INTRODUCTION TO LOGIC - HOMEWORK #3 - **Due Feb 18**

1 IF....THEN [20 POINTS]

You have learned that statements of the form $\varphi \to \psi$ are (vacuously) true whenever φ is false. But this might be different from the ordinary meaning we attribute to statements of the form *if...then*. Collect two examples of natural language statements of the form *if...then* which do not (seem to) conform to the material conditional $\to$.[1] Explain your answers.

2 TRUTH [40 POINTS]

Suppose $V_1(p) = 1$, $V_1(q) = 0$, $V_2(p) = 0$ and $V_2(q) = 0$. Check whether the following hold:

1. $V_1 \models (\neg p) \vee q$
2. $V_1 \models \neg(p \vee q)$
3. $V_2 \models (\neg p) \wedge (\neg q)$
4. $V_2 \models \neg(p \wedge (\neg q))$
5. $V_1 \models p \to q$
6. $V_2 \models (p \to q) \to p$

Use the truth table method.

3 ODD WAYS TO SAY SIMPLE THINGS [40 POINTS]

Consider a propositional language $L_\perp$ whose ingredients are the connective $\to$, the symbol $\perp$ and the atomic propositions $p, q, r\dots$. Well-formed formulas of $L_\perp$ are defined as follows:
(i) any atomic proposition $p, q, r\dots$ is a well-formed formula of $L_\perp$;
(ii) $\perp$ is a well-formed formula of $L_\perp$;
(iii) if φ and ψ are well-formed formulas of $L_\perp$, then $\varphi \to \psi$ is a well-formed formula of $L_\perp$;
(iv) nothing else is a well-formed formula of of $L_\perp$.
While atomic propositions $p, q, r\dots$ can be assigned 1 or 0, the well-formed formula $\perp$ can only be assigned 0. Now answer the following:

> Suppose that instead of saying $\neg\varphi$, we say $\varphi \to \perp$ in language $L_\perp$. Does this make sense? To find out, compare the truth table for $\neg\varphi$ and the truth table for $\varphi \to \perp$. Are they the same or not? What does this tell us? [Hint: the truth-table for $\varphi \to \perp$ has only two rows.]

[1] Here is an example: *If the US economy collapses, the world economy collapses.* Now, as of now the US economy has not (yet) collapsed, so the antecedent of this *if...then* statement is false. But the mere fact that the antecedent is false does not make the entire statement true in so far as ordinary language is concerned. Since the statement in question does not seem (vacuously) true, then we may conclude that the statement does not contain a material conditional, but some other connective.