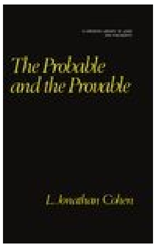

The Probable and The Provable

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CHAPTER

5 The Difficulty about Conjunction

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Abstract

In most civil cases, the plaintiff's contention consists of several component elements. So the multiplication law for the mathematical probability of a conjunction entails that, if the contention as a whole is to be established on the balance of mathematical probability, there must either be very few separate components in the case or most of them must be established at a very high level of probability. Since this constraint on the complexity of civil cases is unknown to the law, the mathematicist analysis is in grave difficulties here. To point out that such component elements in a complex case are rarely independent of one another is no help. Therefore, a mathematicist might claim that the balance of probability is not to be understood as the balance between the probability of the plaintiff's contention and that of its negation, but as the balance between the probability of the plaintiff's contention and that of some contrary contention by the defendant. However, this would misplace the burden of proof. To regard the balance of probability as the difference between prior and posterior probabilities is open to other objections. To claim that the plaintiff's contention as a whole is not to have its probability evaluated at all is like closing one's eyes to facts one does not like.

Keywords: mathematical probability, conjunction, plaintiff, mathematicist analysis, proof, multiplication law

Subject: Philosophy of Science, Epistemology, Metaphysics

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Summary

§17. In most civil cases the plaintiff's contention consists of several component elements. So the multiplication law for the mathematical probability of a conjunction entails that, if the contention as a whole is to be established on the balance of mathematical probability, there must either be very few separate components in the case or most of them must be established at a very high level of probability. Since this constraint on the complexity of civil cases is unknown to the law, the mathematicist analysis is in grave difficulties here. §18. To point out that such component elements in a complex case are rarely independent of one another is no help. §19. A mathematicist might therefore claim that the balance of probability is not to be understood as the balance between the probability of the plaintiff's contention and that of its negation, but as the balance between the probability of the plaintiff's contention and that of some contrary contention by the defendant. However, this would misplace the burden of proof. §20. To regard the balance of probability as the difference between prior and posterior probabilities is open to other objections. §21. To claim that the plaintiff's contention as a whole is not to have its probability evaluated at all is like closing one's eyes to facts one does not like.

§17. The constraint on complexity in civil cases

The rule for civil suits requires a plaintiff to prove each element of his case on the balance of probability. If this probability be construed as a *mathematical* probability, the conjunction principle for such probabilities would impose some curious constraints on the structure of the proof.

The most natural way to construe the requirement of a balance of mathematical probability is as a requirement that the probability of the plaintiff's case, on the facts before the court, be greater than the probability of the defendant's. Then, in accordance with the complementational negation p. 59 ↳ principle for mathematical probability, the probability of each of the plaintiff's factual contentions would have to be greater than ·5 in order to exceed the defendant's relevant probability. But what shall we say then about the probability of the plaintiff's case as a whole—about the probability of the conjunction of his various contentions? It too, presumably, should not fall below ·5, or there would be a balance or probability in favour of at least one of the plaintiff's contentions being false. Justice would hardly be done if a plaintiff were to win on a case that, when considered as a whole, was more probably false than true. Hence on the mathematicist interpretation the court would need to keep a close eye on the separate probabilities of those various contentions, in case they were not high enough to produce a greater than ·5 probability for the conjunction, when this is calculated in accordance with the standard multiplicational principle.

For example, if the case has two independent elements, at least one of the two component contentions must have a substantially higher probability than ·501. Perhaps a car driver is suing his insurance company because it refuses to compensate him after an accident. Suppose the two component issues that are disputed are first, what were the circumstances of the crash, and secondly, what were the terms of the driver's insurance contract. Then, if each of these two issues is determined with a probability of ·71, their joint outcome can be determined with a sufficiently high probability, since $\cdot 71^2$ is greater than ·501. But if one of the component issues is determined with only a ·501 probability, then the other component issue must be determined with a probability of very nearly 1. Otherwise the product of the two probabilities would not be high enough to satisfy the requirements of justice. Or, in other words, if one of the component issues is determined on the balance of probability (whether this balance be understood to lie at ·501, ·51, ·6 or even a higher figure), the other must, in effect, be determined beyond reasonable doubt. But though this constraint seems a necessary consequence of construing the standard in civil cases to require proof on a balance of mathematical probability, it seems to be a rule that is unknown to judges and unrespected by triers of fact.

p. 60 Another unfortunate consequence of applying the conjunction ↳ principle for mathematical probabilities in such a way is a severe constraint on the number of independent component issues in a single case. For example, if a series of independent points are all conjoined in a single allegation, and to establish each point a separate witness (or group of witnesses) is needed, then the higher the number of witnesses (or groups of witnesses) that is needed, the more reliable each witness (or group of witnesses) has to be. So if no witness (or group of witnesses) were ever to have a more than ·9 probability of speaking the truth, no case could ever deserve to win which conjoined more than six component points that were mutually independent and each required a separate witness (or group of witnesses)—since $\cdot 9^7$ is less than ·5. An even lower limit could be imposed on the practicable number of such component issues if the balance of probability was thought to involve a mathematical probability substantially higher than ·501. For example, not more than three such issues would be practicable if the plaintiff had to achieve a level of ·7 in order to win. And a ·7 probability seems scarcely too high a level for the determination of civil issues affecting a man's fortune or reputation, or the conduct of great commerical enterprises. But, if the conception of juridical probabilities as mathematical ones were to force the court to refuse justice in cases involving highly complex issues of fact, that conception would be seriously inexpedient.

No doubt the defendant will often accept several of the plaintiff's component points. If he concentrates his effort on refuting just one or two of the elements in the plaintiff's case, he may well calculate that in practice he will have a better chance of persuading the jury to give judgement against the plaintiff than if he refuses to admit anything and tries laboriously to demolish each of the plaintiff's points in turn. Hence in many complex cases the difficulty about compounding mathematical probabilities would not in fact emerge. The points accepted by the defendant there could each be assigned a mathematical probability of 1, and all that is necessary is that the disputed point or points should compound to a figure higher than ·501, or ·7, or wherever the threshold of balance is conceived to lie. This policy will also minimize the costs that the defendant might have to pay if he loses. Nevertheless it is in principle always open to the defendant to
p. 61 contest each of ↳ the component points in the plaintiff's case, and sometimes it may in practice be in his interest to do so, especially when the trier of fact is a judge sitting without a jury. An insurance company, for example, may wish to fight the plaintiff's interpretation of his contract of insurance, for fear of similar liabilities in other cases. But it may also wish to fight the plaintiff's version of the circumstances of his accident, since the chance of success may be greater; and perhaps, for a plausibly different account of the accident to be shown possible, it may be necessary to disprove several of the plaintiff's allegations. So the difficulties latent in the mathematicist analysis would then become operative.

§18. The independence issue

Four possible ways of trying to circumvent these difficulties will be considered in §§18–21.

One way is to argue that the mathematical probabilities of component points in a civil case are rarely independent. For example, what was actually done by the parties to a contract may well be relevant to determining the terms of the contract. Hence the principle that normally operates here is not the one that is valid only for independent probabilities, viz.

> $p_M[B \& C,A] = p_M[B,A] \times p_M[C,A]$.

Instead it is the more general principle that is valid also for dependent probabilities, viz.

> $p_M[B \& C,A] = p_M[B,A] \times p_M[C,A \& B]$.

Consequently, it may be argued, the mathematical probability of the conjoint outcome need not be much lower than that of each component outcome, since we may suppose $p_M[S,Q \& R]$ to be substantially greater than $p_M[S,Q,]$ where R and S are two component points in the plaintiff's case, and Q the total evidence.

But the trouble with this argument is that quite often the margin of inequality between $p_M[S,Q]$ and $p_M[S,Q \& R]$ in such a case is very slight, or $p_M[S,Q \& R]$ is even less, not greater, than $p_M[S,Q]$. For example, suppose the component issues of a suit for non-performance of contract are the terms of the contract and the actual performance of the defendant. If the actual performance of the defendant constitutes a premiss
p. 62 that is relevant to inferring the terms of the contract then the ↳ plaintiff's allegation of a discrepancy between terms and performance will be harder to prove than if the defendant's actual performance were irrelevant to inferring the terms. More specifically, suppose that the plaintiff in his proof of the terms of contract has to prove both the place where the defendant was to build the plaintiff a house and also the date by which the building was to be completed, and that he also has to prove that no house had been completed at that place by that date. Suppose none of the three probabilities is to be regarded as independent of the others, and the plaintiff proves the component issues of his case so effectively that the probabilities to be multiplied together are ·8, ·8, and ·75. The mathematicist account seems to lead inevitably to the paradoxical conclusion that he should lose his case, since ·8 × ·8 × ·75 is less than ·5.

§19. Does the balance of probability lie between the plaintiff's and the defendant's contentions?

A second way of trying to rescue the mathematicist theory here is to argue that the above-mentioned difficulties arise because the phrase 'the balance of probability' is wrongly construed as denoting the balance between the probability of a certain proposition (or event) and the probability of its negation (or non-occurrence). This construction ensures (since the mathematical probability of not-S is always 1—s when the mathematical probability of S is s) that nothing can be established on the balance of probability if its own probability is less than or equal to ·5. And that in turn severely restricts the extent to which the probabilities of component conclusions can be compounded by multiplication. Therefore, it might be argued, we should not construe the phrase 'the balance of probability' as denoting the balance between the probability of S and the probability of not-S, on Q, where one party to the case asserts S and his opponent asserts not-S, and Q states the facts before the court, but rather as denoting the balance between the probability of S and the probability of, say R, where one party asserts S, the other asserts R, and S and R, though mutually inconsistent, do not exhaust the domain of possibilities. For example, the plaintiff might claim that the defendant was the driver of a car that collided with his own car at 2 a.m. on 20 October 1971,
p. 63 and the defendant might claim that he ↳ was at home in bed at that time on that day (when he might have claimed instead that he was at a party, or that he was working late, or that he was abroad, and so on). It would follow that S might be established 'on the balance of probability' if S was shown to have a probability of, say, ·2 on Q, and R a probability of ·1. A plaintiff could then establish each of any number of independent component points on the balance of probability, in this sense, and the conjunction principle for mathematical probability would still allow him to have established the conjunction of his component claims on the balance of probability, in the same sense. For, where each s_i and r_i are real numbers, if $s_1 \rangle r_1, s_2 \rangle r_2, \ldots, s_n \rangle r_n$, then $(s_1 \times s_2 \times \ldots \times s_n) \rangle (r_1 \times r_2 \times \ldots \times r_n)$.

This way out of the difficulties might fit some kinds of case, such as those that involve more than two parties or where the court is asked for some kind of declaration. But it scarcely fits the standard type of two-party civil case, where the defendant wins if he disproves the plaintiff's allegation. To suppose it fits these cases would be to suppose that the defendant is always required there not merely to counter the plaintiff's allegation, however he may do this, but also to establish some positive claim of his own. Such a supposition introduces a general category of *onus probandi* that does not at present exist, and belongs to a system of law based on inquisitorial objectives rather than to one based on the adversary procedure. Even in the previous paragraph's example the actual issue before the court would be whether or not the defendant was the driver of the car involved in the collision, not whether the defendant was the driver of the car or in bed at the time. Also, if there is direct evidence of a fact alleged by the plaintiff, the case may stand or fall with the reliability of the plaintiff's witness. The issue is then a straightforwardly dichotomous one, between reliability and non-reliability. Or an allegation by the plaintiff may itself be negative in form, e.g. that the defendant never paid him his wages, and then the issue between plaintiff and defendant must again be assigned a straightforward dichotomy of outcomes: were the wages paid or were they not? Similarly, in a suit for libel, one issue may be the truth or falsity of the proposition alleged to be libellous. But all the plaintiff has to do in that connection is to establish the falsehood of this ↳ proposition. If he sets out to establish the truth of some other proposition inconsistent with the alleged libel, it is as a means of establishing the falsehood of the libel, not as an end in itself.

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§20. Does the balance of probability consist in the difference between prior and posterior probabilities?

Thirdly, yet another interpretation for the phrase 'proof on the balance of probability' might be suggested. Perhaps this means not that the probability of the desired conclusion should be greater than that of its negation, nor yet that the probability of the winning party's contention should be greater than that of some contrary contention, but that the facts should be favourably relevant to the desired conclusion. That is, the probability of the desired conclusion on the facts before the court should be greater than the prior probability of that conclusion (rather than equal to, or less than, this). In short, perhaps the requirement is that $p_M[S,Q] > p_M[S]$. The apparent advantage of this interpretation is that we avoid the previous difficulty about burden of proof, and yet, however many independent component points S_l, S_2, ..., S_n we have, the probability of the conjunction S_1 & S_2 & ... & S_n on the facts before the court is always greater than the prior probability of this conjunction if the probability-on-the-facts of each of the component points is greater than its prior probability.

Such an interpretation may in practice even work as well for non-independent outcomes as for independent ones. Nevertheless the interpretation is scarcely tenable. The trouble is that in certain circumstances it allows a plaintiff to prove his over-all case on the balance of probability even if he fails to establish one or more of his component points on its own. For example, suppose the plaintiff has to establish four independent points S_1, S_2, S_3, and S_4. Suppose the prior probabilities of each of these is ·5, and each of S_1, S_2, and S_3 has a ·9 probability on the facts, while S_4 has a ·4 probability on the facts. In these circumstances $p_M[S_1 \& S_2 \& S_3 \& S_4, Q] > p_M[S_1 \& S_2 \& S_3 \& S_4]$, even though $p_M[S_4,Q] < p_M[S_4]$. But the courts would not normally allow a plaintiff to win unless he had established each of his component points on the balance of probability. The latter is a necessary condition for victory as well as a sufficient one. When the necessity of this condition is borne in mind, the ↳ proposed interpretation can easily make the plaintiff's case appear juster than the courts will allow,[1] rather as, when we bear in mind the sufficiency of the condition, the standard mathematicist interpretation—in the way that we have already seen—makes victory often seem less just than they allow.

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Perhaps this particular difficulty in the proposed interpretation could be obviated by supposing some appropriate legal requirement. 'It's a matter of law, not common sense', we may be told, 'that each component point should be established on the balance of probability, quite apart from any need there may be to establish the over-all case.' But such a rule of law requires a rationale, and it is difficult to see where this is to come from if not from an impossibility of ever proving the over-all case on the balance of probability without so proving each component element.

Moreover there is a further difficulty that cannot be obviated in such a way. The proposed interpretation assumes that some positive prior probability is uncontroversially assignable to any contention that comes before the court. But why should this be so? If the level of a probability affects the issue of litigation, justice requires that both parties should have the opportunity to lead evidence relevant to its determination. The very idea of a distinction between prior probabilities that cannot be argued in court, and posterior ones that can, seems to reek of procedural injustice.[2]

§21. Does the plaintiff's contention need evaluation as a whole?

There is also a fourth way of trying to avoid the difficulties generated by the. multiplicative nature of the conjunction principle for mathematical probability. If the rule about balance of probability cannot be
p. 66 reinterpreted so as to escape the ↳ difficulties that confront it, perhaps we should seek to achieve the same end by restricting the rule's range of application. It might be argued that these difficulties arise only if we suppose that in a civil suit the case *as a whole* must win on the balance of mathematical probability. The difficulties all stem, it might be said, from the attempt to compound together into a single over-all figure the various different probabilities relating to disputed component issues. So there is an obvious way to avoid the difficulties while still supposing that the probabilities established by juridical proof are mathematical ones. Where more than one issue of fact is disputed in a case, we seem to be out of trouble if we apply the rule about balance of probability only to each component issue and not to the case as a whole. The plaintiff wins, we might say, if and only if he establishes each component point on the balance of mathematical probability. Hence there is no point in compounding the separate probabilities together, and then the multiplicativeness of the conjunction principle for mathematical probability does not generate any constraints on the practicable number of component issues or on the levels of probability at which these may be resolved.

However, for a mathematicist to evade the difficulty by claiming that the outcome of a complex civil case should not be evaluated as a whole, is rather like closing one's eyes in order to pretend that what one does not see does not exist. For, if nevertheless such an outcome were to be evaluated as a whole in accordance with the principles of mathematical probability, the result that would emerge in very many cases would be the opposite of what the courts themselves would declare. Even though the probabilities of three independent components in a plaintiff's case were, say, ·8, ·8, and ·75, a mathematicist evaluation would give the case as a whole to the defendant, not to the plaintiff. What kind of justice would it be to disregard this fact, if mathematical probability was really what was at stake?[3]

p. 67 The same point emerges particularly starkly in regard to ↳ criminal cases. Presumably on the mathematicist interpretation proof beyond reasonable doubt is proof at a level of probability that is not more than some very small interval short of certainty. But even if each element of the alleged crime is proved at this level—e.g. if it is proved that the accused's finger pressed the trigger, that the victim died as a result of the shot, and that the accused intended such a result—the conjunction of the elements may still not be proved thereby at the appropriate level. What kind of justice would it be to execute a man, or send him to a long term of imprisonment, if the crime as a whole had not been proved to be his responsibility ?

Notes

1 The same difficulty arises for the previous interpretation of 'proof on the balance of probabilities'. If a series of positive contentions by the defendant are to counter the series of contentions by the plaintiff, the plaintiff's over-all case might have a higher probability than the defendant's even if at least one of the plaintiff's component contentions had a lower probability than the defendant's corresponding counter-contention. This kind of difficulty is also rather crippling for any proposal to take favourable relevance as a criterion of confirmation in natural science, as in M. Hesse, *The Structure of*

Scientific Inference (1974), p. 134. Some evidence is allowed to confirm the conjunction of two theories even though it disconfirms one or both: cf. R. Carnap, *Logical Foundations of Probability* (1950), pp. 391 ff.

2 Cf. also §36 (pp. 107 ff.) below, where this issue is discussed at greater length.

3 Much the same question can be raised in reply to those philosophers of science who, like I. Levi, use an analogous argument in defence of a rule permitting acceptance, or belief in the truth, of any proposition exceeding a certain threshold of mathematical probability on the evidence: cf. §88 (pp. 316 ff.) below.