Agenda

PART I: What Are Bayesian Networks?

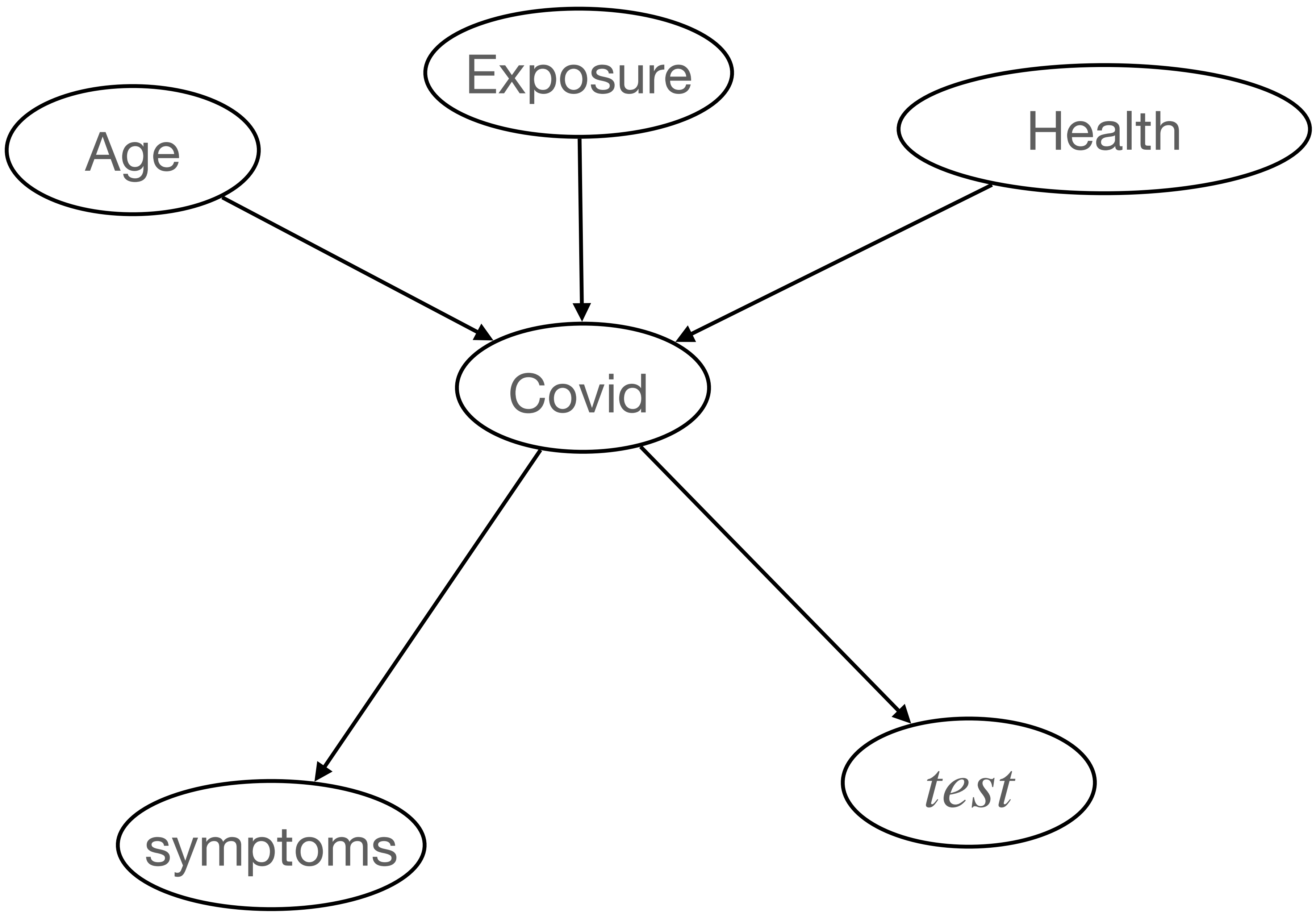
PART II: Group Exercise and Discussion

PART III: Analyzing a Legal Case Using Bayesian Networks

PART I

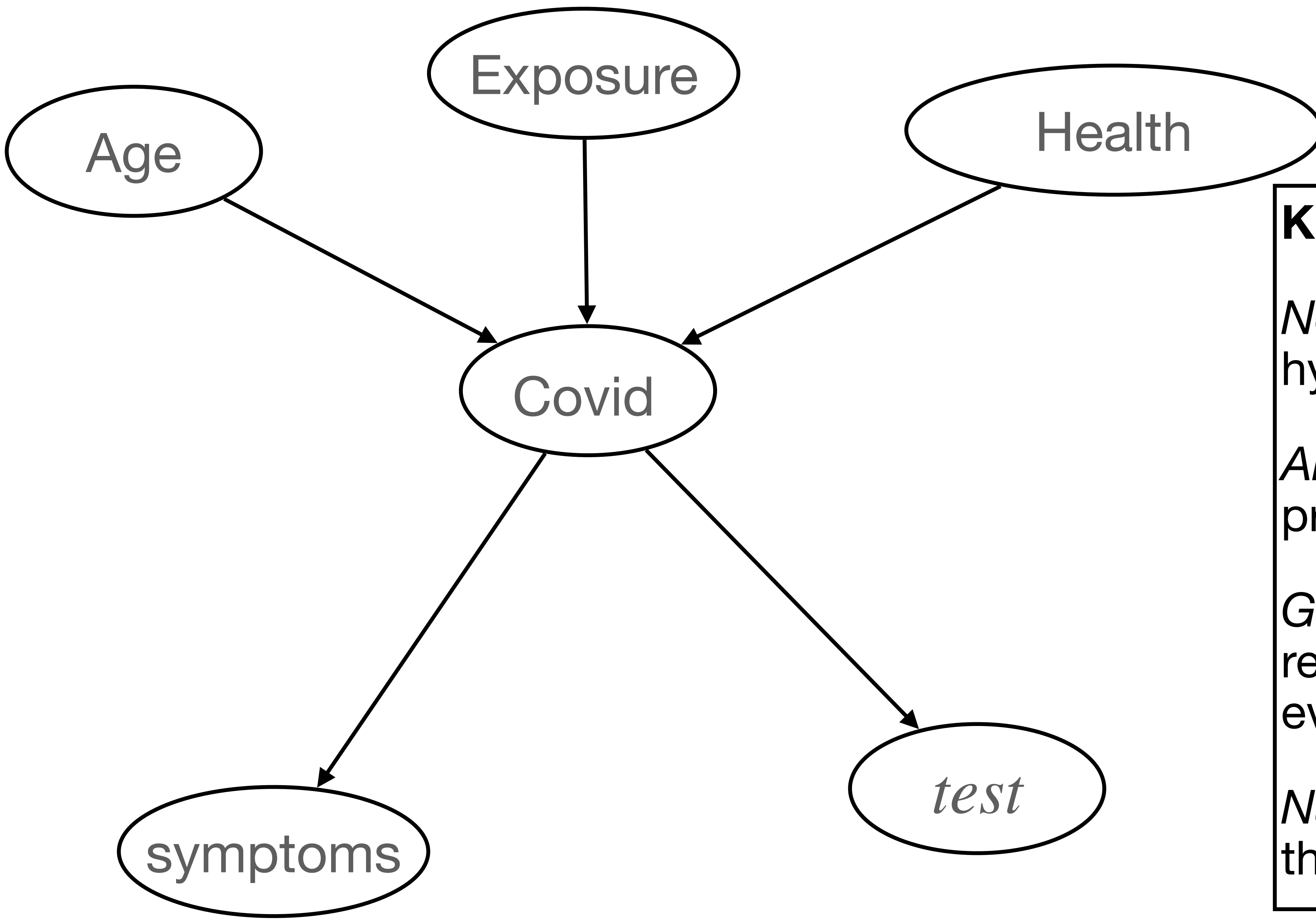
What Are Bayesian Networks?

Example: Bayes Nets for Covid Diagnosis



Norman E Fenton, Scott McLachlan, Peter Lucas, Kudakwashe Dube, Graham A Hitman, Magda Osman, Evangelia Kyrimi, Martin Neil, "A Bayesian network model for personalised COVID19 risk assessment and contact tracing", https://doi.org/10.1101/2020.07.15.20154286

Example: Bayes Nets for Covid Diagnosis



Key ideas:

Nodes: evidence v. hypothesis

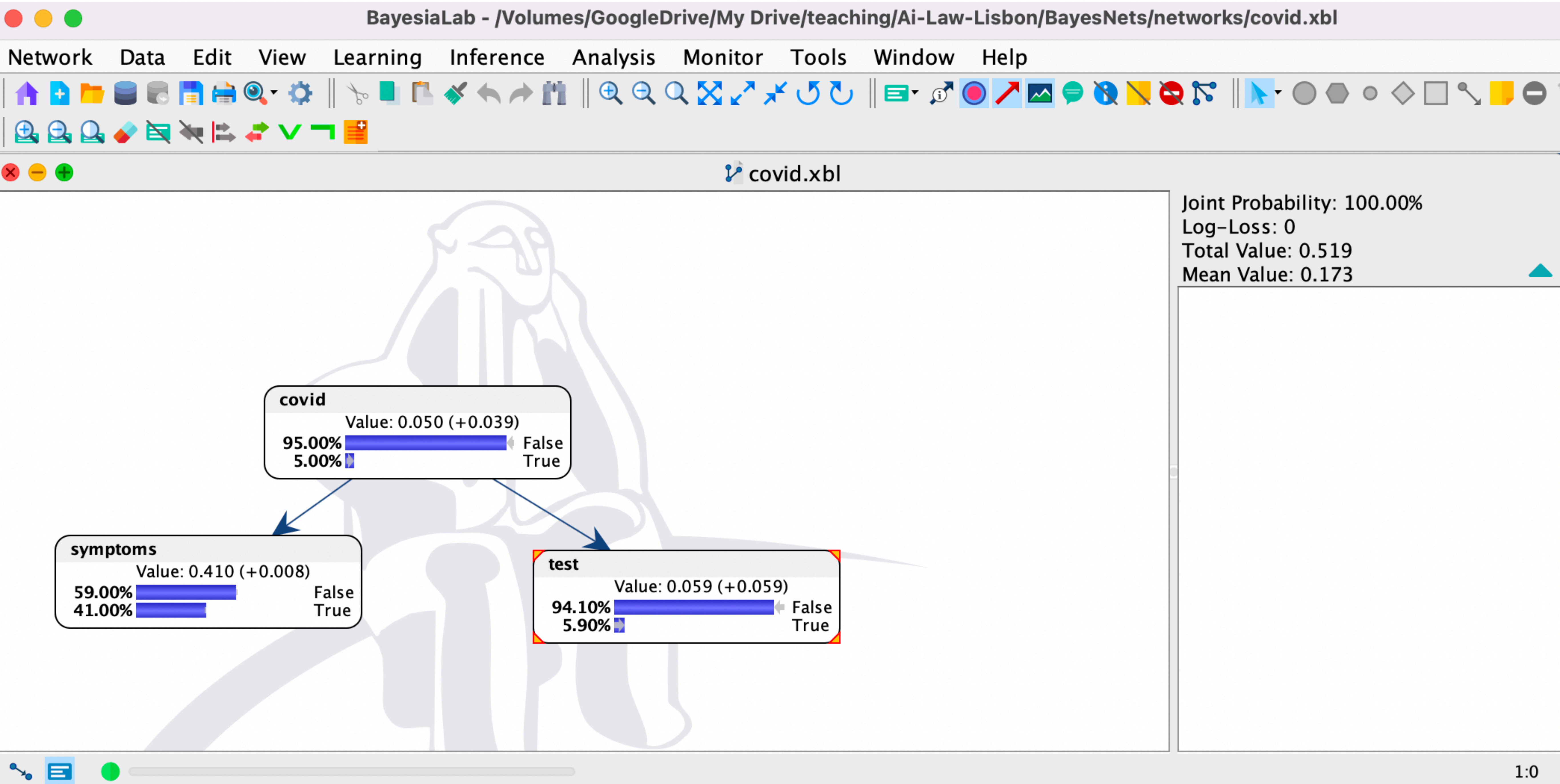
Arrows: diagnostic v. predictive

Graphical part: qualitative relations between evidence and hypothesis

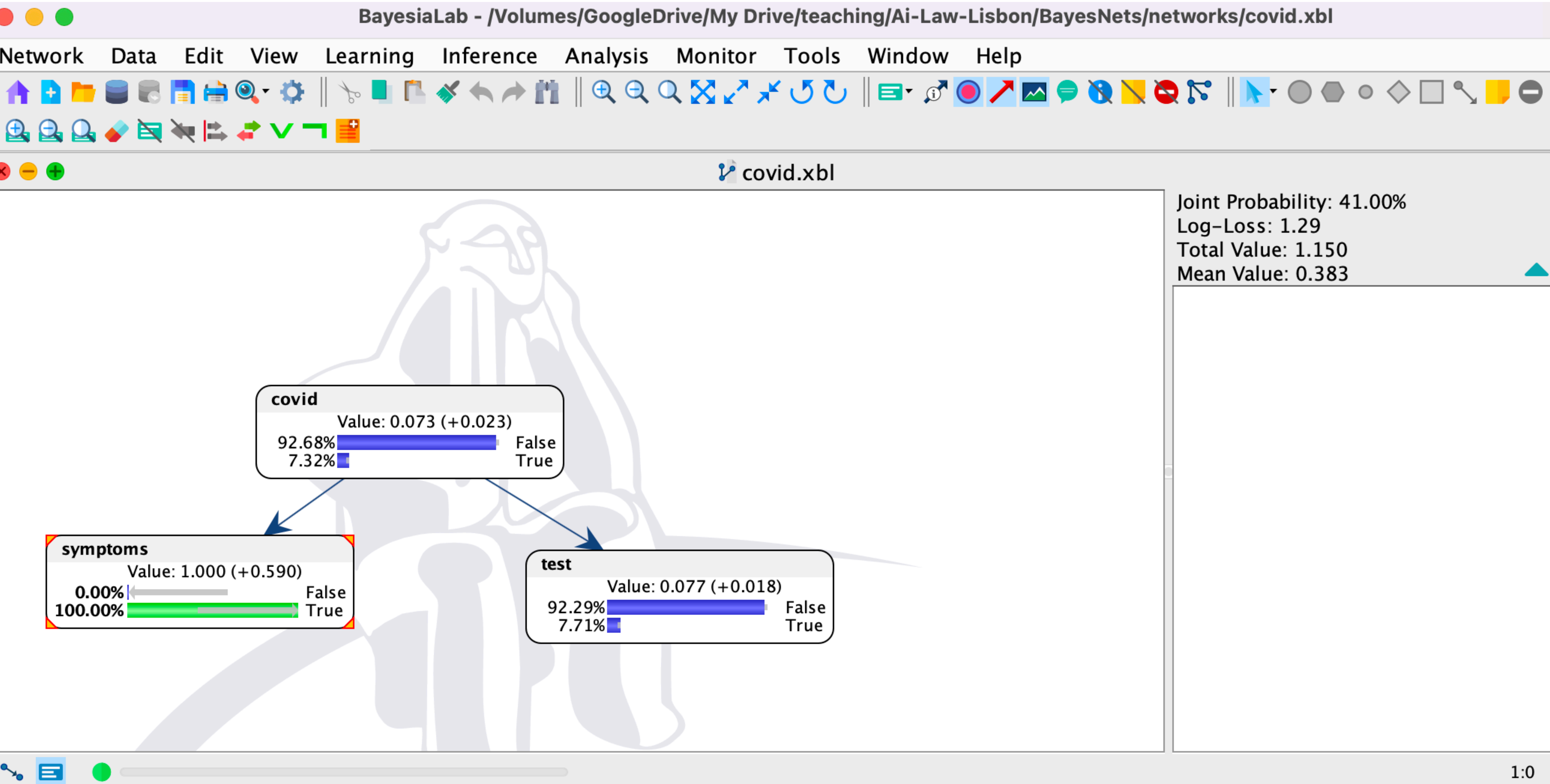
Numerical part: strength of these relations (*more later*)

Norman E Fenton, Scott McLachlan, Peter Lucas, Kudakwashe Dube, Graham A Hitman, Magda Osman, Evangelia Kyrimi, Martin Neil, "A Bayesian network model for personalised COVID19 risk assessment and contact tracing", https://doi.org/10.1101/2020.07.15.20154286

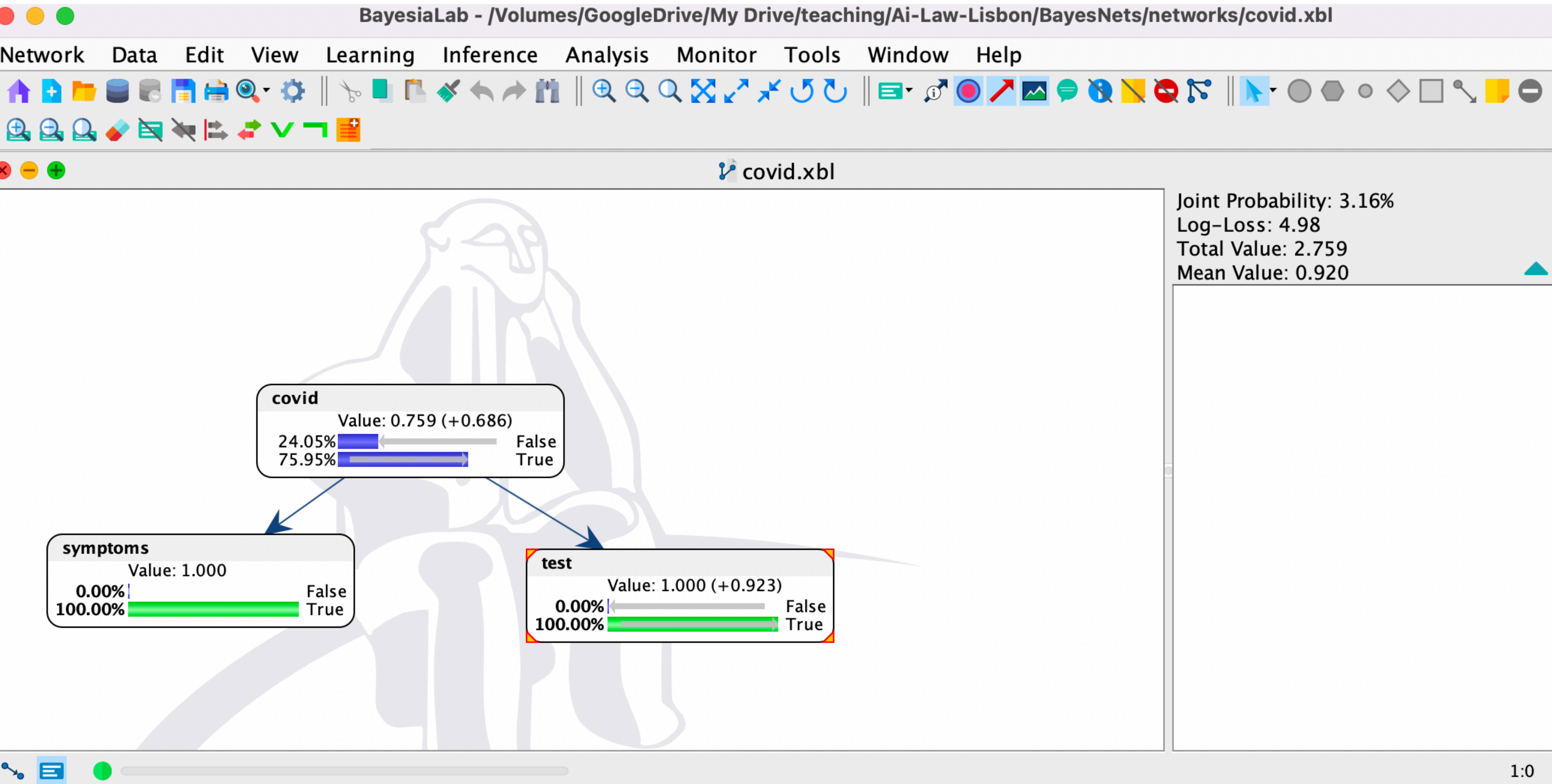
(1) Bayesian Network with BayesiaLab



(2) Bayesian Network with BayesiaLab



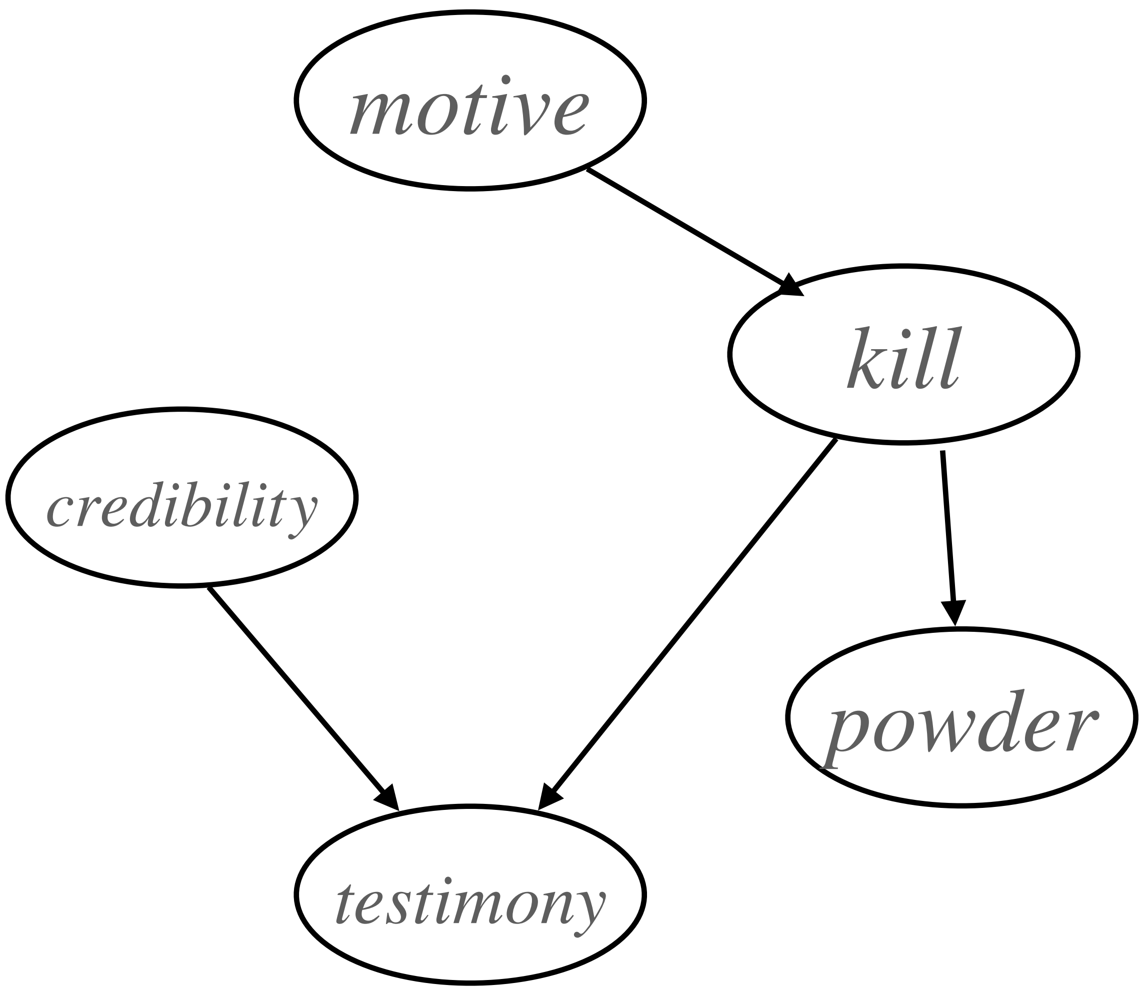
(3) Bayesian Network with BayesiaLab



Graphical Components:
Nodes, Arrows and Idioms

(2) Graphical Components of a Bayesian Network

Arrows



As a first approximation, think of **arrows** as *directions of causal influence* (though this interpretation is debated):

Whether or not the defendant had a motive to kill influences whether or not the defendant killed the victim

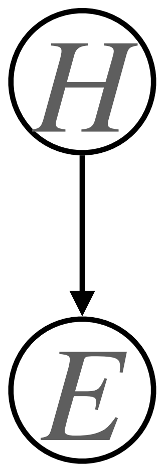
Whether or not the defendant killed the victim influences whether or not gunpowder was found on defendant

Whether or not the defendant killed the victim influences what the witness saw

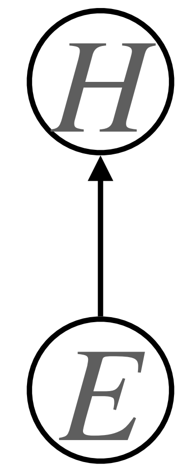
Whether or not the witness is credible influences what the witness says

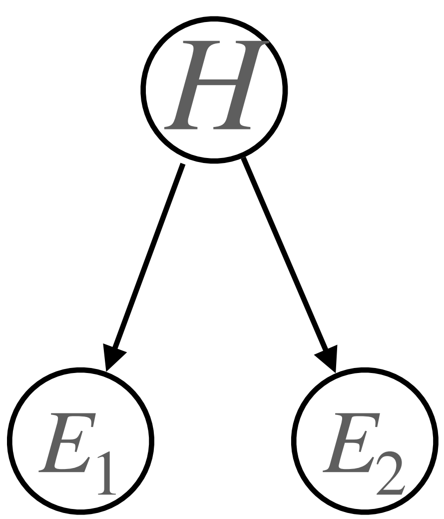
(3a) Graphical *Components of a Bayesian Network*

Idioms (=basic graphical structures)

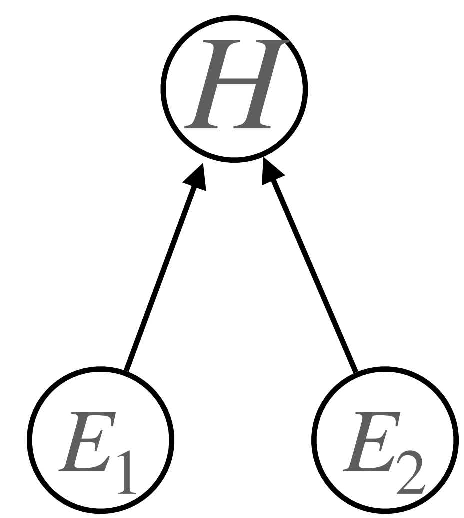


Hypothesis / one piece of evidence



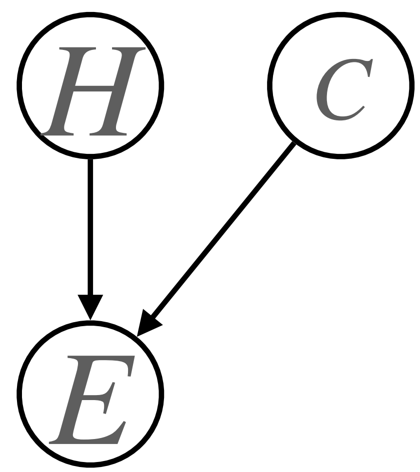


Hypothesis / two piece of evidence

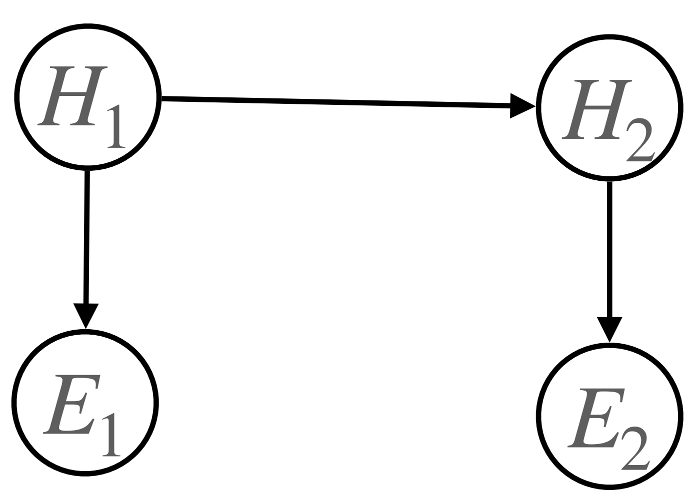


(3b) Graphical *Components of a Bayesian Network*

Idioms (=basic graphical structures)



Evidence / Hypothesis *plus* **Credibility**



Rebuttal: hypotheses H1 and H2 are incompatible

Basic Idioms Can Be Combined and Form More Complex Graphs

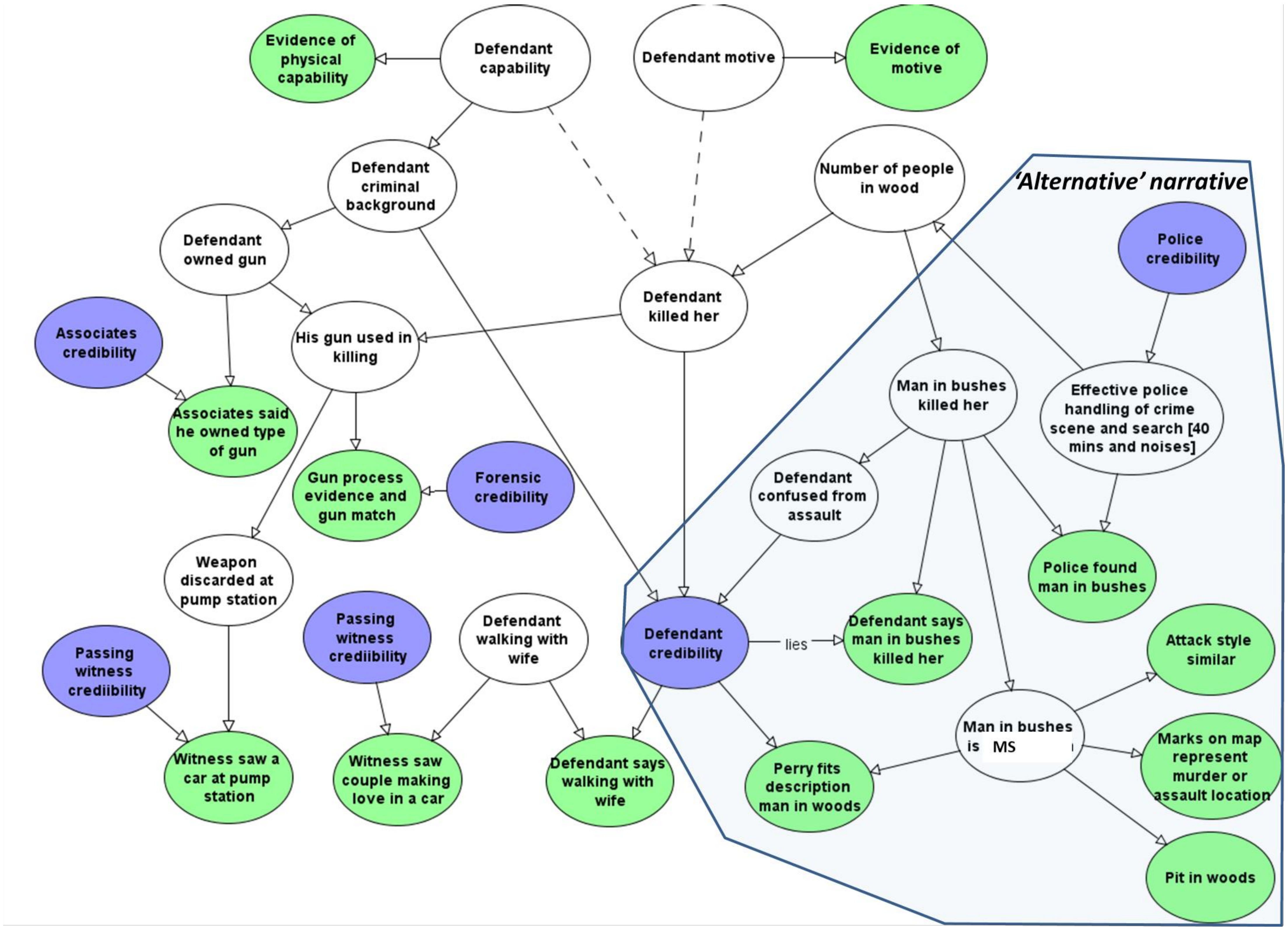


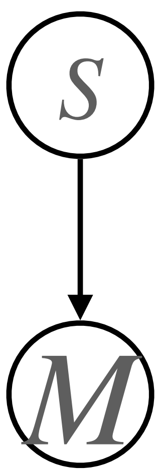
Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Numerical Component: Probability Tables

Examples of Bayesian Networks for Assessing DNA Evidence and Eyewitness Evidence

Example 1: DNA Match Evidence (M) Source Hypothesis (S)

Graph



Probabilities

$P(S = yes) =$ prior

$P_0(M = yes \mid S = yes) = 1$

$P(M = yes \mid S = no) = RMP$

Random Match Probability

Probability Tables

S=yes	Prior
S=no	1-prior

	S=yes	**S=no**
M=yes	100%	RMP
M=no	0%	1-RMP

Bayes' theorem needed to calculate P(S = yes | M = yes), as follows:

$$P_0(S = yes \mid M = yes) = \frac{P(M = yes \mid S = yes)}{P(M = yes)} P(S = yes)$$

$$= \frac{P(M = yes \mid S = yes)}{P(M = yes \mid S = yes)P(S = yes) + P(M = yes \mid S = no)P(S = no)} P(S = yes)$$

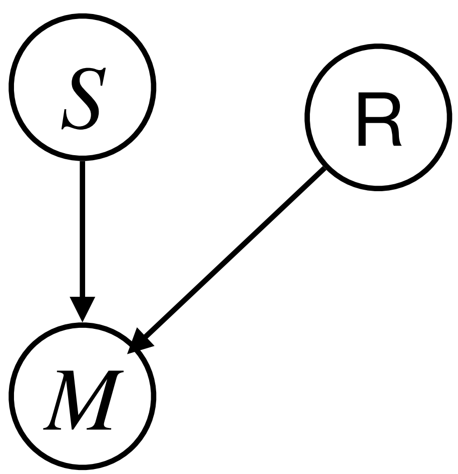
Aside
How Are Random Match Probabilities Calculated?

See Charles H. Brenner's "Forensic mathematics of DNA matching" available at https://dna-view.com/profile.htm

<table>
<tr><th colspan="2">DNA Profile</th><th colspan="4">Allele frequency from database</th><th colspan="2">Genotype frequency for locus</th></tr>
<tr><th>Locus</th><th>Alleles</th><th>times allele observed</th><th>size of database</th><th colspan="2">Frequency</th><th>formula</th><th>number</th></tr>
<tr><td rowspan="2">CSF1PO</td><td>10</td><td>109</td><td rowspan="2">432</td><td>p=</td><td>0.25</td><td rowspan="2">$2pq$</td><td rowspan="2">0.16</td></tr>
<tr><td>11</td><td>134</td><td>q=</td><td>0.31</td></tr>
<tr><td rowspan="2">TPOX</td><td>8</td><td rowspan="2">229</td><td rowspan="2">432</td><td rowspan="2">p=</td><td rowspan="2">0.53</td><td rowspan="2">p^2</td><td rowspan="2">0.28</td></tr>
<tr><td>8</td></tr>
<tr><td rowspan="2">THO1</td><td>6</td><td>102</td><td rowspan="2">428</td><td>p=</td><td>0.24</td><td rowspan="2">$2pq$</td><td rowspan="2">0.07</td></tr>
<tr><td>7</td><td>64</td><td>q=</td><td>0.15</td></tr>
<tr><td rowspan="2">vWA</td><td>16</td><td rowspan="2">91</td><td rowspan="2">428</td><td rowspan="2">p=</td><td rowspan="2">0.21</td><td rowspan="2">p^2</td><td rowspan="2">0.05</td></tr>
<tr><td>16</td></tr>
<tr><td></td><td></td><td></td><td colspan="4">profile frequency=</td><td>0.00014</td></tr>
</table>

Example 2: DNA Match + Test Reliability

Graph



Probabilities

$P(S = yes)$ = prior for S

$P(R = yes)$ = prior for R

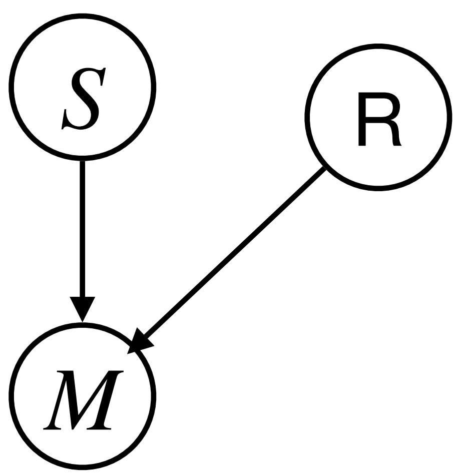
Probability Tables

S=yes	Prior (low?)
S=no	1-prior

R=yes	Prior (high?)
R=no	1-prior

Example 2: DNA Match + Test Reliability

Graph



Probabilities

$P(S = yes)$ = prior for S

$P(R = yes)$ = prior for R

$P_0(M = yes \,|\, S = yes \;\&\; R = yes) = 1$

$P(M = yes \,|\, S = no \;\&\; R = yes) = RMP$

Random Match Probability

$P_0(M = yes \,|\, S = yes \;\&\; R = no) = 0.5$

$P(M = yes \,|\, S = no \;\&\; R = no) = 0.5$

Probability Tables

S=yes	Prior (low?)
S=no	1-prior

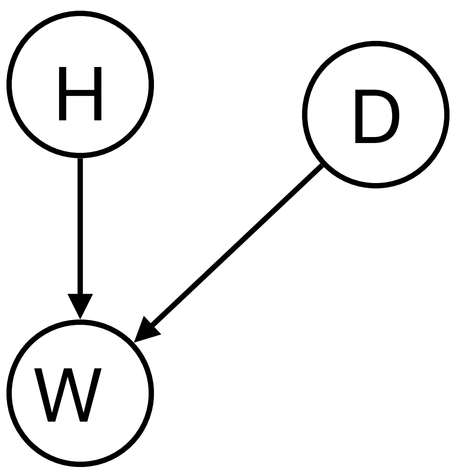
R=yes	Prior (high?)
R=no	1-prior

	S=yes & R=yes	**S=no & R=yes**	**S=yes & R=no**	**S=no & R=no**
M=yes	100%	RMP	50%	50%
M=no	0%	1-RMP	50%	50%

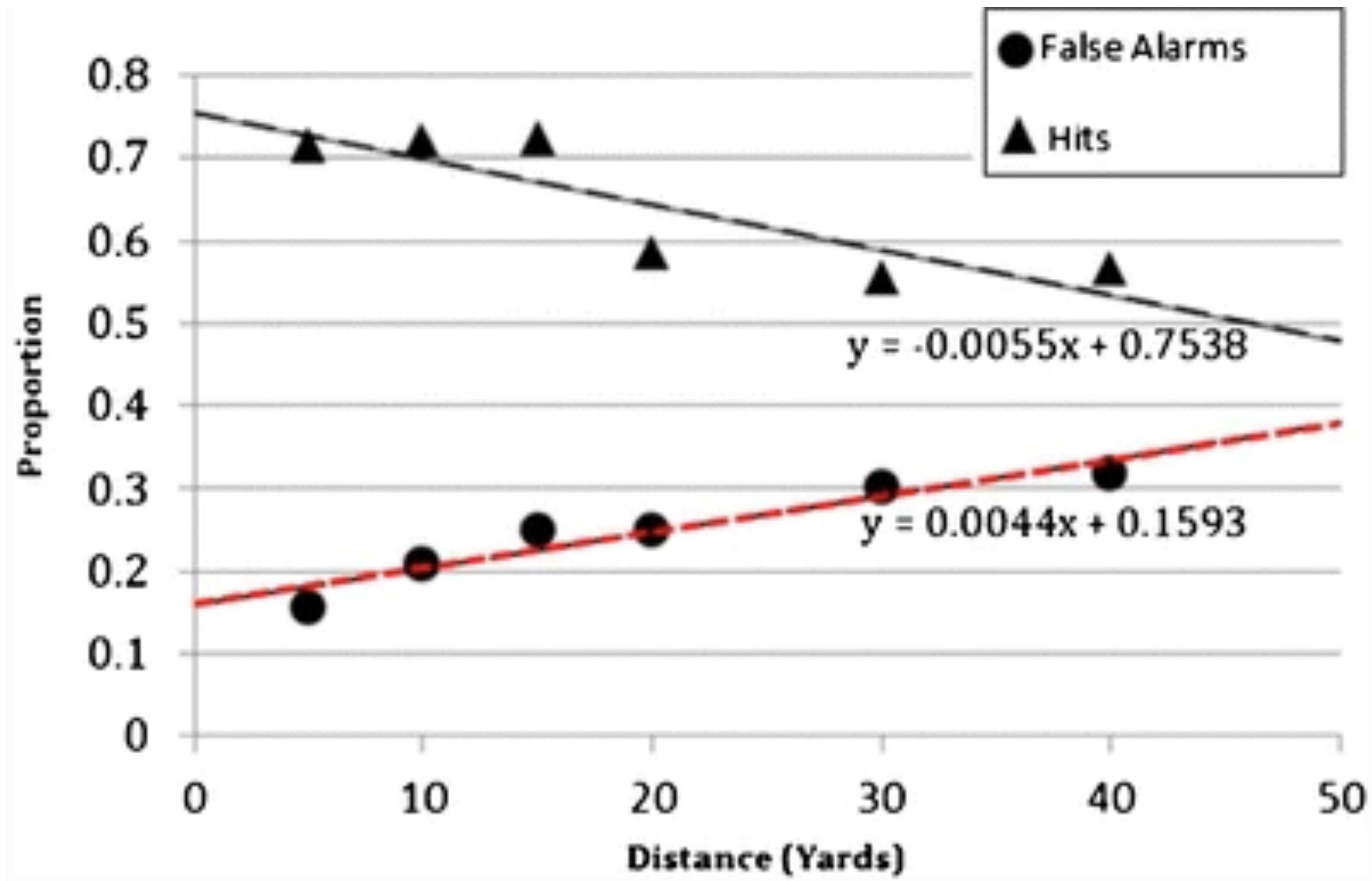
Bayes' theorem needed to calculate P(S = yes | M = yes).
But manual calculations quickly become unmanageable!

Example 3: Eyewitness and Distance

Graph



Node D can represent a continuous variable for “distance”. We can use psychological findings to fill in the numbers in the probability table



Lampinen, James Michael, Erickson, William Blake, Moore, Kara N., & Hittson, Aaron (2014), “Effects of distance on face recognition: implications for eyewitness identification”, *Psychonomic Bulletin & Review*, 21.

Bayesian Networks Summary

(1) **Qualitative**: *A graphical representation of relationships between pieces of evidence and hypothesis*

(2) **Numerical:** The *strength of these relationships is expressed numerically with probabilities tables*

(3) **Reasoning**: *Able to calculate probabilities of hypotheses based on evidence using Bayes' theorem (or dedicated software)*

PART II
Group Exercise and Discussion

Consider this Stylized Legal Case

Chris is shot (clearly murder) on an island.

There are 100 possible perpetrators. One of them is Fred.

Gun shot residue is found on Fred's hands same day as the shooting took place.

> There are two possible explanations: Fred shot Chris or Fred was at the shooting range the same day. Both explanations can be true. Given the gun shot residue, it is impossible that both are false.

Fred goes to the shooting range 4 days a week.

Daniela, a woman who works at the shooting range, is asked if she saw Fred on the day in question, and she says that he was not at the range that day.

> Daniela's accuracy in correctly identifying and remembering Fred is 99%. In other words, if Fred was at the shooting range that day, there is a 1% chance that she will incorrectly report that he was not there, and if he was not, there is a 99% chance that she will correctly report that he was not there.

What is the probability that Fred shot Chris?

Group Exercise

- Start with an **informal analysis** of the case: what are the main pieces of evidence? How would a judge or a lawyer analyze this case? How strong is the evidence against Fred? Is there a reasonable doubt about Fred's guilt?

- Sketch how a **graph of a Bayesian network** (nodes and arrows) could look like. Is there only one possible graph or multiple graphs seem appropriate here?

- Fill in the **probability tables** with the right numbers. Do you have all the numbers you need or are some numbers missing?

Informal Reasoning: *Do you Agree?*

It is **initially** unlikely that Fred shot Chris. There were a lot of other people on the island who could have done that.

After finding **gun powder's residue on Fred**, it is still not very likely that Fred shot Chris. Fred goes to the shooting range every week (4 out of 7 days). We would expect him to have gun powder on his hands the same day he went to the shooting range.

> One question might be: can the gun powder be washed away easily? Assume gunpowder does not survive more than one day.

Daniela's testimony changes things. She is highly reliable (99%). If the hypothesis that Fred was at the shooting range that day is ruled out, the most likely explanation is that Fred did indeed shot Chris.

Questions for Discussion

Feel Free to Add Your Own!

1. Can Bayesian networks be helpful to judges?

   If not, why not. If yes, in what ways exactly?

2. Will different people come up with different graphs for a Bayesian networks?

   If yes, wouldn't such subjectivity be a problem?

3. Where do the numbers needed to fill the probability tables come from?

PART III
Analysis of a Legal Case Using Bayesian Networks

Tasks of a Judge

(1) *Gatekeeping:* apply exclusionary rules about relevance, hearsay, character evidence, privileges, etc.

(2) *Assess* the evidence for and against the defendant, and then finally decide

(2) Seek evidence and asks questions

(4) Write down a written opinion that lays down in detail the reasoning that supports to the decision

Simonshaven case

If You Were a Judge Writing the Opinion, How Would You Organize Your Analysis?

Informal Analysis of the Case

(NB: *Matters of fact only*)

(1) Identify factual propositions (=hypotheses) under dispute.

These can be ultimate *probanda* or intermediate propositions.

(2) Identify key pieces of evidence which favor or oppose the factual propositions under dispute

(3) Make an assessment of the case as a whole, all things considered.

This can can require an assessment of the balance of the evidence for/against the accused or an assessment of whether a reasonable doubt about guilt exists.

The Analysis That Follows Is Taken From this Paper

Analyzing the Simonshaven Case using Bayesian Networks

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Barbaros Yet, Department of Industrial Engineering, Hacettepe Universitesi,Turkey

David Lagnado, Department of Experimental Psychology, University College London

Full Bayesian Network

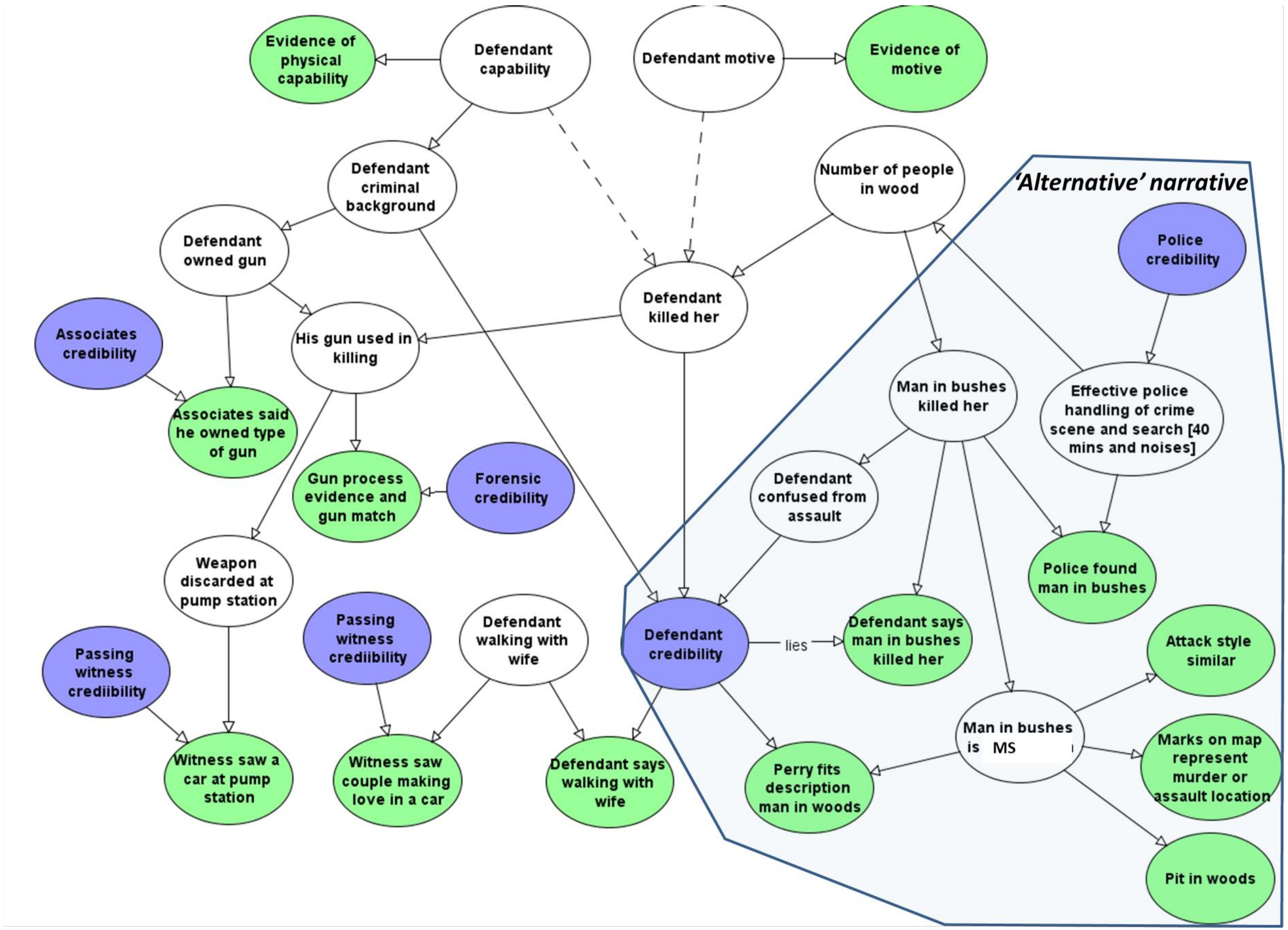
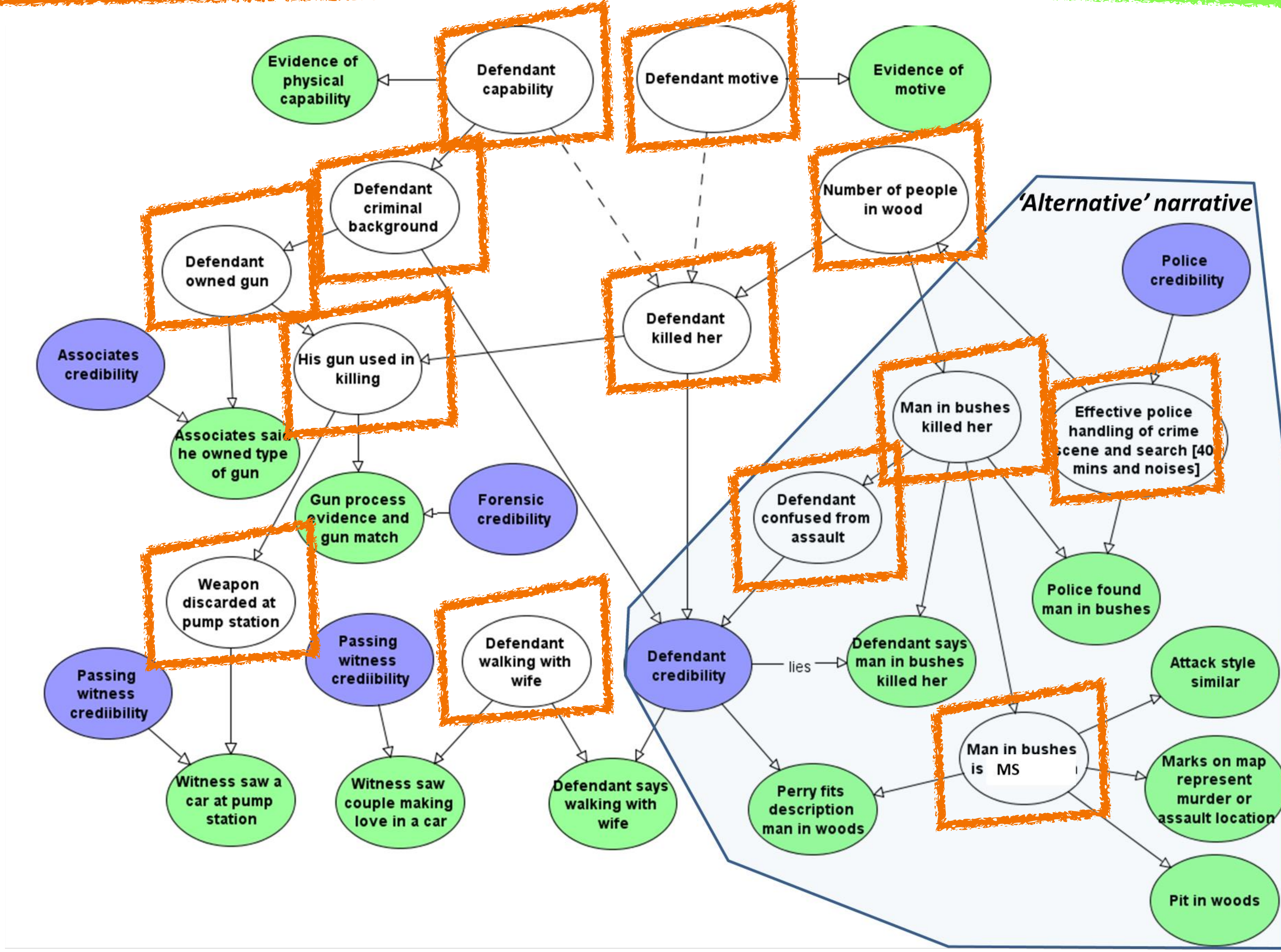


Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Hypothesis Nodes v. *Evidence* Nodes



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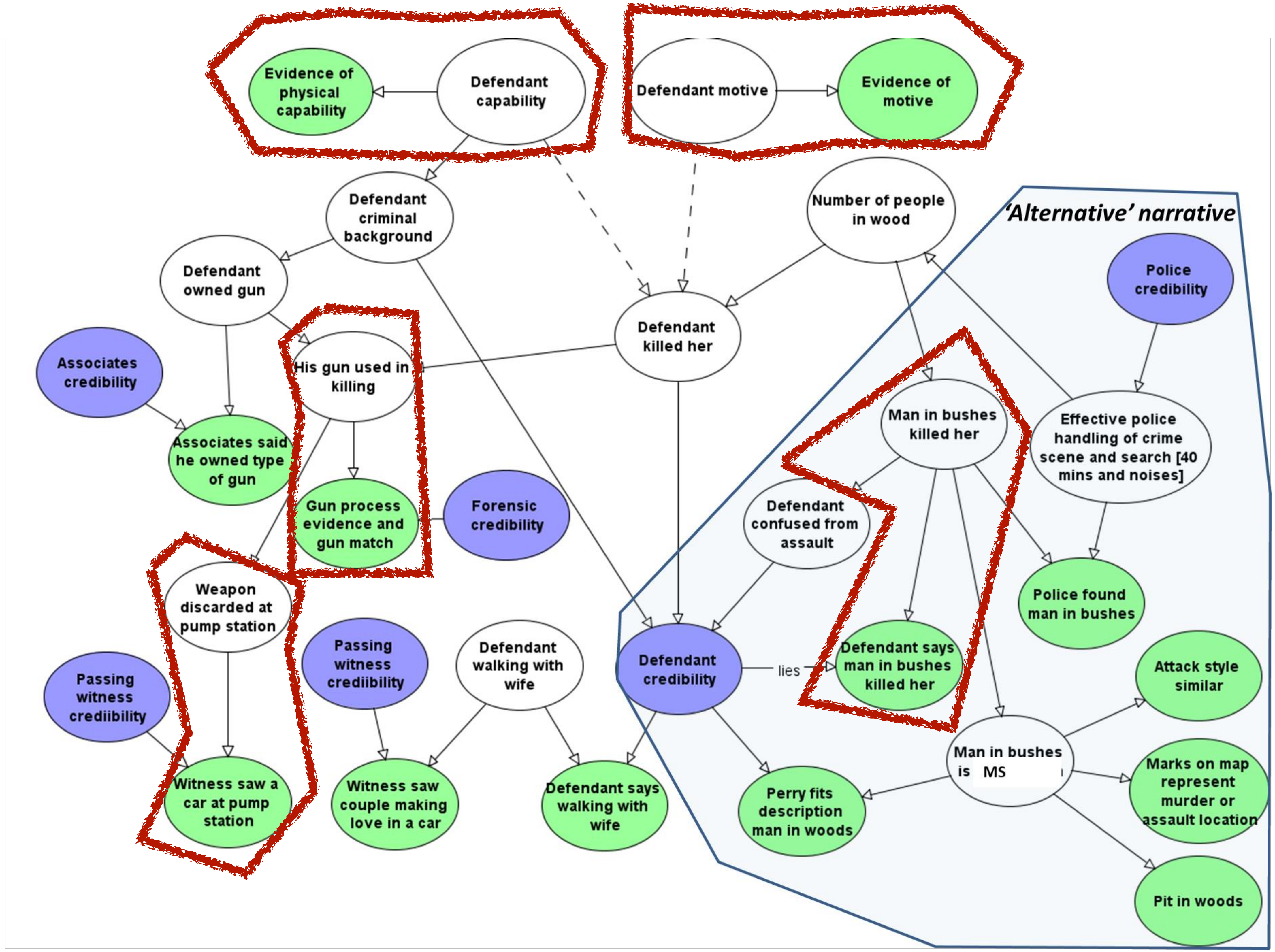
Examples: Evidence/Hypothesis Idioms



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Examples: Evidence Credibility Idiom

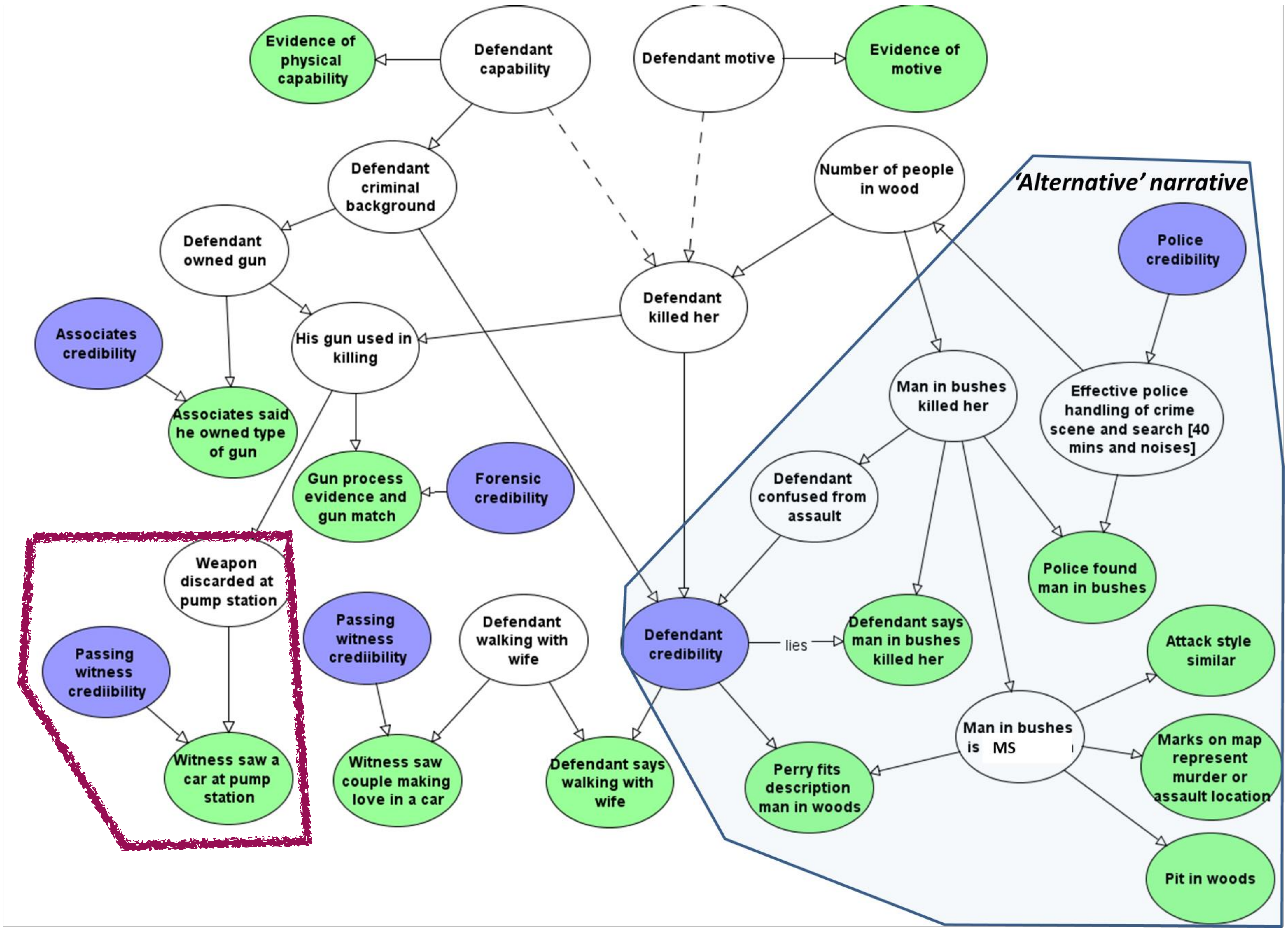
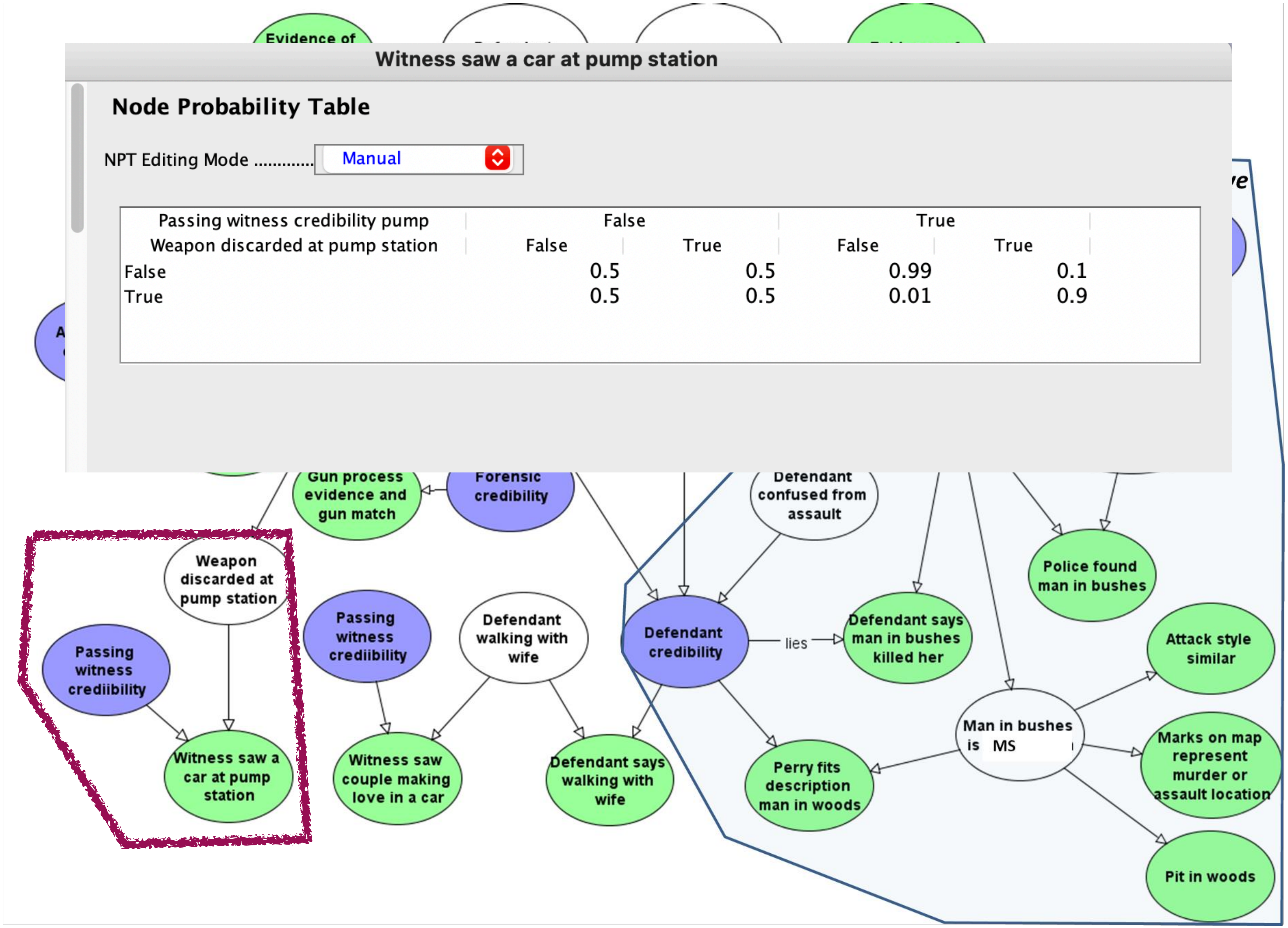


Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Examples of Probability Tables

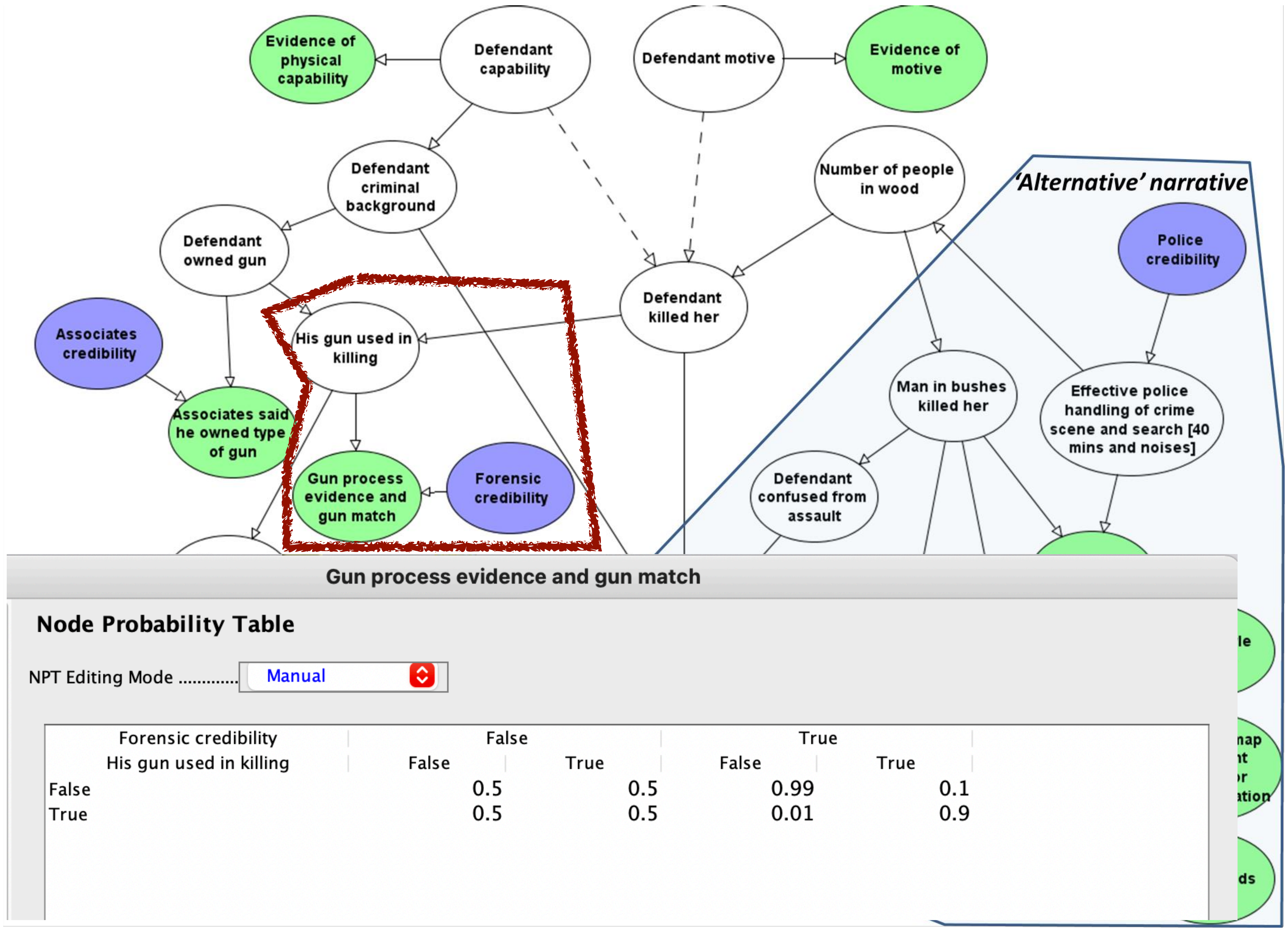
Weapon Discarded at Pump Station?



Passing witness credibility pump	False		True	
Weapon discarded at pump station	False	True	False	True
False	0.5	0.5	0.99	0.1
True	0.5	0.5	0.01	0.9

Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Gun Match Evidence



Forensic credibility	False		True	
His gun used in killing	False	True	False	True
False	0.5	0.5	0.99	0.1
True	0.5	0.5	0.01	0.9

Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Did the Defendant Kill the Victim?

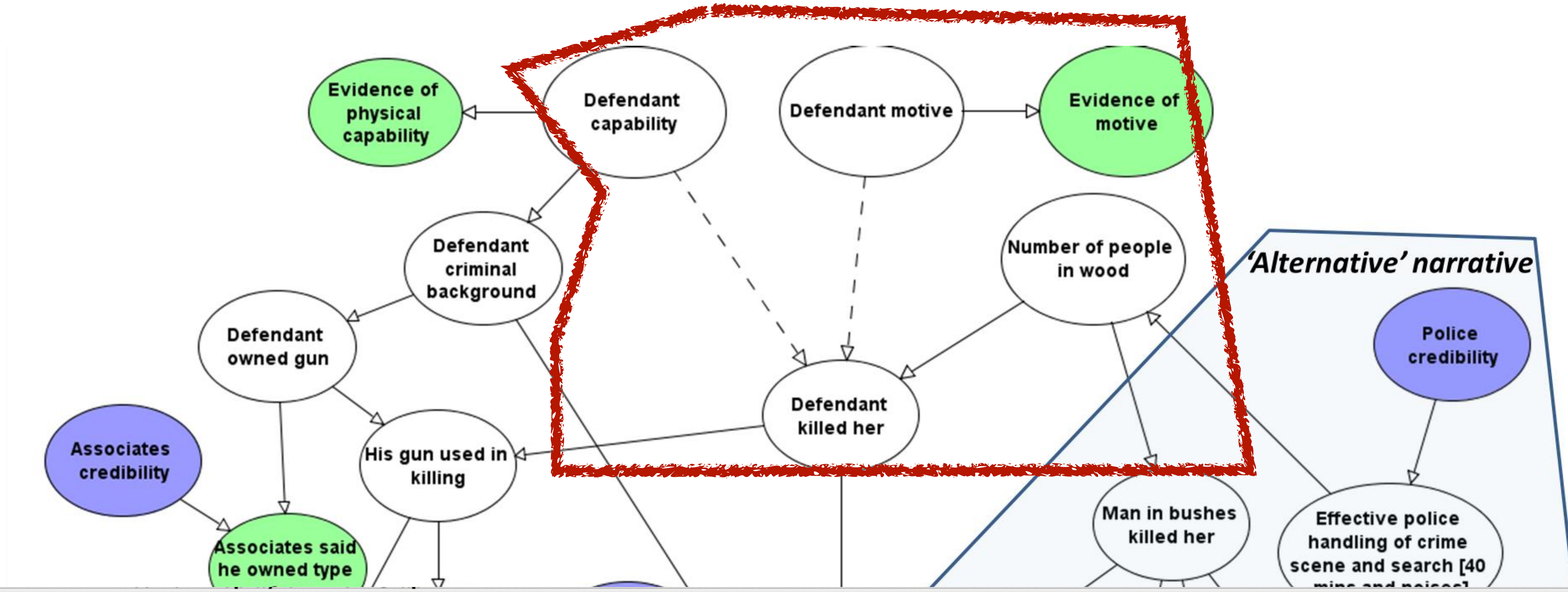


Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Defendant killed her

Node Probability Table

NPT Editing Mode .... Manual

Defendant motive and capability	False			True		
Number of people in wood	10	20	100	10	20	100
False	0.9	0.95	0.99	0.1	0.2	0.5
True	0.1	0.05	0.01	0.9	0.8	0.5

How Many People Were in the Woods?

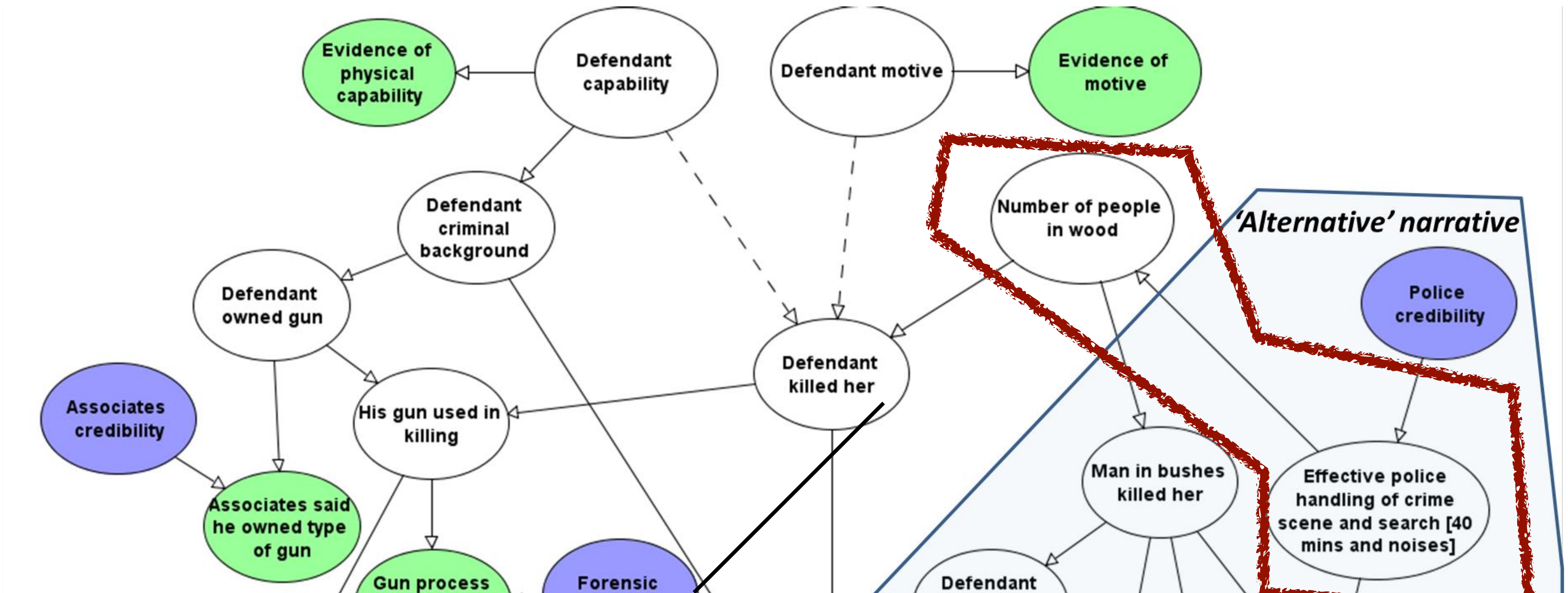


Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

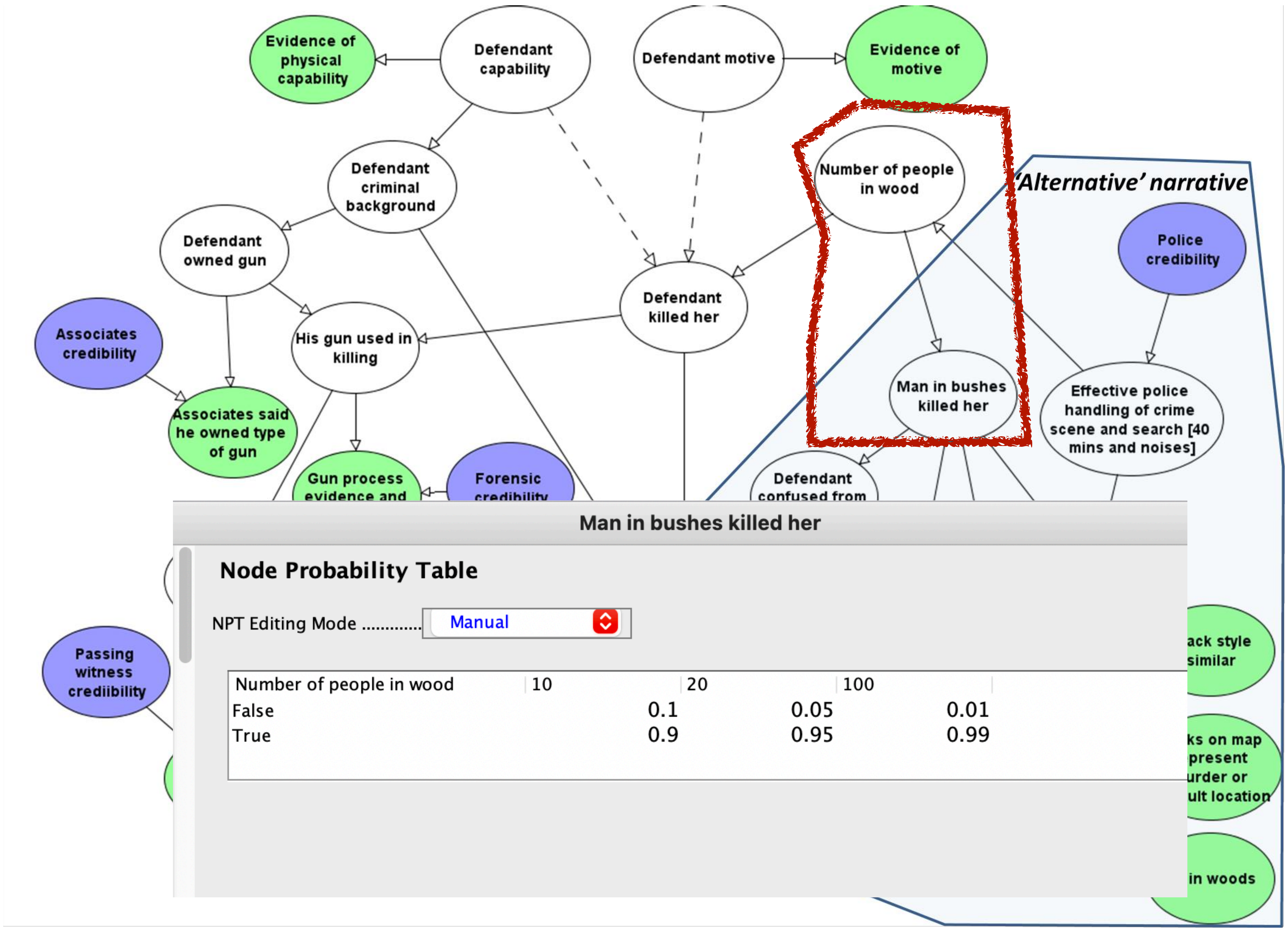
Number of people in wood

Node Probability Table

NPT Editing Mode .... Manual

Effective police handling of crime scene and search [40 mins and noises]	False	True
10	0.0625	0.9
20	0.3125	0.1
100	0.625	0.0

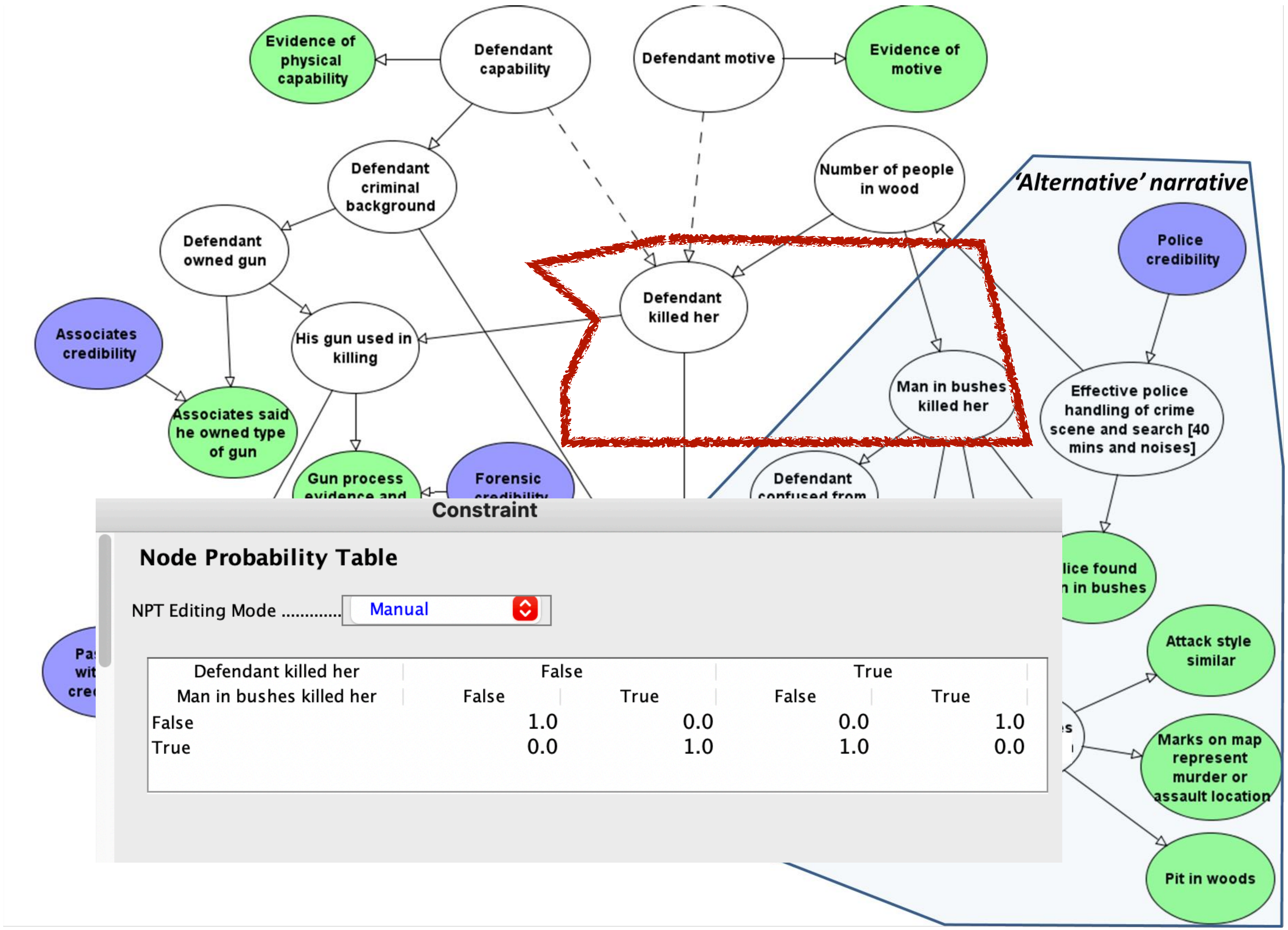
Did The Man in the Bushes Kill the Victim?



Number of people in wood	10	20	100
False	0.1	0.05	0.01
True	0.9	0.95	0.99

Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Incompatible Hypotheses



Defendant killed her	False		True	
Man in bushes killed her	False	True	False	True
False	1.0	0.0	0.0	1.0
True	0.0	1.0	1.0	0.0

Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Was *Perry Sultan* in the Woods?

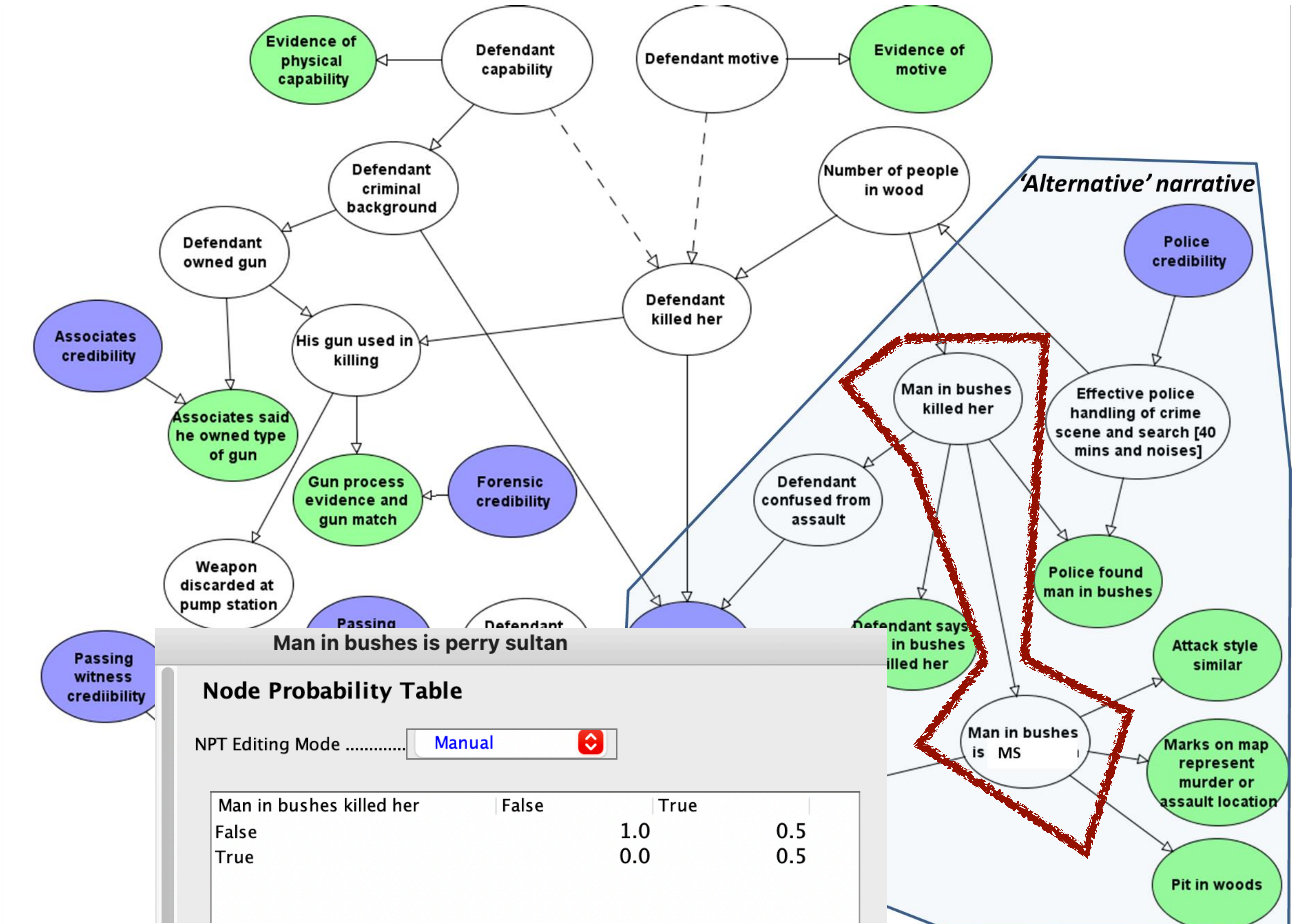


Figure 8 Full Simonshaven model, subdivided into the prosecution and alternative narratives

Defendant's credibility

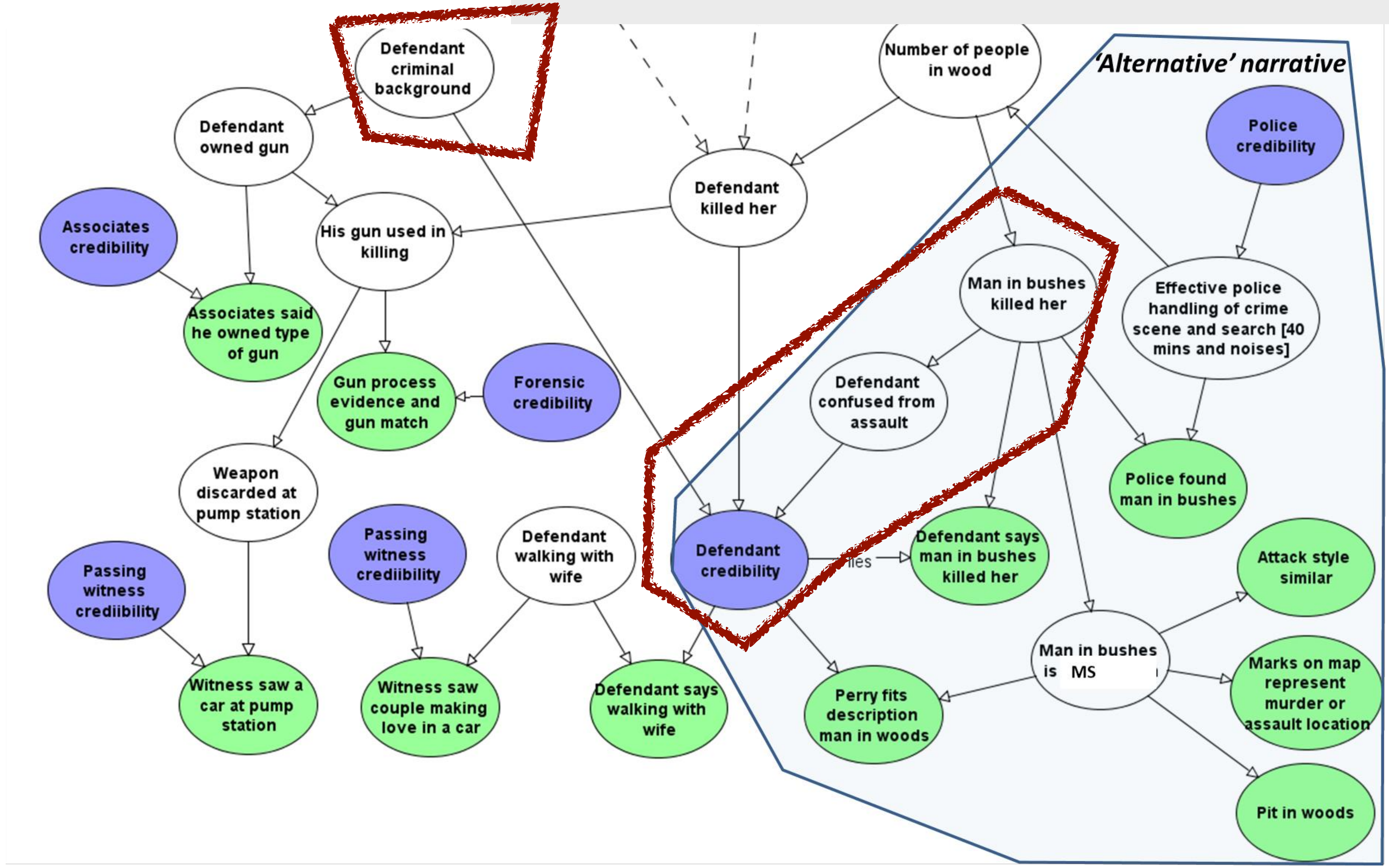
Defendant credibility

Node Probability Table

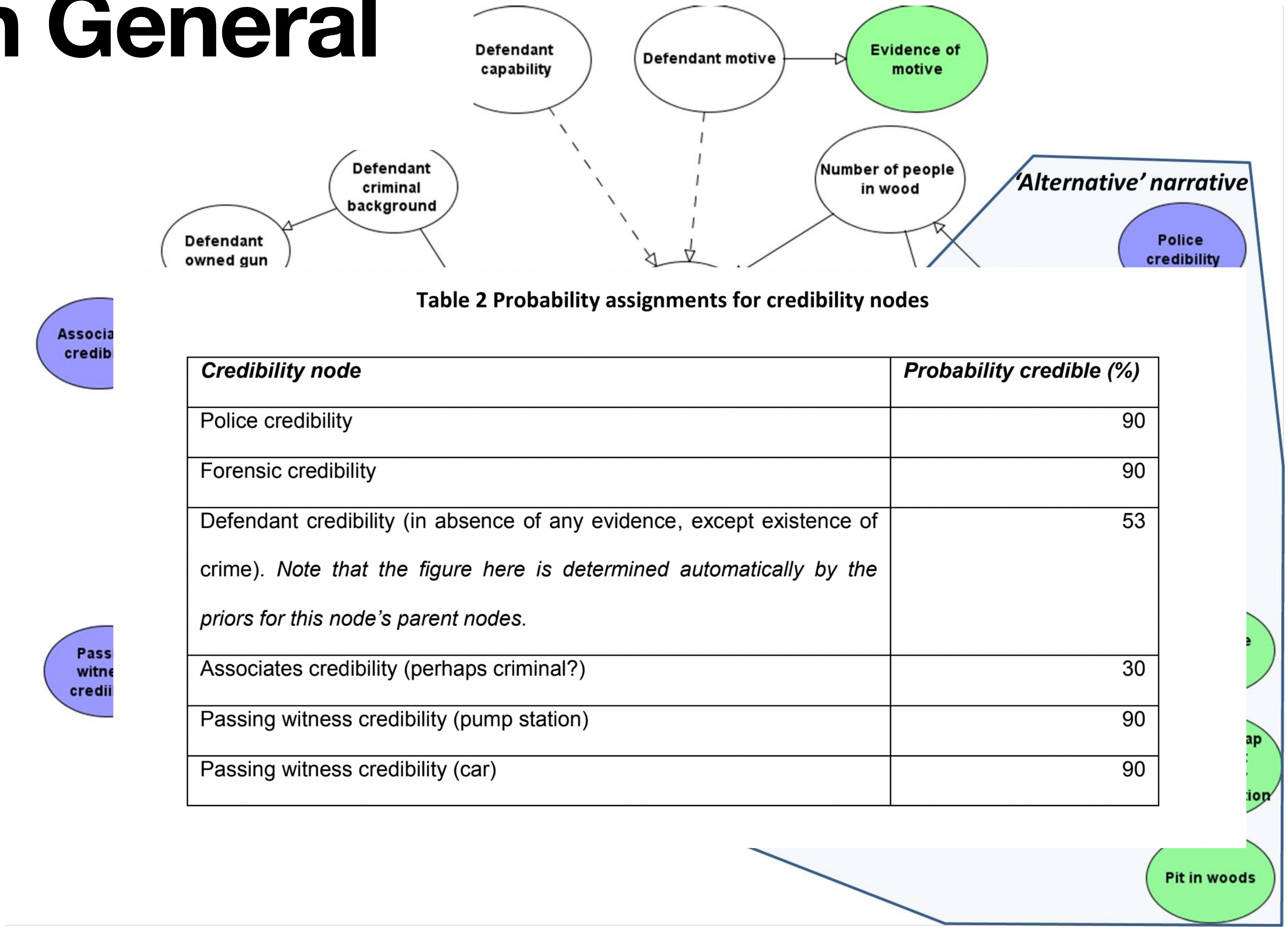
NPT Editing Mode .... Manual

Defendant killed her	False				True			
Defendant criminal background	False		True		False		True	
Defendant confused from assault	False	True	False	True	False	True	False	True
False	0.1	0.5	0.1	0.6	0.9	0.9	0.99	0.99
True	0.9	0.5	0.9	0.4	0.1	0.1	0.01	0.01

ull Simonshaven model, subdivided into the prosecution and alternative narratives



Credibility in General



ull Simonshaven model, subdivided into the prosecution and alternative narratives

Table 2 Probability assignments for credibility nodes

Credibility node	***Probability credible (%)***
Police credibility	90
Forensic credibility	90
Defendant credibility (in absence of any evidence, except existence of crime). *Note that the figure here is determined automatically by the priors for this node's parent nodes.*	53
Associates credibility (perhaps criminal?)	30
Passing witness credibility (pump station)	90
Passing witness credibility (car)	90

Changes in Probability as Evidence is Added

Table 3 Changes to probability of guilt, and defendant credibility, as evidence is entered in model (P refers to prosecution evidence and D to defence evidence)

Evidence (cumulative)	***Probability defendant guilty (%)*** *[rounded down]*	***Probability defendant credible*** *[rounded down]*
None	1	55
Evidence physical capability and Evidence of motive (P)	21	41
Associates said he owned type of gun + witness saw car at pump station (P)	53	25
Gun process evidence and gun match (P)	93	5
Witness saw couple making love on car (P) but defendant says walking with wife at time (D)	96	< 1
Police failed to find man in bushes and poor handling of crime scene (D)	80	2
Various bits of MS evidence {attack style, marks on map, pit in woods} and fact that defendant says man in bushes killed her (D)	46	6
MS does not fit suspect's description of the man in the woods (P)	74	4

Sensitivity Analysis: What If We Had Assigned Different Numbers?