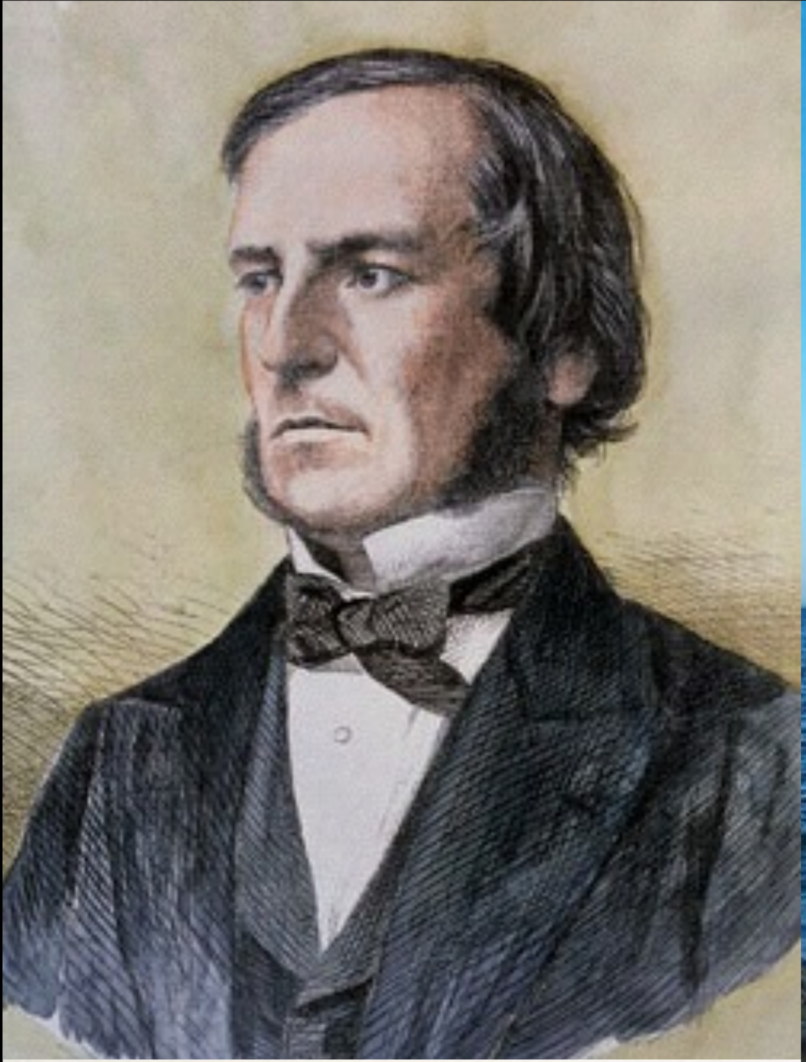
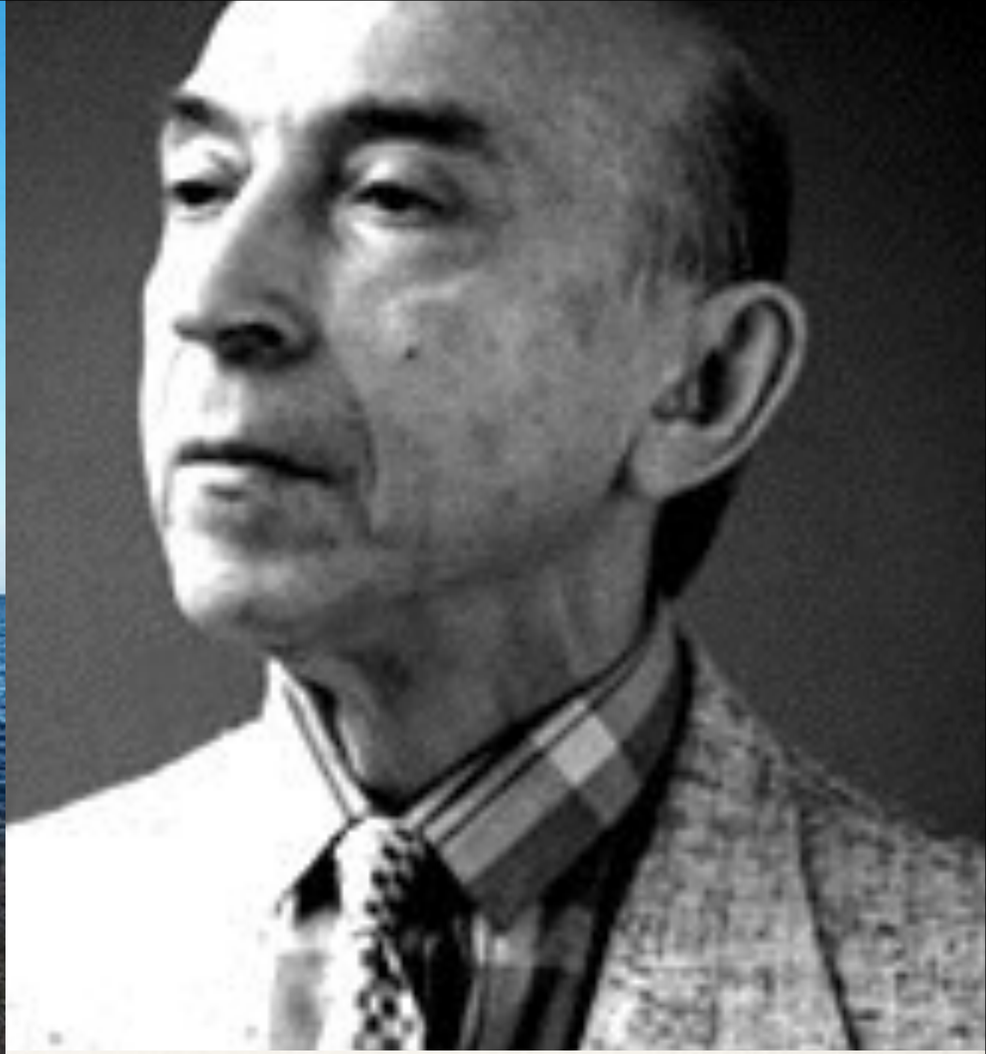

George Boole


Graham Priest

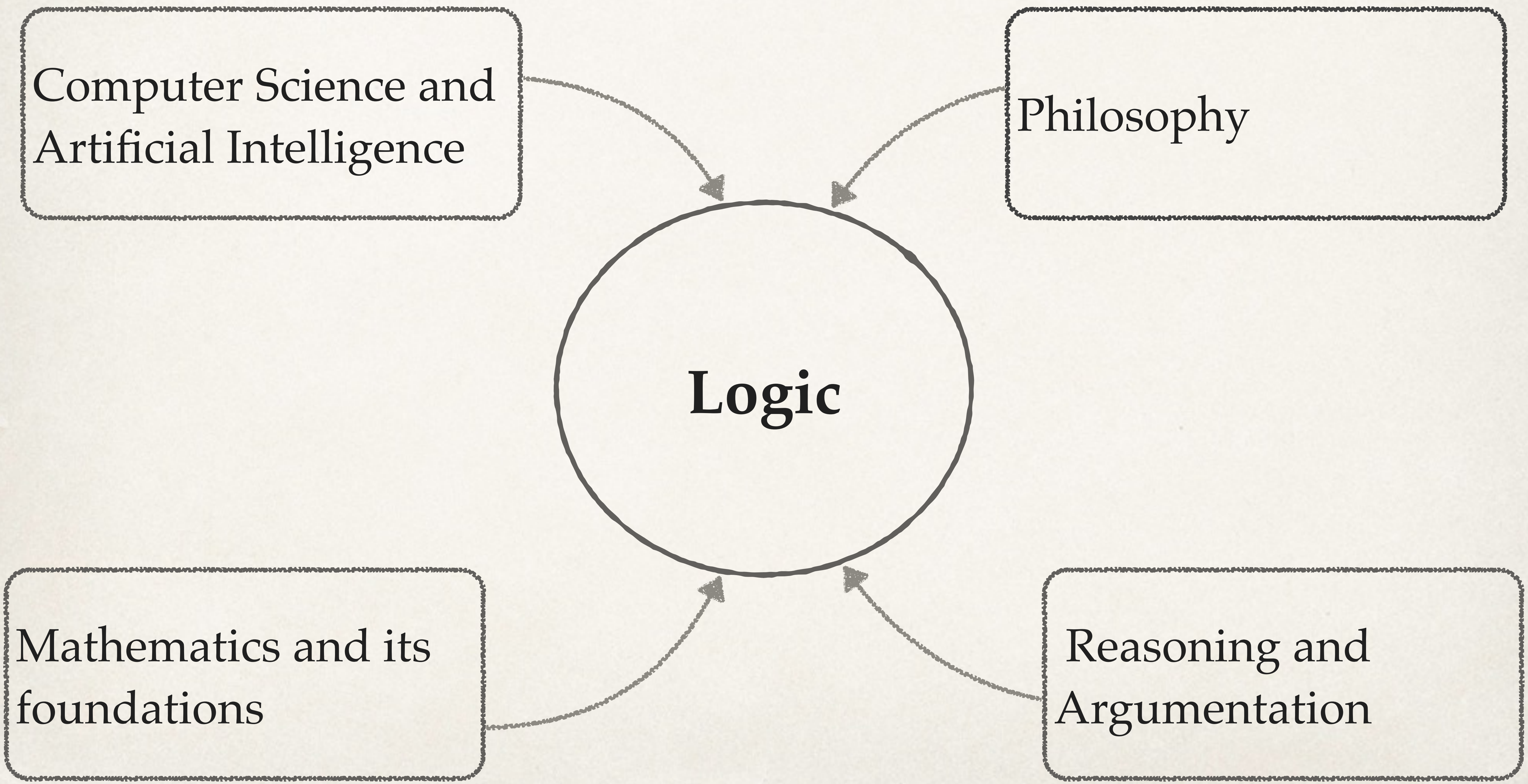

Lofti Zadeh

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

Week 1 — Friday Class

Logic is Many Things



Recall —
What to Expect from this Course

Learn about **propositional, predicate, modal, and inductive logic**

Learn some **history** and **philosophy of logic** along the way

Learn how to write **formal proofs**, both semantic and syntactic proofs

Learn about **logical puzzles** and **paradoxes**

The Firmest Principle of Logic

Principle of Non-Contradiction (PNC):

not-(A **and** (**not**-A))

A and not-A cannot both be true (at the same time)

Why should we accept PNC?

A “Semantic” (Algebraic) Argument for PNC: *Statements Can Be Assigned Only 0’s or 1’s*

Let X stand for some statement. Clearly, ***X and X*** is the same as ***X***. So, in algebraic notation, we can write ***X*X=X***. (This was Boole’s idea, namely that “***and***” can be understood as the operation of multiplication.)

Now, if ***X*X=X*** holds, then ***X*** can only have value ***0*** or ***1***.

By algebra, ***X*X=X*** implies ***X-(X*X)=0***, which implies ***X*(1-X)=0***.

We can interpret ***X*(1-X)=0*** as saying that ***X and not-X*** is false, where multiplication stands for “***and,***” the expression “***1-X***” stands for “***not-X,***” and “***0***” stands for falsity. In other words, ***X*(1-X)=0*** says that contradictions are false, and this is (a version of) **PNC**.

Historical Aside: George Boole's **Algebra of Logic** (mid 19th century)

- Statements have value 0 or 1 only
- The connective "**and**" is understood as *multiplication*
- The connective "**not**" is understood as *subtraction*

A "Syntactic" Argument for PNC: *A Contradiction Implies Anything*

1. A and not-A	By assumption
2. A	from 1
3. A or B	from 2
4. not-A	from 1
5. B	from 3, 4

Similar to the syntactic proofs you'll do in week 3 of the course.

This shows that *from a contradiction anything follows*. But if so, **our logical system would become trivial** because it would yield an argument for anything. Hence, some might conclude that we must reject contradictions.

But Have we Really Established that the Principle of Non-Contradiction Holds?

Maybe Some Contradictions Exist…

A contradiction is a statement like
A and (not-A)

Consider the sentence

"this sentence is not true"

If the sentence is true, then it is not true.

If the sentence is not true, then is true.

Thus, the sentence *"this sentence is not true"* is true ***and*** it is not true. Contradiction!

One solution: Require that statements cannot (self-)refer to their own truth value.

This solution does not require us to admit of contradictions.

Another solution: Admit that there are contradictions.

Here we need a logic that can handle contradictions.

Can There Be a Logic that Admits of Contradictions?

Certain Logicians Believe that Some Contradictions Are True

Dialethism:
The view that some contradictions are true

Paraconsistent Logic:
The type of logic needed to avoid the effect of "from the contradiction anything follows"

Graham Priest

Another Important Logical Principle

Principle of Excluded Middle (PEM):

(A **or** (**not**-A))

"Either A is true or not-A is true."

Contrast it with **Principle of Non Contradiction (PNC):**

not-(A **and** (**not**-A))

"A and not-A cannot both be true (at the same time)."

Are PNC and PEM equivalent or not?

A "Semantic" (Algebraic) Argument for PEM

Let X stand for some statement. Clearly, the following holds by algebra:

$$(1-X)+X=1$$

Let addition stand for "*either…or,*" the expression "***1-X***" for "***not-X,***" and "***1***" for truth. In other words, ***(1-X)+X=1*** asserts that ***either not-X or X*** is true, and this is (a version of) **PEM**.

Note that algebraically **PNC** looks like this:

$$X*(1-X)=0$$

Classical and Non-Classical Logic

Classical Logic:

Both the Principle of Non Contradiction *and* the Principle of Excluded Middle hold.

Intuitionistic Logic:

Only the principle of Non Contradiction holds. The Principle of Excluded Middle *does not* hold.

Paraconsistent Logic:

It avoids the effect of "*from the contradiction anything follows*" so that even the Principle of Non Contradiction *need not* hold.

Non-Classical

A Semantic Principle

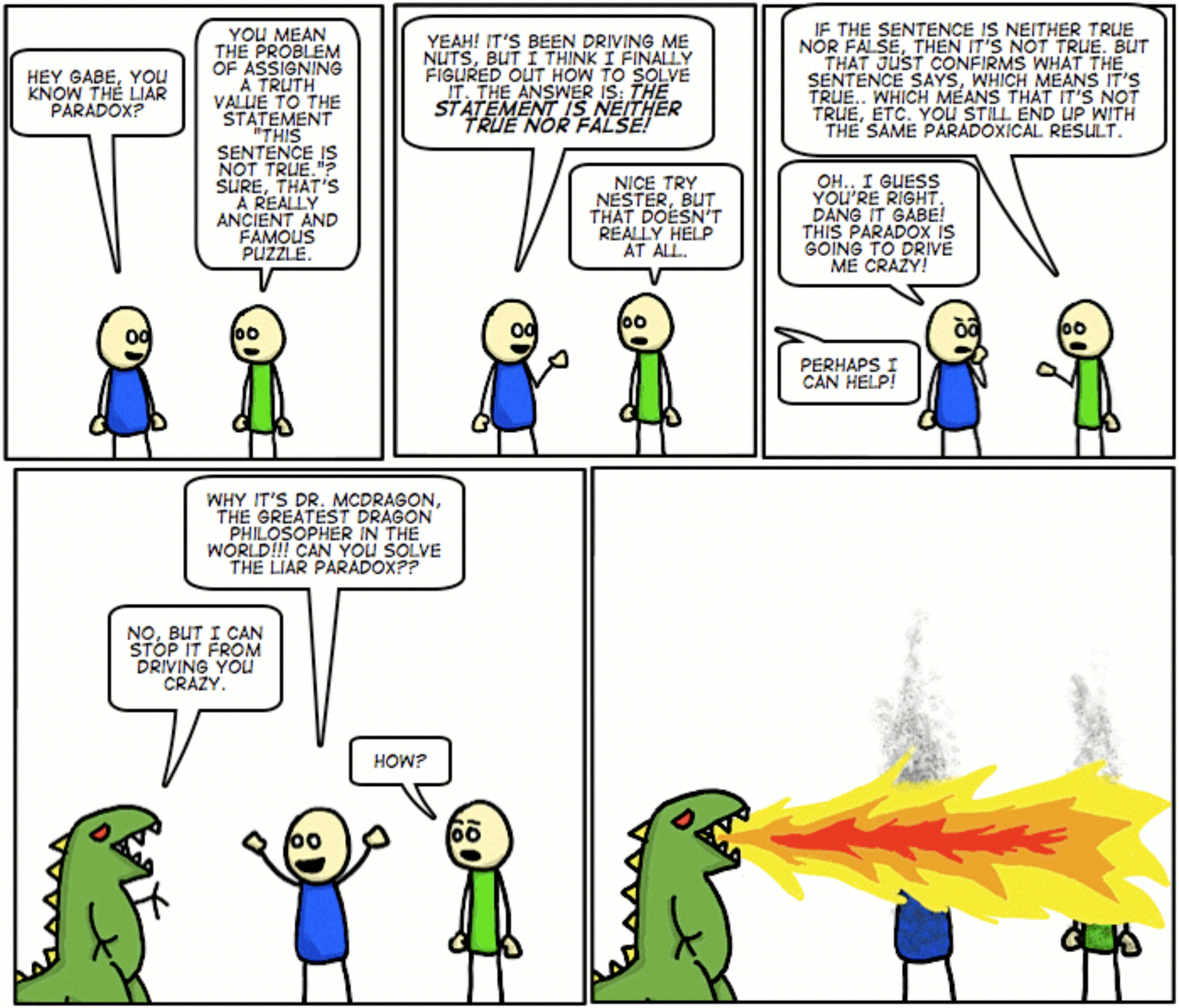
Principle of Bivalence (PB):

Every statement A is either true or false

Every statement can be assigned 1 or 0

Can We Solve the Liar Paradox by Denying the Principle of Bivalence?

The liar paradox is the one triggered by the statement "this statement is not true"


HEY GABE, YOU KNOW THE LIAR PARADOX?
YOU MEAN THE PROBLEM OF ASSIGNING A TRUTH VALUE TO THE STATEMENT "THIS SENTENCE IS NOT TRUE."? SURE, THAT'S A REALLY ANCIENT AND FAMOUS PUZZLE.
YEAH! IT'S BEEN DRIVING ME NUTS, BUT I THINK I FINALLY FIGURED OUT HOW TO SOLVE IT. THE ANSWER IS: *THE STATEMENT IS NEITHER TRUE NOR FALSE!*
NICE TRY NESTER, BUT THAT DOESN'T REALLY HELP AT ALL.
IF THE SENTENCE IS NEITHER TRUE NOR FALSE, THEN IT'S NOT TRUE. BUT THAT JUST CONFIRMS WHAT THE SENTENCE SAYS, WHICH MEANS IT'S TRUE.. WHICH MEANS THAT IT'S NOT TRUE, ETC. YOU STILL END UP WITH THE SAME PARADOXICAL RESULT.
OH.. I GUESS YOU'RE RIGHT. DANG IT GABE! THIS PARADOX IS GOING TO DRIVE ME CRAZY!
PERHAPS I CAN HELP!
WHY IT'S DR. MCDRAGON, THE GREATEST DRAGON PHILOSOPHER IN THE WORLD!!! CAN YOU SOLVE THE LIAR PARADOX??
NO, BUT I CAN STOP IT FROM DRIVING YOU CRAZY.
HOW?

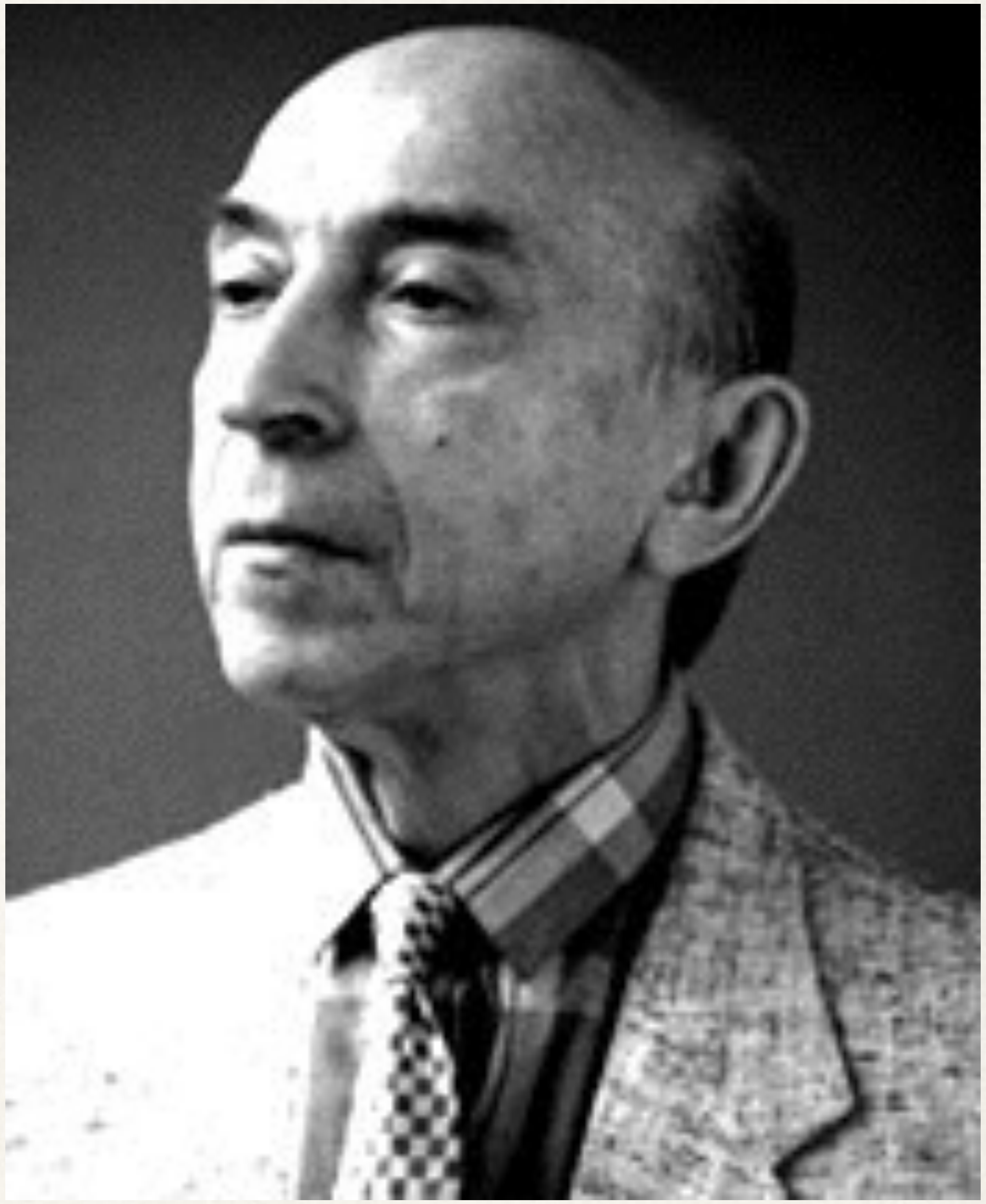
Fuzzy and Multivalued Logics

Suppose statements are assigned truth values between 0 and 1.

Suppose the statement "this statement is not true" is assigned value 0.5.

Does this solve the paradox?

The statement would be **half-true** and *therefore also* **half-false**.



Zadeh, Inventor of Fuzzy Logic

The Three Principles Compared

Principle of Non Contradiction (PNC):

not-(A **and** (**not**-A))

A and not-A cannot both be true (at the same time)

Principle of Excluded Middle (PEM):

(A **or** (**not**-A))

Either A is true or not-A is true

They can be expressed through logical formulas. They are syntactically expressible.

Principle of Bivalence (PB):

Every statement A is either true or false

It is a semantic principle. It is not syntactically expressible.

And What About This?

Consider

"the next sentence is not true"

"the previous sentence is true"

If the first sentence is true, the second sentence is not true. Now, the second sentence asserts that the first sentence is true, so if the second sentence is not true, the **the first sentence is not true.**

If the first sentence is not true, the second sentence is true. (**Uhm…**) Now, the second sentence asserts that the first sentence is true, so **the first sentence is true**.