A Peculiarity of Our Propositional Language

<u>Notational Convention</u>:

We shall consider negated formulas of the form

$$\neg\phi$$

as abbreviations of

$$\phi\rightarrow\bot$$

An Application of →E

$$\frac{\phi \quad \neg\phi}{\bot}\ \to E$$

Given our notational convention, this is a correct application of rule →E

An Application of →E

$$\frac{\phi \quad \neg\phi}{\bot}\ \to\!\mathbf{E}$$

Given our notational convention, this is a correct application of rule →E

$$\frac{\phi \quad \phi\to\bot}{\bot}\ \to\!\mathbf{E}$$

An Application of →I

$[\phi]^i$

.

.

.

$\bot$

$\dfrac{\qquad\qquad}{\neg\phi}\ \rightarrow\mathbf{I}^i$

$[\phi]^i$

.

.

.

$\bot$

$\dfrac{\qquad\qquad}{\phi\rightarrow\bot}\ \rightarrow\mathbf{I}^i$

Given
our notational convention,
this is a correct application
of rule

Deriving the PNC

$$\dfrac{\dfrac{\dfrac{[\phi \wedge \neg \phi]^1}{\phi}\wedge E \qquad \dfrac{[\phi \wedge \neg \phi]^1}{\neg \phi}\wedge E}{\bot}\to E}{\neg(\phi \wedge \neg \phi)}\to I^1$$

Deriving the PNC

$$\dfrac{\dfrac{\dfrac{[\phi \wedge \neg \phi]^1}{\phi}\wedge E \qquad \dfrac{[\phi \wedge \neg \phi]^1}{\neg \phi}\wedge E}{\dfrac{\bot}{\neg(\phi \wedge \neg \phi)}\to I^1}\to E$$

Note the use of our notational convention in the application of rules $\to$**E** and $\to$**I**