Syntax

Definition of formulas of ***Lp***

Semantics

Truth: $V \vDash \psi$

Validity: $\vDash \psi$

Logical Consequence: $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

Intro to Logic- Midterm Review (1)

What We've Done So Far

Key notions

Syntax and **Semantics** of propositional logic

- Definition of formulas
- $\mathbf{V} \models \psi$
- $\models \psi$
- $\phi_1, \phi_2, \ldots, \phi_k \models \psi$

Derivations in propositional logic

- Derivation rules
- $\vdash \psi$
- $\phi_1, \phi_2, \ldots, \phi_k \vdash \psi$

Syntax of Propositional Logic

Ingredients of the Propositional Language

1. Basic (*atomic*) statements (**propositions**):

$$\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \ldots$$

2. Operators to build more statements:

"**not** ... "	becomes	$\neg \ldots$
"... **and** ..."	becomes	$\ldots \wedge \ldots$
"... **or** ..."	becomes	$\ldots \vee \ldots$
"**if** ... **then**"	becomes	$\ldots \longrightarrow \ldots$
" ... **if and only if** ..."	becomes	$\ldots \leftrightarrow \ldots$

Inductive Definition of Formulas in the Language of Propositional Logic (**Lp**)

Base case:

p, q, r … are formulas of **Lp**.

ϕ and ψ are not formulas; they are **schemata for formulas**.

Inductive cases (or inductive steps):

If ϕ is a formula of **Lp**, then $\neg\phi$ is a formula of **Lp**

If ϕ and ψ are formulas of **Lp**, then **(**$\phi \land \psi$**)** is a formula of **Lp**

If ϕ and ψ are formulas of **Lp**, then **(**$\phi \lor \psi$**)** is a formula of **Lp**

If ϕ and ψ are formulas of **Lp**, then **(**$\phi \rightarrow \psi$**)** is a formula of **Lp**

Final clause:

Nothing else is a formula of **Lp**

Terminology

Formulas such as

p, q, r, …

are called **atomic formulas**.

(They are also called atomic propositions)

Formulas *of the form*

$\neg\phi$

$(\phi \wedge \psi)$

$(\phi \vee \psi)$

$(\phi \rightarrow \psi)$

are called **complex** or **molecular** formulas.

Semantics of Propositional Logic

Truth: $\mathbf{V} \vDash \psi$

How to determine whether a formula is true (=has value 1) or false (=has value 0) relative to a valuation V

If the formula is atomic, check which value (1 or 0) valuation **V assigns to the atomic formula in question.**

If the formula is complex, then the valuation **V will assign a unique value (1 or 0) to the formula according to the truth tables for ¬, ∧, ∨, and →**

Truth Tables for the Connectives

$\neg$	φ
0	1
1	0

φ	$\wedge$	ψ
1	**1**	1
1	**0**	0
0	**0**	1
0	**0**	0

φ	$\vee$	ψ
1	**1**	1
1	**1**	0
0	**1**	1
0	**0**	0

φ	$\rightarrow$	ψ
1	**1**	1
1	**0**	0
0	**1**	1
0	**1**	0

Notation and Terminology

$V(\psi)= 1$ *iff* $V \vDash \psi$ *iff* ψ is true relative to V *iff* V makes ψ true

$V(\psi)=0$ *iff* $V \nvDash \psi$ *iff* ψ is false relative to V *iff* V makes ψ false

NB: The expressions separated by *iff* are equivalent to one another. So, e.g., writing "$V(\psi)= 1$" or "$V \vDash \psi$" or "V makes ψ true" is the same.

Validity: $\vDash \psi$

$\vDash \psi$ *iff* **all valuations *V*'s make ψ true**

Examples

ϕ	$\vee$	$\neg$	ϕ
1	1	0	1
0	1	1	0

$\neg$	$(\phi$	$\wedge$	$\neg$	$\phi)$
1	1	0	0	1
1	0	0	1	0

Logical Consequence: $\phi_1, \phi_2, \ldots \phi_k \vDash \psi$

$\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

iff **all valuations *V*'s that make $\phi_1, \phi_2, \ldots, \phi_k$ true make ψ true**

iff **for all *V*'s [*if V* makes $\phi_1, \phi_2, \ldots, \phi_k$ true, *then V* makes ψ true]**

NB: Logical consequence is expressed as an *if-then* statement, so whenever the antecedent is false, logical consequence will hold vacuously

Checking whether $\phi_1, \phi_2, \ldots\phi_k \vDash \psi$

(1) *S*elect *all* valuations **V**'s which make $\boldsymbol{\phi_1, \phi_2, \ldots, \phi_k}$ ***all*** true.

(2) Check whether those valuations **V**'s which you have selected in **(1)** are such that they ***all*** make ψ true.

If YES, then $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

If NO, then $\phi_1, \phi_2, \ldots, \phi_k \nvDash \psi$

NB: If no V can make $\phi_1, \phi_2, \ldots, \phi_k$ *all* true, then $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$ will hold vacuously

Checking that $\neg q, p \to q \models \neg p$

$\neg q$	$(p$	$\to$	$q)$	$\neg p$
0	1	**1**	1	**0**
1	1	**0**	0	**0**
0	0	**1**	1	**1**
1	0	**1**	0	**1**

We only need to check the last line of the table because this is where $\neg q$, $p \to q$ are both true.

Checking that $\neg p, p \to q \not\models \neg q$

$\neg p$	$(p$	$\to$	$q)$		$\neg q$
0	1	1	1		0
0	1	0	0		1
1	0	1	1		0
1	0	1	0		1

Not all valuations that make true both $\boldsymbol{p \to q}$ *and* $\boldsymbol{\neg p}$ *also make true the conclusion* $\boldsymbol{\neg q}$.

Checking that $\neg p, p \models \neg p$

$\neg \mathbf{p}$	$\mathbf{p}$		$\neg \mathbf{p}$
0	1		0
1	0		1

There is no valuation that makes both ***¬p and p*** *so $\neg p, p \models \neg p$ holds (<u>vacuously</u>)*