Please write your answer as clearly, meticulously and neatly as possible. Make every step in your reasoning fully explicit. Answers lacking in clarity and precision will lose points.

PART 1: Terminology [30pt]

1. What are the two functions of logic? Illustrate with examples.

2. Complete the following:

   (a) $\varphi \models \psi$ means that ...
   (b) $\vdash \psi$ means that ...

3. Explain and justify the notational convention N.C. about negated formulas.

PART 2: Doing Things With Logic[40pt]

4. Find a formula of your choice that is always true (i.e. a tautology) and for which you can construct a derivation. N.B: The formula cannot be the principle of non-contradiction. Once you have found the formula, please (a) check that it is in fact a tautology using the truth table method, and then also (b) construct a derivation for it.

5. Give an example of the following argument pattern and check its validity using the truth table method:

   If A, then B

   If B, then C

   not-C

   _____

   not-A

6. Construct a derivation of $(((\varphi \to \psi) \land (\psi \to \sigma)) \land \varphi) \to \sigma$

7. What is the difference between the principle of non-contradiction and the principle of excluded middle? Once you have stated the two principles, you should do two things. First, show that both principles are tautologies. Second, show that if you drop the principle of bivalence, the principle of excluded middle is no longer a tautology.

PART 3: THINKING ABOUT LOGIC [30PT]

8. Do you think using the principle of bivalence is always justified? Explain why (not).

9. Is it possible that $\varphi \models \psi$ and $\varphi \models \neg\psi$?

10. In what way studying logic was a waste of your time and in what way studying logic was useful for you. Explain your thoughts about this very precisely!

PART 4: SOMETHING EXTRA [40PT]

11. Suppose $V(\varphi) = V(\neg\psi)$. Determine the truth value (or truth values) of $(\varphi \wedge \psi)$.

12. Suppose $V(\varphi) \geq V(\neg\psi)$. Determine the truth value (or truth values) of $\neg(\varphi \vee \neg\psi)$.

13. Construct a derivation of $(((\varphi \to \psi) \wedge (\psi \to \sigma)) \wedge \neg\sigma) \to \neg\varphi$

14. Construct a derivation of $(\varphi \wedge \neg\neg\neg\varphi) \to \neg(\varphi \wedge \neg\neg\neg\varphi)$