikka's explanation is that $\mathbf{B}_i(p \wedge \neg \mathbf{B}_j p)$ is inconsistent only if $i = j$.

Moore's Explanation (and Block's)

'*p* but I don't believe that *p*' is odd whenever it is asserted.

Asserting a sentence presupposes believing that sentence (at least asserting it honestly)

The Two Accounts Compared

Hintikka If '*p* but I don't believe that *p*' is **believed**, then it is inconsistent

Moore If '*p* but I don't believe that *p*' is **asserted**, then it is inconsistent

Hintikka's explanation is less demanding than Moore's (why?).

Objection to Moore's explanation:
The sentence '*p* but I cannot believe that *p*' is not odd.

Upshot: There are assertions whose content need not be believed by the speaker.

Epistemic Variant of Moore's Sentence

p but I don't know that *p*

$$p \wedge \neg \mathbf{K}_i p$$

Under Hintikka's account:

$\mathbf{K}_i(p \wedge \neg \mathbf{K}_i p)$ is inconsistent (exercise).

$\mathbf{B}_i(p \wedge \neg \mathbf{K}_i p)$ is consistent (exercise).

What about this?

p but **you** do not know that *p*.
(epistemically inconsistent when addressed to anyone)

Fitch's Knowability Paradox (1)

P1: $\forall\varphi(\varphi \rightarrow \Diamond \mathbf{K}\varphi)$

P2: $\exists\varphi(\varphi \wedge \neg\mathbf{K}\varphi)$

Thus, $p \wedge \neg\mathbf{K}p$, by existential instantiation.

Put $\varphi := p \wedge \neg\mathbf{K}p$

Thus, $(p \wedge \neg\mathbf{K}p) \rightarrow \Diamond\mathbf{K}(p \wedge \neg\mathbf{K}p)$.

C1 Thus, $\Diamond\mathbf{K}(p \wedge \neg\mathbf{K}p)$.

However

Assume $\mathbf{K}(p \wedge \neg\mathbf{K}p)$ for contradiction.

$\mathbf{K}p \wedge \mathbf{K}\neg\mathbf{K}p$, by distributivity of $\mathbf{K}$.

$\mathbf{K}p \wedge \neg\mathbf{K}p$, by veridicality of $\mathbf{K}$.

$\neg\mathbf{K}(p \wedge \neg\mathbf{K}p)$, by reductio rule.

$\Box\neg\mathbf{K}(p \wedge \neg\mathbf{K}p)$, by necessitation rule.

C2 $\neg\Diamond\mathbf{K}(p \wedge \neg\mathbf{K}p)$.

Fitch's Knowability Paradox (2)

P1: $\forall\varphi(\varphi \rightarrow \Diamond\mathbf{K}\varphi)$

P2: $\exists\varphi(\varphi \wedge \neg\mathbf{K}\varphi)$

C1: Thus, $\Diamond\mathbf{K}(p \wedge \neg\mathbf{K}p)$.

C2: $\neg\Diamond\mathbf{K}(p \wedge \neg\mathbf{K}p)$.

C2 contradicts **C1**, which follows from **P1** and **P2**.

So the negation of **P2** is the case, namely $\forall\varphi(\varphi \rightarrow \mathbf{K}\varphi)$.

Or the negation of **P1** is the case, namely $\exists\varphi(\varphi \wedge \neg\Diamond\mathbf{K}\varphi)$.

Philosophical conclusion

Knowability thesis (**P1**) and non-omniscience (**P2**) yield:

the thesis that every truth is known (idealism?);

or the thesis that there is an unknowable truth (mysticism?).

Some Philosophical Claims in Epistemic Logic

$\varphi \wedge \Diamond \neg \mathbf{K}_i \varphi$ (realism)
$\varphi \rightarrow \Box \mathbf{K}_i \varphi$ (idealism)
$\varphi \rightarrow \Diamond \mathbf{K}_i \varphi$ (*ens et verum convertuntur*)
$\varphi \rightarrow \neg \Diamond \mathbf{K}_i \varphi$ (epistemic nihilism, e.g., Gorgias)

Gettier's Problem

Contra Knowledge as justified true belief.

Example 1 Suppose one is justified in holding φ true.

Thus, one is justified in holding $\varphi \vee \psi$ true.

(*assumption:* derivation rule preserves justification).

By chance, φ is fale, but ψ is true.

So, $\varphi \vee \psi$ is a justified true belief, but ...

Plan

Formalizing information using modal logic.

Three Notions of Information

- Being informative (as opposed to trivial).
- Becoming informed.
- Being informed (=holding the information that)

A Modal Logic of Being Informed

- Agent i is informed that (holds the information that) φ.
  $\mathbf{I}_i\varphi$
- φ is consistent with what i is informed of.
  $\mathbf{U}_i\varphi$
  (the information that i holds can be consistently updated with φ)

A Modal Logic of Being Informed (1)

Satisfies:

- Distributivity Axiom: $\mathbf{I}_i(\varphi \to \psi) \to (\mathbf{I}_i\varphi \to \mathbf{I}_i\psi)$.
- Consistency: $\mathbf{I}_i\varphi \to \mathbf{U}_i\varphi$ (seriality).
  Keep in mind the distinction between 'being informed' and 'becoming informed'. One can *become* informed of contradictory information, but not *being* informed of contradictory information.
- Veridicality: $\mathbf{I}_i\varphi \to \varphi$ (reflexivity).
  Keep in mind the distinction between 'holding the information **that** φ' and 'holding φ **as** information'. The latter need not satisfy veridicality, but the formes does.
- Brower's axiom: $\varphi \to \mathbf{I}_i\mathbf{U}_i\varphi$ (symmetry).
  No clear argument yet (sorry!).
- Trasmissibility: $\mathbf{I}_i\mathbf{I}_j\varphi \to \mathbf{I}_i\varphi$ (theorem).

A Modal Logic of Being Informed (2)

Satisfies:

- Distributivity Axiom: $\mathbf{I}_i(\varphi \rightarrow \psi) \rightarrow (\mathbf{I}_i\varphi \rightarrow \mathbf{I}_i\psi)$.
- Consistency: $\mathbf{I}_i\varphi \rightarrow \mathbf{U}_i\varphi$ (seriality).
- Veridicality: $\mathbf{I}_i\varphi \rightarrow \varphi$ (reflexivity).
- Trasmissibility: $\mathbf{I}_i\mathbf{I}_j\varphi \rightarrow \mathbf{I}_i\varphi$ (theorem).
- Brower's axiom: $\varphi \rightarrow \mathbf{I}_i\mathbf{U}_i\varphi$ (symmetry).

Does not satisfy:

- $\mathbf{I}_i\varphi \rightarrow \mathbf{I}_i\mathbf{I}_i\varphi$ (transitivity). Information can be held by artificial agents. So 'being informed that' is not a mental or conscious state. Hence, introspection-like arguments shall not apply.

Epistemic vs. Information Logic

1. Epistemic logic does not contain the symmetry axiom $\varphi \to \Box\Diamond\varphi$. Information logic does.
2. Epistemic logic contains the **KK** axiom, but information logic does not.

- Information logic can be seen as a logic for artificial agents (=agents without mental or conscious states)
- We can understand information as **knowledge without the knowing subject**.

Omniscience Problem and Information

Problem: Information logic is not immune from omniscience problem or information overload.

Replies:

- The *informed* artificial agent can be a Turing Machine, which can prove all the propositional tautologies.
- Inputting logical tautologies into an information base does not change its information content.
- All propositional tautologies are not informative:
  '$\vdash \varphi$ implies $\vdash \mathbf{I}\varphi$' is a shorthand for
  '$\vdash \varphi$ implies $P(\varphi) = 1$ implies $\mathit{Inf}(\varphi) = 0$ implies $\vdash \mathbf{I}\varphi$'

Against $\mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\varphi$

- The causes of the Gettier problem may be due to the 'justification' or 'belief' part in the definition of knowledge.

  Suggestion:
  abandoning $\mathbf{K}_i\varphi \rightarrow \mathbf{B}_i\varphi$ and endorsing $\mathbf{K}_i\varphi \rightarrow \mathbf{I}_i\varphi$.
- This would open up an *information based approach to epistemology*, rather than a doxastic based approach to epistemology.
- This would solve the Gettier problem.