Syntax

*Inductive definition of formulas of **Lp***

Semantics

Truth: $V \vDash \psi$

Validity: $\vDash \psi$

Logical Consequence: $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

PHIL 50 - Review (I)

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What We've Done So Far

Key notions

WEEK 2: **Syntax** and **Semantics** of propositional logic

- Inductive definition of formulas
- $\mathbf{V} \vDash \psi$
- $\vDash \psi$
- $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

WEEK 3: **Derivations** in propositional logic

- Derivation rules
- $\vdash \psi$
- $\phi_1, \phi_2, \ldots, \phi_k \vdash \psi$

Topics Covered in Week 2

Syntax of Propositional Logic

Ingredients of the Propositional Language

1. Basic (*atomic*) statements (**propositions**):

$$\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \ldots$$

2. Operators to build more statements:

"**not** … "	becomes	$\neg \ldots$
"… **and** …"	becomes	$\ldots \wedge \ldots$
"… **or** …"	becomes	$\ldots \vee \ldots$
"**if** … **then**"	becomes	$\ldots \rightarrow \ldots$
" … **if and only if** …"	becomes	$\ldots \leftrightarrow \ldots$

Inductive Definition of Formulas in the Language of Propositional Logic (**Lp**)

Base case:

p, q, r … are formulas of **Lp**.

ϕ and ψ are not formulas; they are **schemata for formulas**.

Inductive cases (or inductive steps):

If ϕ is a formula of **Lp**, then $\neg\phi$ is a formula of **Lp**

If ϕ and ψ are formulas of **Lp**, then **(**$\phi \wedge \psi$**)** is a formula of **Lp**

If ϕ and ψ are formulas of **Lp**, then **(**$\phi \vee \psi$**)** is a formula of **Lp**

If ϕ and ψ are formulas of **Lp**, then **(**$\phi \rightarrow \psi$**)** is a formula of **Lp**

Final clause:

Nothing else is a formula of **Lp**

Terminology

Formulas such as

p, q, r, …

are called **atomic formulas**.

(They are also called atomic propositions)

Formulas _of the form_

$\neg\phi$

$(\phi \wedge \psi)$

$(\phi \vee \psi)$

$(\phi \to \psi)$

are called **complex** or **molecular** formulas.

Semantics of Propositional Logic

Truth: $\mathbf{V} \vDash \psi$

How to determine whether a formula is true (=has value 1) or false (=has value 0) relative to a valuation V

If the formula is atomic, check which value (1 or 0) valuation **V assigns to the atomic formula in question.**

If the formula is complex, then the valuation **V will assign a unique value (1 or 0) to the formula according to the truth tables for ¬, ∧, ∨, and →**

Truth Tables for the Connectives

$\neg$	φ
$\mathbf{0}$	1
$\mathbf{1}$	0

φ	$\wedge$	ψ
1	$\mathbf{1}$	1
1	$\mathbf{0}$	0
0	$\mathbf{0}$	1
0	$\mathbf{0}$	0

φ	$\vee$	ψ
1	$\mathbf{1}$	1
1	$\mathbf{1}$	0
0	$\mathbf{1}$	1
0	$\mathbf{0}$	0

φ	$\to$	ψ
1	$\mathbf{1}$	1
1	$\mathbf{0}$	0
0	$\mathbf{1}$	1
0	$\mathbf{1}$	0

Notation and Terminology

$V(\psi) = 1$ *iff* $V \vDash \psi$ *iff* ψ is true relative to V *iff* V makes ψ true

$V(\psi) = 0$ *iff* $V \nvDash \psi$ *iff* ψ is false relative to V *iff* V makes ψ false

NB: The expressions separated by *iff* are equivalent to one another. So, e.g., writing "$V(\psi)= 1$" or "$V \vDash \psi$" or "V makes ψ true" is the same.

Validity: $\vDash \psi$

$\vDash \psi$ *iff* **all valuations *V*'s make ψ true**

Examples

ϕ	$\vee$	$\neg$	ϕ
1	1	0	1
0	1	1	0

$\neg$	$(\phi$	$\wedge$	$\neg$	$\phi)$
1	1	0	0	1
1	0	0	1	0

Logical Consequence: $\phi_1, \phi_2, \ldots \phi_k \vDash \psi$

$\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

iff **all valuations *V*'s that make $\phi_1, \phi_2, \ldots, \phi_k$ true make ψ true**

iff **for all *V*'s [*if V* makes $\phi_1, \phi_2, \ldots, \phi_k$ true, *then V* makes ψ true]**

NB: Logical consequence is expressed as an *if-then* statement, so whenever the antecedent is false, logical consequence will hold vacuously

Checking whether $\phi_1, \phi_2, \ldots \phi_k \vDash \psi$

(1) Select *all* valuations **V**'s which make $\boldsymbol{\phi_1, \phi_2, \ldots, \phi_k}$ ***all*** true.

(2) Check whether those valuations **V**'s which you have selected in **(1)** are such that they ***all*** make ψ true.

If YES, then $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$

If NO, then $\phi_1, \phi_2, \ldots, \phi_k \nvDash \psi$

NB: If no V can make $\phi_1, \phi_2, \ldots, \phi_k$ *all* true, then $\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$ will hold vacuously

Checking that $\neg q, p \rightarrow q \models \neg p$

$\neg q$	$(p$	$\rightarrow$	$q)$	$\neg p$
0	1	**1**	1	**0**
1	1	**0**	0	**0**
0	0	**1**	1	**1**
1	0	**1**	0	**1**

We only need to check the last line of the table because this is where $\neg q$, $p \rightarrow q$ are both true.

Checking that $\neg p, p \to q \not\models \neg q$

$\neg p$	$(p$	$\to$	$q)$		$\neg q$
0	1	1	1		0
0	1	0	0		1
1	0	1	1		0
1	0	1	0		1

*Not all valuations that make true both **p →q** and **¬p** also make true the conclusion **¬q**.*

Checking that $\neg p, p \vDash \neg p$

$\neg$**p**	**p**		$\neg$**p**
0	1		0
1	0		1

There is no valuation that makes both ***¬p and p*** *so*
$\neg p, p \vDash \neg p$ holds (vacuously)