Syntax

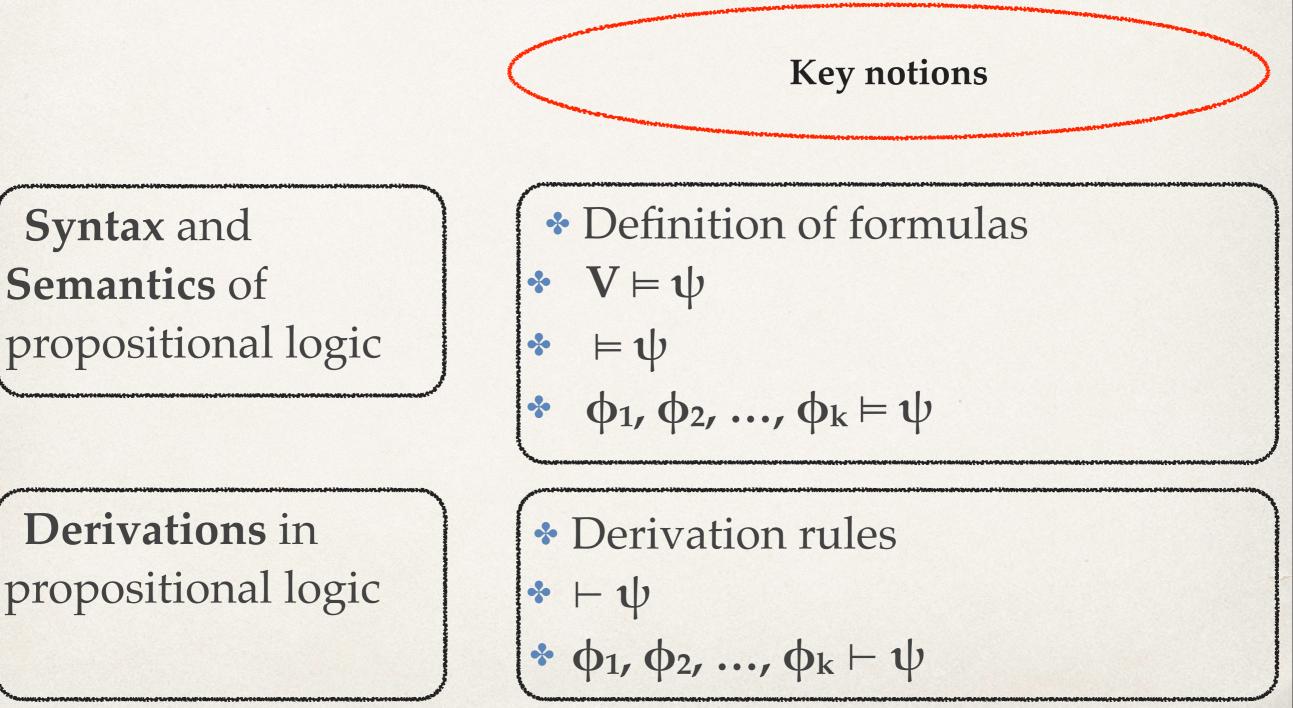
Definition of formulas of Lp

Semantics

Truth: $V \models \psi$ *Validity:* $\models \psi$ *Logical Consequence:* $\phi_1, \phi_2, ..., \phi_k \models \psi$

Intro to Logic- Midterm Review (1)

What We've Done So Far



Syntax of Propositional Logic

Ingredients of the Propositional Language

1 Basic (*atomic*) statements (**propositions**):

 $\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \dots$

2 Operators to build more statements:

" not "	becomes	⊸
"… and …"	becomes	
"… or …"	becomes	· · · · V · · · ·
" if then "	becomes	$\ldots \rightarrow \ldots$
" if and only if"	becomes	$\ldots \leftrightarrow \ldots$

Inductive Definition of Formulas in the Language of Propositional Logic (**Lp**)

Base case:

p, q, r ... are formulas of Lp.

Inductive cases (or inductive steps):

φ and ψ are
 not formulas; they are
 schemata for
 formulas.

If ϕ is a formula of **Lp**, then $\neg \phi$ is a formula of **Lp** If ϕ and ψ are formulas of **Lp**, then ($\phi \land \psi$) is a formula of **Lp** If ϕ and ψ are formulas of **Lp**, then ($\phi \lor \psi$) is a formula of **Lp** If ϕ and ψ are formulas of **Lp**, then ($\phi \rightarrow \psi$) is a formula of **Lp**

Final clause:

Nothing else is a formula of Lp



Formulas such as

```
p, q, r, ...
are called atomic formulas.
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(They are also called atomic propositions)

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Formulas <u>of the form</u>

\neg \phi

(\phi \land \psi)

(\phi \lor \psi)

(\phi \rightarrow \psi)

are called complex or molecular formulas.
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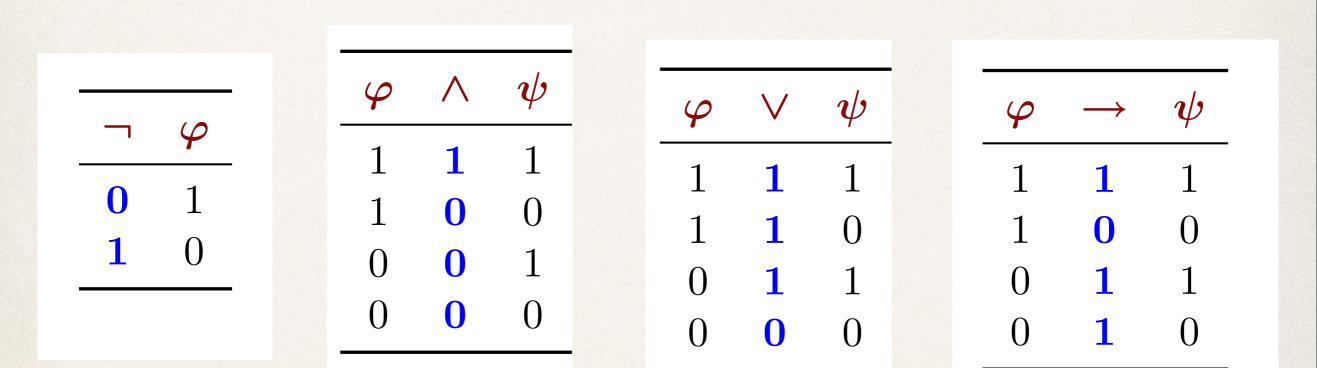
Semantics of Propositional Logic

Truth: $V \models \psi$

How to determine whether a formula is true (=has value 1) or false (=has value 0) relative to a valuation V If the formula is atomic, check which value (1 or 0) valuation **V assigns to the atomic formula in question**.

If the formula is complex, then the valuation V will assign a unique value (1 or 0) to the formula according to the truth tables for \neg , \land , \lor , and \rightarrow

Truth Tables for the Connectives



Notation and Terminology

 $V(\psi)=1$ *iff* $V \models \psi$ *iff* ψ is true relative to V *iff* V makes ψ true

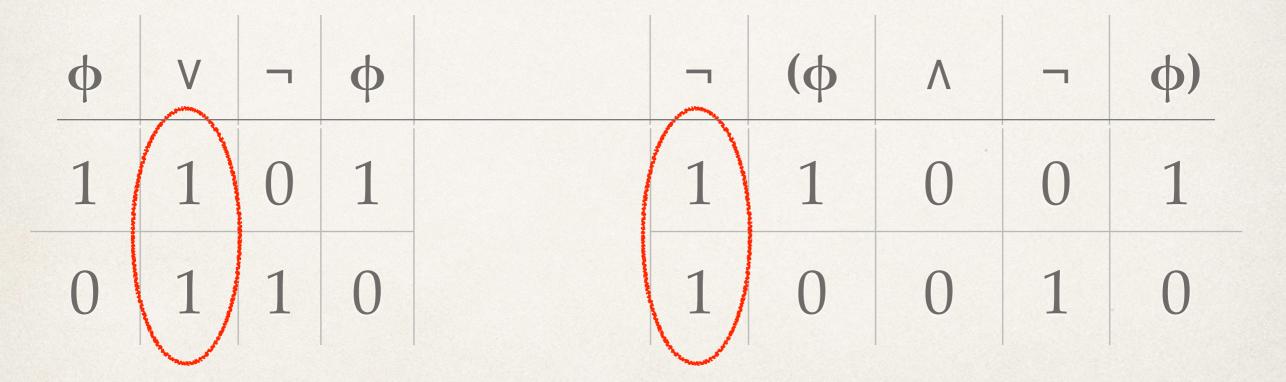
 $V(\psi)=0$ *iff* $V \neq \psi$ *iff* ψ is false relative to V *iff* V makes ψ false

NB: The expressions separated by *iff* are equivalent to one another. So, e.g., writing " $V(\psi)=1$ " or " $V \models \psi$ " or "V makes ψ true" is the same.



$\models \psi$ *iff* all valuations V's make ψ true

Examples



Logical Consequence: $\phi_1, \phi_2, \dots \phi_k \models \psi$

 $\phi_1, \phi_2, ..., \phi_k \vDash \psi$

iff all valuations V's that make $\phi_1, \phi_2, ..., \phi_k$ true make ψ true

iff for all V's [*if* V makes $\phi_1, \phi_2, ..., \phi_k$ true, *then* V makes ψ true]

NB: Logical consequence is expressed as an *if-then* statement, so whenever the antecedent is false, logical consequence will hold vacuously

Checking whether $\phi_1, \phi_2, \dots \phi_k \models \psi$

(1) Select *all* valuations V's which make $\phi_1, \phi_2, ..., \phi_k$ *all* true.

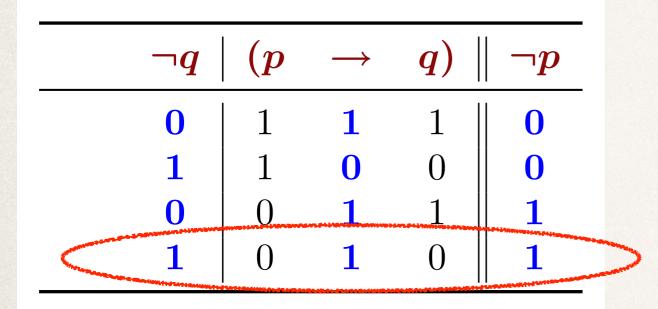
(2) Check whether those valuations V's which you have selected in (1) are such that they *all* make ψ true.

If YES, then $\phi_1, \phi_2, ..., \phi_k \models \psi$

If NO, then $\phi_1, \phi_2, ..., \phi_k \neq \psi$

NB: If no V can make $\phi_1, \phi_2, ..., \phi_k$ *all* true, then $\phi_1, \phi_2, ..., \phi_k \models \psi$ will hold vacuously

Checking that $\neg q, p \rightarrow q \models \neg p$



We only need to check the last line of the table because this is where $\neg q$, $p \rightarrow q$ are both true.

Checking that $\neg p, p \rightarrow q \nvDash \neg q$

¬p	(p	\rightarrow	q)	٦q	
0	1	1	1	0	
0	1	0	0	1	
 1	0	1	1	0	
1	0	1	0	1	

Not all valuations that make true both $p \rightarrow q$ and $\neg p$ also make true the conclusion $\neg q$.

Checking that $\neg p, p \models \neg p$

¬p	р	¬p
0	1	0
1	0	1

There is no valuation that makes both $\neg p$ *and* p *so* $\neg p$, $p \models \neg p$ *holds* (*vacuously*)