MATH ON TRIAL – COLLINS CASE

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1. COLLINS – MAIN OPINION

Five objections to the introduction of statistical evidence:
(a) statistical estimates were mere guesses;
(b) appeal to independence for application of product rule was unjustified;
(c) the couple involved in the crime might not have had the distinctive features F;
(d) distinctive features F might not be unique;
(e) numbers might have distracted and confused the jury.

Suppose objections (a) and (b) are addressed. Consider this argument from the prosecutor
(step 1) Distinctive features F have a probability of 1 in 12 million in California couples.
(step 2) An innocent couple could have features F with a 1 in 12 million probability.
(step 3) Collins who have features F are innocent with a 1 in 12 million probability.
(step 4) Collins are guilty with a high probability (the complement of 1 in 12 million).
What is wrong with this argument? Which step is problematic?

QUESTION: What proposition, exactly, can be assigned a probability of 1 in 12 million? Is it the probability of innocence? If not, how do we use the figure 1 in 12 million to arrive at the probability of guilt/innocence?

A suggestion is contained in the mathematical appendix in *Collins* (see below).

2. COLLINS – MATHEMATICAL APPENDIX

The Court's calculations rely on the *binomial distribution* formula. To illustrate, let p be the probability of event E, on the supposition that the probability of E does not change as we iterate our experiment (e.g. the probability of getting heads does not change as we keep tossing our coin). The binomial distribution formula allows us to calculate the probability of getting a k number of occurrences of E over n iterations, with event E having a probability of p (e.g. the binomial distribution allows us to calculate the probability of getting a k number of heads over an n number of coin tosses, with the event 'heads' having a probability of one half), as follows:

$$\binom{n}{k} p^k \, (1-p)^{n-k}\,.$$

The Court was interested in the probability that there are a k number of couples with features F among California couples (assuming F has probability 1 in 12 million; so objections (a) and (b) above are set aside here). In particular, he Court was interested in the probability that *exactly one couple* among California couples had features F, so this is the case $k = 1$. By putting $k = 1$, the binomial distribution formula becomes:

$np\,(1-p)^{n-1}$.

QUESTION: How did the Court use the binomial distribution formula to arrive at the 0.4 probability? Did the Court use the correct formula here? Is possessing features F among California couples like getting heads on a series of coin tosses? Why not?

3. COLLINS AND BAYES' THEOREM

Let us stipulate that
(a) the guilty couple, in fact, fits the description D (blond, ponytail, mustache, etc.);
(b) the Collins match description D; and
(c) D has a frequency of 1 in 12,000,000.
Let M stand for *the Collins match the description D* and let G stand for *the Collins are guilty*. Bayes' theorem tells us that

$$P(G|M) = \frac{P(M|G)}{P(M)}P(G) = \frac{P(M|G)}{P(M|G)P(G) + P(M|\neg G)P(\neg G)}P(G).$$

We can assume—simplifying a bit!—that
$P(G) = \frac{1}{n}$, with n the population of, say, Los Angeles and vicinities (maybe 6 million people?);
$P(M|G) = 1$; and
$P(M|\neg G) = \frac{1}{12{,}000{,}000}$.
So we have

$$P(G|M) = \frac{1}{\frac{1}{n} + \frac{1}{12{,}000{,}000} \times \frac{n-1}{n}} \times \frac{1}{n} = \frac{1}{1 + \frac{1}{12{,}000{,}000} \times (n-1)}.$$

With $n = 6,000,000$, we get

$$P(G|M) \approx \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}.$$