Today we Begin with the Simplest Logical System: **Propositional Logic**

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Syntax: rules to build well-formed formulas

Semantics: rules to assign (truth) values to these formulas

SYNTAX of the Propositional Language

Ingredients of the Propositional Language

1. Basic (*atomic*) statements (**propositions**):

$$\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \ldots$$

2. Operators to build more statements:

"**not** … "	becomes	$\neg \ldots$
"… **and** …"	becomes	$\ldots \wedge \ldots$
"… **or** …"	becomes	$\ldots \vee \ldots$
"**if** … **then**"	becomes	$\ldots \rightarrow \ldots$
" … **if and only if** …"	becomes	$\ldots \leftrightarrow \ldots$

Well-Formed Formulas

The **language** $\mathcal{L}_{\mathtt{P}}$ is a set of formulas satisfying:

1. All the basic propositions are in $\mathcal{L}_{\mathtt{P}}$:

$$\boldsymbol{p} \in \mathcal{L}_{\mathtt{P}}, \quad \boldsymbol{q} \in \mathcal{L}_{\mathtt{P}}, \quad \boldsymbol{r} \in \mathcal{L}_{\mathtt{P}}, \quad \ldots$$

2. If $\varphi \in \mathcal{L}_{\mathtt{P}}$ and $\psi \in \mathcal{L}_{\mathtt{P}}$, then

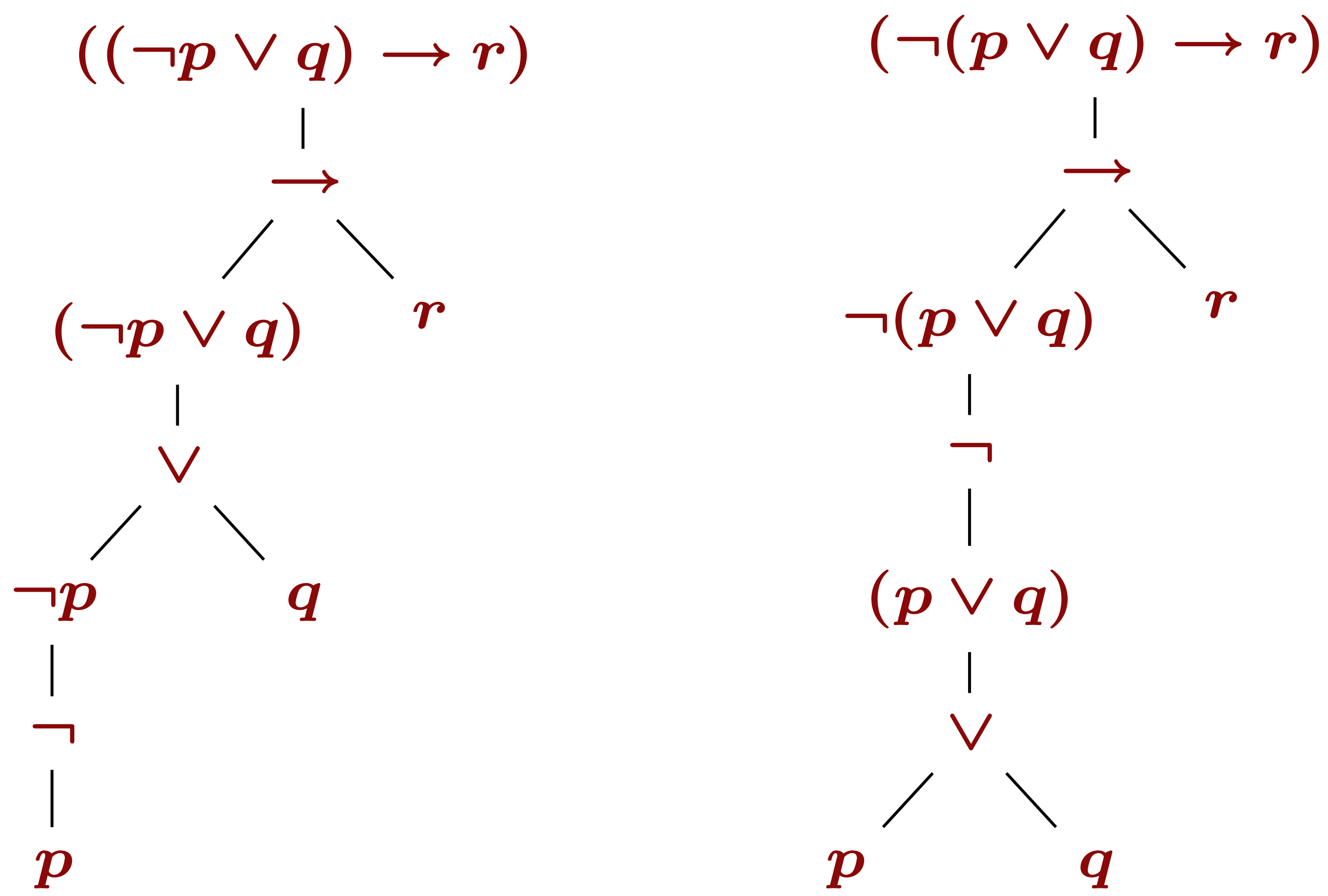
$$\neg\varphi \in \mathcal{L}_{\mathtt{P}}, \qquad (\varphi \wedge \psi) \in \mathcal{L}_{\mathtt{P}}, \qquad (\varphi \longrightarrow \psi) \in \mathcal{L}_{\mathtt{P}},$$
$$(\varphi \vee \psi) \in \mathcal{L}_{\mathtt{P}}, \qquad (\varphi \leftrightarrow \psi) \in \mathcal{L}_{\mathtt{P}}.$$

3. Nothing else is in $\mathcal{L}_{\mathtt{P}}$.

In practice, we will avoid parenthesis if they are not necessary.

Formulas as Trees

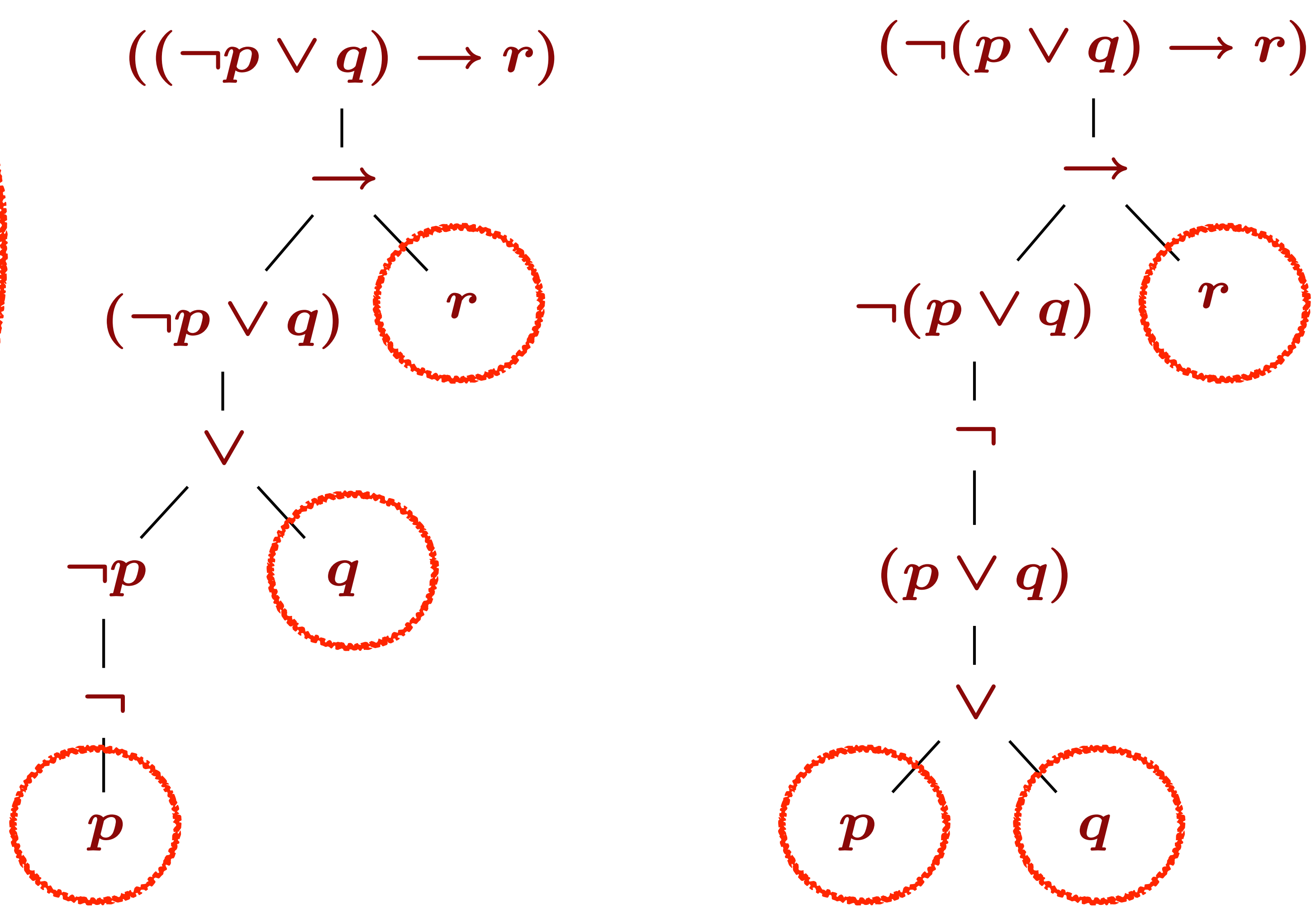
The construction of a formula can be seen as building a **tree**.



Formulas as Trees

The construction of a formula can be seen as building a **tree**.

The formulas that are circled in red are **basic (or atomic) formulas**



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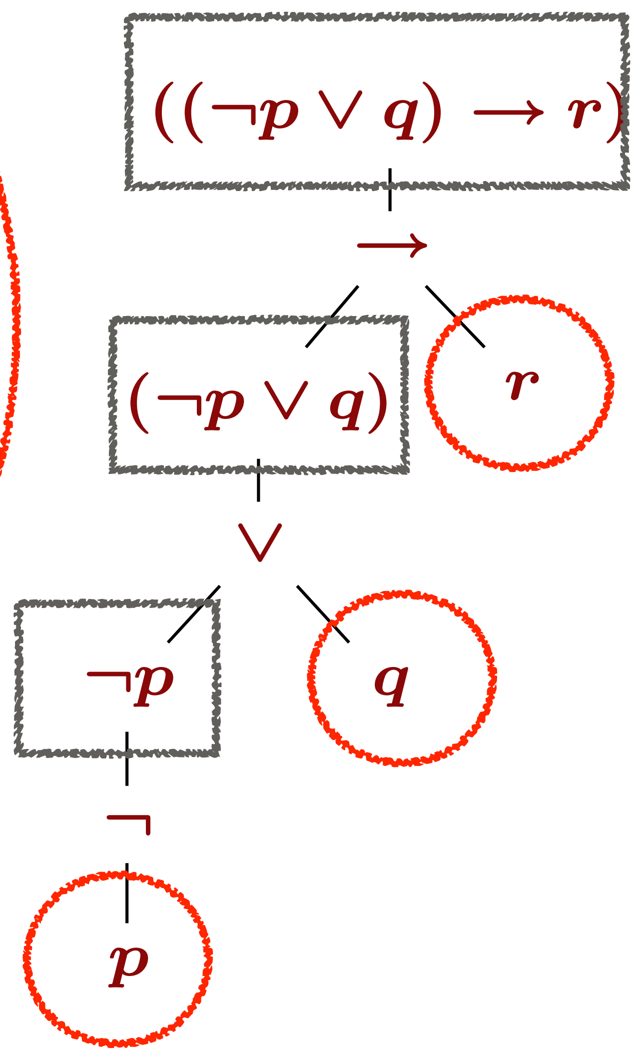

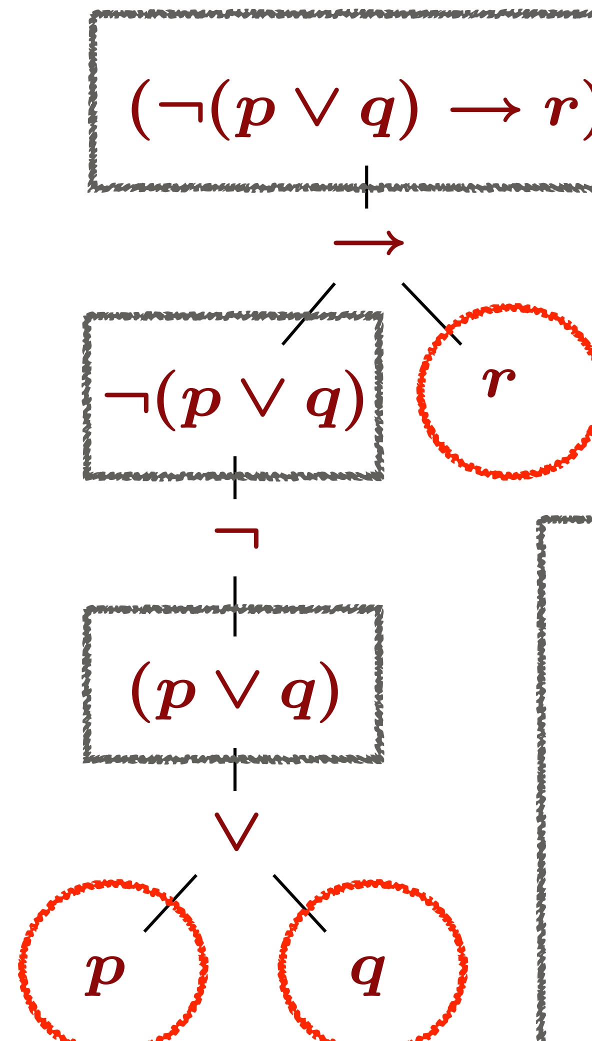

The formulas within a grey rectangle are **more complex (or molecular) formulas**