

# Admirable Consequence

(*consequentia mirabilis*)

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Consider the proposition  
“I exist”

Let's assume for the sake of  
argument that its negation  
holds, i.e. “I do not exist.”

If I do not exist, in order to  
entertain the proposition “I  
do not exist” I need to  
exist, whence “I exist.”

$$(\neg\phi \rightarrow \phi) \rightarrow \phi$$



*Descartes (sort of...)*

# Establishing $\vdash (\neg\phi \rightarrow \phi) \rightarrow \phi$

$[\neg\phi \rightarrow \phi]^1 \quad [\neg\phi]^2$

$\rightarrow \mathbf{E}$

$\phi \quad [\neg\phi]^2$

$\rightarrow \mathbf{E}$

$\perp$

$\mathbf{RAA}^2$

$\phi$

$\rightarrow \mathbf{I}^1$

$(\neg\phi \rightarrow \phi) \rightarrow \phi$

The derivation of  $(\neg\phi \rightarrow \phi) \rightarrow \phi$   
crucially rests upon **RAA**