Truth	*Validity*	*Logical Consequence*	*Equivalence*
$V \vDash \psi$	$\vDash \psi$	$\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$	$\phi \equiv \psi$

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

Week 2 — Friday Class

Overview of Key Notions

- <u>Truth</u> $V \vDash \psi$ *iff* **valuation V makes ψ true**

- <u>Validity</u> $\vDash \psi$ *iff* **all valuations V's make ψ true**

- <u>Logical Consequence</u>

$\phi_1, \phi_2, \ldots, \phi_k \vDash \psi$ *iff* **all valuations V's that make $\phi_1, \phi_2, \ldots, \phi_k$ true make ψ true**

iff **for all valuations V's [if V makes $\phi_1, \phi_2, \ldots, \phi_k$ true, V makes ψ true]**

- <u>Logical equivalence</u> $\phi \equiv \psi$ *iff* $\phi \vDash \psi$ **and** $\psi \vDash \phi$

What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a formula **relative to ONE Valuation**

$$V \vDash \psi$$

How can we evaluate a formula **relative to ALL valuations?**

How to Think About a Valuation

For any (atomic) formula in the language a valuation ***V*** tells us whether the formula is true (value **1**) or false (value **0**).

You can think of **V** as **selecting one possible complete description of the world as a whole** (in so far as the world is describable through language)

So, each **V** represent one possible selection of a complete description of the world.

How **MANY** Valuations Functions?

With **one** atomic proposition, there are **two** possible valuations.

With **two** atomic propositions, there are **four** possible valuations.

With **three** atomic propositions, there are **2^3=8** possible valuations.

With **n** atomic propositions, there are **2^n** possible valuations.

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1		1		1		1
1		1		0		0
0		0		1		1
0		0		0		0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1		1	1	1		1
1		1	0	0		0
0		0	1	1		1
0		0	1	0		0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1	1	1	1	1		1
1	0	1	0	0		0
0	0	0	1	1		1
0	0	0	1	0		0

Evaluating One Formula Relative to ALL Valuations

$(p$	$\wedge$	$(p$	$\to$	$q))$	$\to$	q
1	1	1	1	1	**1**	1
1	0	1	0	0	**1**	0
0	0	0	1	1	**1**	1
0	0	0	1	0	**1**	0

Evaluating One Formula Relative to ALL Valuations

If a formula is true regardless of the selection of the valuation function, the formula is true no matter what the world is like.

$(p$	$\wedge$	$(p$	$\rightarrow$	$q))$	$\rightarrow$	q
1	1	1	1	1	**1**	1
1	0	1	0	0	**1**	0
0	0	0	1	1	**1**	1
0	0	0	1	0	**1**	0

Always true

$\neg$	$\neg$	p
1	0	1
0	1	0

Sometimes true

Classification of Formulas

- Those that are never true (**contradiction**):

$$p \wedge (\neg p), \ldots$$

- Those that can be true (**satisfiable**):

$$(\neg p) \vee q, \ldots$$

- Those that are always true (**valid**, **tautology**):

$$(p \wedge (p \rightarrow q)) \rightarrow q, \ldots$$

If the formula φ is valid, we write $\models \varphi$

The expression
$V \models \phi$
means that ϕ is true *relative to ONE valuation*.
Instead, the expression
$\models \phi$
means that ϕ is true *relative to ALL valuations*.

Validity of PEM and PNC

ϕ	$\vee$	$\neg$	ϕ
1	1	0	1
0	1	1	0

$\neg$	$(\phi$	$\wedge$	$\neg$	$\phi)$
1	1	0	0	1
1	0	0	1	0

If we assume that formulas can take value 0 or 1 (i.e. **principle of bivalence**), then PEM and PNC are both **valid**.

We can write:

$\vDash \phi \vee \neg\phi$

and

$\vDash \neg(\phi \wedge \neg\phi)$

What Happens to PEM and PNC if we Drop Bivalence?

For you to
discover in the
homework

Establishing the Equivalence

$(\phi \rightarrow \psi) \equiv \neg\phi \vee \psi$

$(\phi$	$\rightarrow$	$\psi)$	$\equiv$	$(\neg$	ϕ	$\vee$	$\psi)$
1	1	1		0	1	1	1
1	0	0		0	1	0	0
0	1	1		1	0	1	1
0	1	0		1	0	1	0

Establishing the Equivalence

$$(\phi \wedge \psi) \equiv \neg(\neg\phi \vee \neg\psi)$$

$(\phi$	$\wedge$	$\psi)$	$\equiv$	$\neg$	$(\neg$	ϕ	$\vee$	$\neg$	$\psi)$
1	1	1		1	0	1	0	0	1
1	0	0		0	0	1	1	1	0
0	0	1		0	1	0	1	0	1
0	0	0		0	1	0	1	1	0

So…Connectives Can Be Inter-defined!

Useful equivalences

$(\phi \to \psi) \equiv \neg\phi \vee \psi$

$(\phi \wedge \psi) \equiv \neg(\neg\phi \vee \neg\psi)$

$(\phi \leftrightarrow \psi) \equiv (\phi \to \psi) \wedge (\psi \to \phi)$

These equivalences show that we only need ∨ and ¬ to express all other connectives such as ∧, → and ↔

What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a formula **relative to ONE Valuation**

How to evaluate a formula **relative to ALL valuations**

Can we get an account of (deductively) **valid argument**?

Deductively Valid Arguments

<u>Informally speaking</u>, an *argument* is said to be **deductively valid**

if and only if

whenever **the premises** are **true**, the **conclusion** is **always true**.

<u>Given the semantics of propositional logic</u>, an *argument* is said to be **deductively valid**

if and only if

whenever **all** valuations that make **true** the **premises** make **true** the **conclusion**.

This definition is **system-relative**; it applies within the system of propositional logic.

Recall Modus Ponens

Premise 1: If you take the medication, then you will get better
Premise 2: You are taking the medication

Conclusion: You will get better

Modus Ponens:

If p, then q
p

---

q

Modus Ponens:

$p \rightarrow q$
q

---

q

Is Modus Ponens Valid?

p	$(p$	$\rightarrow$	$q)$	q
1	1	1	1	1
1	1	0	0	0
0	0	1	1	1
0	0	1	0	0

We only need to check the first line of the table because this is where the premises are all true.

We can write

$p, p \rightarrow q \vDash q$

Recall Modus Tollens

Premise 1: If you take the medication, then you will get better
Premise 2: You are NOT getting better

Conclusion: You are NOT taking the medication

Modus Tollens:

If ***p***, then ***q***
not-q

---

not-p

Modus Tollens:

$p \rightarrow q$
$\neg q$

$\neg p$

Is Modus Tollens Valid?

$\neg q$	$(p$	$\to$	$q)$	$\neg p$
0	1	**1**	1	**0**
1	1	**0**	0	**0**
0	0	**1**	1	**1**
1	0	**1**	0	**1**

We only need to check the last line of the table because this is where the premises are all true.

We can write

$$\neg q, p \to q \models \neg p$$

Is “Denying the Consequent” a Valid Argument Pattern?

Premise 1: If the money supply increases by less than 5%, inflation will decrease
Premise 2: The money supply does NOT increase by less than 5%

Conclusion: Inflation will NOT decrease

$$\begin{array}{l} \mathbf{p \rightarrow q} \\ \mathbf{\neg p} \\ \hline \mathbf{\neg q} \end{array}$$

If you construct the appropriate truth table, you see that this argument patters is NOT valid.

Truth Table Method to Check "Denying the Consequent"

¬p	(p	→	q)		¬q
0	1	1	1		0
0	1	0	0		1
1	0	1	1		0
1	0	1	0		1

Not all valuations that make true the premises ***p →q*** *and* ***¬p*** *make true the conclusion* ***¬q****. So "Denying the Consequent" is not valid in propositional logic.*

Validity is Relative to the Logical System

We could formally establish - **within the system of propositional logic -** that *Modus Ponens* and *Modus Tollens* are valid argument patterns, while *Denying the Consequent* is not.

But how significant is this result? Should we be convinced by it?

We should always bear in mind that **formal proofs of validity are relative to a logical system**.

But **is our logical system adequate** for what we want it to do, e.g. accounting for good reasoning?

A Clarification on Truth-Functional Connectives

Truth-Functional Connectives

A **one-place** connective **C** is used truth-functionally whenever the truth value of the formula **Cϕ** is **a function of** (is completely determined by) the truth value of the constituent formula **ϕ.** *An **example** of a one-place truth functional connective is ¬.*

A **two-place** connective **C** is used truth-functionally whenever the truth value of the formula **(ϕ C ψ)** is **a function of** (is completely determined by) the truth values of the constituent formulas **ϕ** and **ψ.** *An **example** of a two-place truth functional connective is ∧.*

And similarly for any **n-ary** connective…

AND-THEN Is Not a Truth-Functional Connective

Truth-functional connective

φ	$\wedge$	ψ
1	**1**	1
1	**0**	0
0	**0**	1
0	**0**	0

Truth-functional connective

φ	$\rightarrow$	ψ
1	**1**	1
1	**0**	0
0	**1**	1
0	**1**	0

Not a truth-functional connective

ϕ	AND-THEN	ψ
1	????	1
1	0	0
0	0	1
0	0	0

In one case, assigning truth values to ϕ and ψ does not determine the truth value of "ϕ **AND-THEN** ψ". *The temporal order of* ϕ *and* ψ *matters, not merely their truth values.*

Other Examples of Non-Truth Functional Connectives

"I avoid the lecture"
BECAUSE
"the instructor is confusing"

ϕ	BECAUSE	ψ
1	????	1
1	0	0
0	0	1
0	0	0

"Wittgenstein wrote his thesis"
WHILE
"he was fighting in the Great War"

ϕ	WHILE	ψ
1	????	1
1	0	0
0	0	1
0	0	0