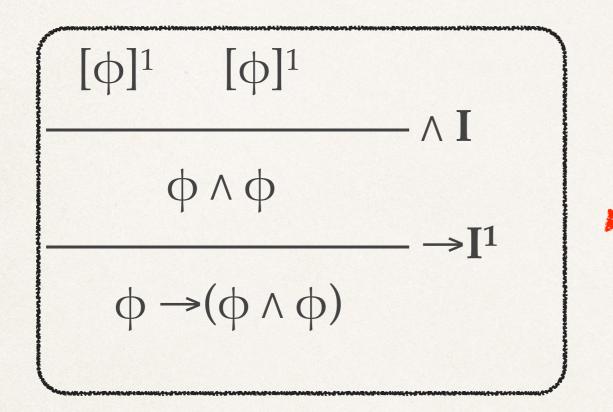# Two Somewhat Trivial Claims

Idempotency $\vdash \phi \rightarrow (\phi \wedge \phi)$ **and** $\vdash (\phi \wedge \phi) \rightarrow \phi$

Commutativity $\vdash (\phi \wedge \psi) \rightarrow (\psi \wedge \phi)$ **and** $\vdash (\psi \wedge \phi) \rightarrow (\phi \wedge \psi)$

Establishing $\vdash \phi \rightarrow (\phi \wedge \phi)$

$$\frac{\dfrac{[\phi]^1 \qquad [\phi]^1}{\phi \wedge \phi} \wedge \mathbf{I}}{\phi \rightarrow (\phi \wedge \phi)} \rightarrow \mathbf{I}^1$$

The same assumption ϕ is used twice, although both instances are cancelled at once by one application of $\rightarrow \mathbf{I}$

Establishing $\vdash (\phi \wedge \psi) \rightarrow (\psi \wedge \phi)$

$$\cfrac{\cfrac{[\phi \wedge \psi]^1}{\psi} \wedge E \qquad \cfrac{[\phi \wedge \psi]^1}{\phi} \wedge E}{\cfrac{\psi \wedge \phi}{(\phi \wedge \psi) \rightarrow (\psi \wedge \phi)} \rightarrow I^1} \wedge I$$