What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a formula **relative to ONE Valuation**

$$V \models \psi$$

How can we evaluate a formula **relative to ALL valuations?**

How MANY Valuations Functions?

With **one** atomic proposition, there are **two** possible valuations.

With **two** atomic propositions, there are **four** possible valuations.

With **three** atomic propositions, there are **2^3=8** possible valuations.

With **n** atomic propositions, there are **2^n** possible valuations.

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1		1		1		1
1		1		0		0
0		0		1		1
0		0		0		0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1		1	1	1		1
1		1	0	0		0
0		0	1	1		1
0		0	1	0		0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1	1	1	1	1		1
1	0	1	0	0		0
0	0	0	1	1		1
0	0	0	1	0		0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\rightarrow$	$q))$	$\rightarrow$	q
1	1	1	1	1	**1**	1
1	0	1	0	0	**1**	0
0	0	0	1	1	**1**	1
0	0	0	1	0	**1**	0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1	1	1	1	1	**1**	1
1	0	1	0	0	**1**	0
0	0	0	1	1	**1**	1
0	0	0	1	0	**1**	0

Always true

$\neg$	$\neg$	p
1	0	1
0	1	0

Sometimes true

Classification of Formulas

- Those that are never true (**contradiction**):

$$p \wedge (\neg p), \ldots$$

- Those that can be true (**satisfiable**):

$$(\neg p) \vee q, \ldots$$

- Those that are always true (**valid**, **tautology**):

$$(p \wedge (p \to q)) \to q, \ldots$$

If the formula φ is valid, we write $\models \varphi$

The expression
$V \models \phi$
means that ϕ is true
relative to ONE valuation.
Instead, the expression
$\models \phi$
means that ϕ is true
relative to ALL valuations.