PHI 169 - CRITICAL REASONING - PROBABILITY EXAM #1 SOLUTIONS

- (a) By looking up on Google, one finds:
 - total US population in 1974 was 213.9 million
 - total US population in 2001 was 285 million

By looking at the statistics from FBJS, one finds:

- US population incarcerated in 1974 was 1.819 million
- US population incarcerated in 2001 was 5.618 million

The required probabilities can now be calculated, keeping in mind that $P = (....) = \frac{PART}{WHOLE}$

- in 1974 $P(I) = \frac{1.819}{213.9} = 0.0085039731 \approx 0.8\%$
- in 2001 $P(I) = \frac{5.618}{285} = 0.0197122807 \approx 2\%$

The probability increased by a factor of $\frac{0.02}{0.008} = 2.5$

- (b) The rule is $P(A|B) = \frac{\# \text{ elements in } A \cap B}{\# \text{ elements in } B}$. Our B is I ("a person, male or female, is incarcerated"). Similarly, A&B in this case is $WM \cap I$. So, we have
 - $P(WM|I) = \frac{\text{\# elements in } WM \cap I}{\text{\# elements in } I}$

We know from the FBJS that in 2001 the set I counts 5.618 million people and set $WM \cap I$ counts 1.978 million. So $P(WM|I) = \frac{\# \text{ elements in } WM \cap I}{\# \text{ elements in } I} = \frac{1.978}{5.618} \approx 0.35 \approx 35\%$.

- (c) Follow the same procedure but replace WM with BM. From the FBJS we know that in 2001 black males incarcerated were 1.936 million. So, $P(BM|I) = \frac{1.936}{5.618} \approx 0.34 \approx 34\%$.
- (d) The person is more likely to be white male even though by a slight margin. By the calculations in (b) and (c), we discovered that $P(BM|I) \approx 34\%$ and $P(WM|I) \approx 35\%$.
- (e) The claim about racial disproportion is P(I|BM) > P(I|WI), although P(BM|I) < P(WM|I). Consider US Census from 2000. The US population was 281,421,906 and blacks make up 12%, that is, approximately 34 million. Half should be black males, that is, approximately 17 million. So, $P(I|BM) = \frac{\# \text{ elements in } I \cap BM}{\# \text{ elements in } BM} = \frac{1.936}{17} = 0.11388235294 \approx 11\%$

Non-hispanic white in 2000 were roughly roughy 70% of the US population, that is, approximately, 195 million. Half should be white males, that is, approximately 97.5 million.

So,
$$P(I|WM)=\frac{\text{\# elements in }WM\cap I}{\text{\# elements in }WM}=\frac{1.978}{97.5}=0.02028717948\approx 2\%$$

Black males were more likely to be incarcerated than white males by a factor of $\frac{11}{2} = 5.5$. This is what is meant by racial disproportion among incarcerated people.