

What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a
formula **relative to**
ONE Valuation

How to evaluate a
formula **relative to**
ALL valuations

Can we get an
account of
(deductively) **valid**
argument?

Deductively Valid Arguments

Informally speaking, an *argument* is said to be **deductively valid**

if and only if

whenever **the premises** are **true**, the **conclusion** is **always true**.

Given the semantics of propositional logic, an *argument* is said to be **deductively valid**

if and only if

whenever **all** valuations that make **true** the **premises** make **true** the **conclusion**.

Recall Modus Ponens

Premise 1: If you take the medication, then you will get better

Premise 2: You are taking the medication

Conclusion: You will get better

Modus Ponens:

If p , then q

p

q

Is Modus Ponens Valid?

| p | $(p \rightarrow q)$ | q |
|----------|---------------------|----------|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 1 | 0 |

Is Modus Ponens Valid?

| p | $(p \rightarrow q)$ | q |
|-----|---------------------|-----|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 1 | 0 |

We only need to check the first line of the table because this is where the premises are all true.

We can write

$$p, p \rightarrow q \models q$$

Recall Modus Tollens

Premise 1: If you take the medication, then you will get better

Premise 2: You are NOT getting better

Conclusion: You are NOT taking the medication

Modus Tollens:

If p , then q

$\text{not-}q$

$\text{not-}p$

Modus Tollens:

$p \rightarrow q$

$\neg q$

$\neg p$

Is Modus Tollens Valid?

| $\neg q$ | $(p \rightarrow q)$ | $\neg p$ |
|----------|---------------------|----------|
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |

Is Modus Tollens Valid?

| $\neg q$ | $(p \rightarrow q)$ | $\neg p$ |
|----------|---------------------|----------|
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |

We only need to check the last line of the table because this is where the premises are all true.

We can write

$$\neg q, p \rightarrow q \models \neg p$$