INTRODUCTION TO LOGIC - HOMEWORK #8 - Due May 7

This assignment is similar in structure to the final. There are ten questions in total plus five questions for extra credit. This assignment and the final exam are graded on a 100pt scale.

PART 1: TERMINOLOGY [30PT]

1. What is *modus ponens*? What is *modus tollens*? Give examples in English.
2. Complete the following:
   (a) $V \models \psi$ means that ...
   (b) $\models \psi$ means that ...
3. Explain the derivation rule $\rightarrow I$ and the derivation rule $\rightarrow E$. What is the difference?

PART 2: DOING THINGS WITH LOGIC[40PT]

4. This consists of three parts. First, give an example of a formula that is always true and show semantically (i.e. using truth tables) that the formula you've given is, in fact, always true. Second, give an example of a formula that is always false and show semantically (i.e. using truth tables) that the formula you've given is, in fact, always false. Finally, give an example of a formula that is sometimes (but not always) true and show semantically (i.e. using truth tables) that the formula you've given is, in fact, sometimes (but not always) true.
5. How do you check the validity of an argument using the truth table method? Please be as precise as possible. Give an example.
6. Construct a derivation of $((\varphi \wedge \psi) \rightarrow \sigma) \rightarrow (\psi \rightarrow (\varphi \rightarrow \sigma))$
7. This consists of two parts. First, construct a derivation of the principle of non-contradiction, and second, show semantically (i.e. using truth tables) that the principle of non-contradiction is a tautology.

PART 3: THINKING ABOUT LOGIC [30PT]

8. Explain why truth-tables give us the meaning of the connectives. Give an illustration using a connective of your choice.
9. Explain why the semantics of propositional logic (as defined in this course) assumes the principle of bivalence. Given bivalence, show why it is (not) possible that

$$V(p) = 1 + V(\neg p)$$

   What happens if we drop the principle of bivalence?
10. What is the Wason's selection task and what does it tell us about the connective $\rightarrow$?

PART 4: SOMETHING EXTRA [50PT]

11. Suppose $V(\varphi) = 1 - V(\psi)$. Determine the truth value of $(\varphi \wedge \psi)$.
12. Suppose $V(\varphi) \geq V(\psi)$. Determine the truth value of $\neg(\varphi \vee \neg\psi)$.
13. Construct a derivation of $((\varphi \rightarrow \psi) \wedge \neg\psi) \rightarrow \neg\varphi$
14. Construct a derivation of $\varphi \rightarrow \neg\neg\varphi$
15. Construct a derivation of $\neg\neg\neg\varphi \rightarrow \neg\varphi$