

INTRODUCTION TO LOGIC - MIDTERM EXAM - DUE MARCH 19 BY MIDNIGHT

**Submission:** Please type your answers or write them neatly. Messy or illegible answers will not be accepted. Upload your midterm as one single file through the link on the course website. You need to have a Google email account to upload your file. Message me in Slack if you face technical difficulties.

Ex. 1 [10pt.] Break down the following formulas by using trees :

(a)  $(p \wedge \neg r) \vee ((\neg p) \rightarrow r)$

(b)  $(p \wedge \neg(r \vee \neg p)) \rightarrow r$

Ex. 2 [20pt.] Let  $V_1(p) = V_2(r) = 0$  and let  $V_1(r) = V_2(p) = 1$ . Check the following:

(c)  $V_1 \models (p \wedge \neg r) \vee ((\neg p) \rightarrow r)$

(d)  $V_2 \models (p \wedge \neg(r \vee \neg p)) \rightarrow r$

Ex. 3 [20pt.] Check whether the following are tautologies, contradictions or neither:

(e)  $p \vee \neg q$

(f)  $(p \wedge q) \rightarrow (p \rightarrow q)$

Ex. 4 [20pt.] Check whether the following arguments are valid:

(g) Premise (1)  $p \vee q$

Premise (2)  $\neg p \rightarrow \neg q$

Premise (3)  $p \rightarrow q$

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(Conclusion)  $p$

(h) Premise (1)  $p \vee q$

Premise (2)  $\neg p \rightarrow \neg q$

Premise (3)  $p \rightarrow q$

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(Conclusion)  $q$

Ex. 5 [30pt.] Answer the following puzzles on the next page ...

- (i) Titus was tried for murder. The prosecutor said: *If the defendant is guilty, then he had an accomplice.* The defense attorney responded: *That's not true!*. Why was this the worst thing the defense attorney could have said? [*Hint: Keep in mind the meaning of if...then according to the truth table for  $\rightarrow$ .*]
- (j) This case takes place on an imaginary island populated only by Lehman faculty, Lehman administrators and Lehman students. Faculty always tell the truth (*really?*), administrators always lie (*never trust them!*), and students sometimes lie and sometimes tell the truth (*no offense!*). Three people from the island, Dane, Laio, and Uvus, were being tried for stealing lab equipment from a Lehman laboratory. Shame! It is known that the crime was committed by only one of them. It is also known that the one who committed the crime was a Lehman faculty member, and the only faculty member among them. The three defendants made the following statements:
- Dane*: I am innocent.  
*Laio*: That is true.  
*Uvus*: Laio is not a student.

Which one is guilty? [*Hint: This is not straightforward. Be meticulous!*]