

**INTRODUCTION TO LOGIC - HOMEWORK #4 - Due March 3**

**1 TAUTOLOGY, CONTRADICTION OR NEITHER? [30 POINTS]**

Use truth tables to check whether the following are tautologies, contradictions or neither:

1.  $p \vee \neg p$
2.  $\neg(p \vee \neg p) \rightarrow q$
3.  $\neg(\neg p \wedge q) \rightarrow q$
4.  $\neg(p \vee \neg q) \rightarrow (\neg q \vee q)$

**2 BIVALENCE SURRENDERED [30 POINTS]**

We have seen that the Principle of Non-Contradiction  $\neg(\varphi \wedge \neg\varphi)$  and the Principle of Excluded Middle  $(\varphi \vee \neg\varphi)$  are valid provided we maintain the Principle of Bivalence. What happens if we do away with it? Let's suppose that formulas can be assigned three truth values, namely 1, 0, and 0.5. And let's suppose that negation, conjunction and disjunction behave as follows:

$\neg$	$\varphi$
0	1
1	0
0.5	0.5

$\varphi$	$\wedge$	$\psi$
1	1	1
1	0.5	0.5
1	0	0
0.5	0.5	1
0.5	0.5	0.5
0.5	0	0
0	0	1
0	0	0.5
0	0	0

$\varphi$	$\vee$	$\psi$
1	1	1
1	1	0.5
1	1	0
0.5	1	1
0.5	0.5	0.5
0.5	0.5	0
0	1	1
0	0.5	0.5
0	0	0

Use the truth table method to check whether or not the Principle of Non-Contradiction and the Principle of Excluded Middle are still valid when there are three truth values.

**3 LOGICAL CONSEQUENCE [40 POINTS]**

Please give an example in English of the following argument patterns, and then check whether they are valid argument patterns:

(a)  $p \vee q$   
 $\neg q$   
 \_\_\_\_\_  
 $p$

(b)  $(p \vee q) \rightarrow r$   
 $p$   
 \_\_\_\_\_  
 $r$

Motivate your answers. [Hint: Use truth tables to check validity]