The Semantic and the Syntactic Perspective

✤ <u>Logical Consequence</u>

$$\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$$

iff

all valuations *V*'s that make $\phi_1, \phi_2, \ldots, \phi_k$ true also make ψ true

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✤ <u>Derivability</u>

$$\phi_1, \phi_2, \ldots, \phi_k \vdash \psi$$

iff

there is a derivation whose assumptions are $\phi_1, \phi_2, \ldots, \phi_k$ and whose conclusion is ψ

What Does a Derivation Look Like?

$$\cfrac{\cfrac{\cfrac{\cfrac{[\varphi]^1}{\varphi \vee \neg\varphi} \vee I \quad [\neg(\varphi \vee \neg\varphi)]^2}{\cfrac{\bot}{\neg\varphi} \to I^1}}{\varphi \vee \neg\varphi} \vee I \quad [\neg(\varphi \vee \neg\varphi)]^2}{\cfrac{\bot}{\varphi \vee \neg\varphi} RAA^2} \to E$$