### TruthValidityLogical ConsequenceEquivalence $V \vDash \psi$ $\vDash \psi$ $\phi_1, \phi_2, \dots, \phi_k \vDash \psi$ $\phi \equiv \psi$

#### PHIL 50 - Introduction to Logic

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Week 2 — Friday Class

#### **Overview of Key Notions**

- \* <u>Truth</u>  $V \vDash \psi$  *iff* valuation V makes  $\psi$  true
- \* <u>Validity</u>  $\models \psi$  *iff* all valuations V's make  $\psi$  true
- Logical Consequence

 $\phi_1, \phi_2, ..., \phi_k \models \psi$  *iff* all valuations *V*'s that make  $\phi_1, \phi_2, ..., \phi_k$  true make  $\psi$  true

*iff* for all valuations V's [if V makes  $\phi_1, \phi_2, ..., \phi_k$  true, V makes  $\psi$  true]

• Logical equivalence  $\phi \equiv \psi$  iff  $\phi \models \psi$  and  $\psi \models \phi$ 

## What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a formula **relative to ONE Valuation** 

 $V \vDash \psi$ 

How can we evaluate a formula relative to ALL valuations?

#### How to Think About a Valuation

For any (atomic) formula in the language a valuation *V* tells us whether the formula is true (value **1**) or false (value **0**).

You can think of V as selecting one possible complete description of the world as a whole (in so far as the world is describable through language)

So, each V represent one possible selection of a complete description of the world.

#### How MANY Valuations Functions?

With one atomic proposition, there are **two** possible valuations.

With **two** atomic propositions, there are **four** possible valuations.

With **three** atomic propositions, there are **2^3=8** possible valuations.

With **n** atomic propositions, there are **2^n** possible valuations.

<i>(p</i>	$\wedge$	( <i>p</i>	$\rightarrow$	q))	$\rightarrow$	$\boldsymbol{q}$
1		1		1		1
1		1		0		0
0		0		1		1
0		0		0		0

( <i>p</i>	$\wedge$	( <i>p</i>	$\rightarrow$	q))	$\rightarrow$	$oldsymbol{q}$
1		1	1	1		1
1		1	0	0		0
0		0	1	1		1
0		0	1	0		0

<i>(p</i>	$\wedge$	( <i>p</i>	$\rightarrow$	q))	$\rightarrow$	$oldsymbol{q}$
1	1	1	1	1		1
1	0	1	0	0		0
0	0	0	1	1		1
0	0	0	1	0		0

<i>(p</i>	$\wedge$	( <i>p</i>	$\rightarrow$	q))	$\rightarrow$	$\boldsymbol{q}$
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0

If a formula is true regardless of the selection of the valuation function, the formula is true no matter what the world is like.

	( <i>p</i>	$\wedge$	( <i>p</i>	$\rightarrow$	q))	$\rightarrow$	$oldsymbol{q}$	
	1	1	1	1	1	1	1	Always true
		0		0		1	0	
	0	0	0	1	1	1	1	
)	0	0	0	1	0	1	0	
								-
			_	_	p			
			1	0	1			Sometimes true
			0	1	0			Jonneumestrue

#### Classification of Formulas

• Those that are never true (contradiction):

 $p \wedge (\neg p), \ldots$ 

• Those that can be true (**satisfiable**):

 $(\neg p) \lor q, \dots$ 

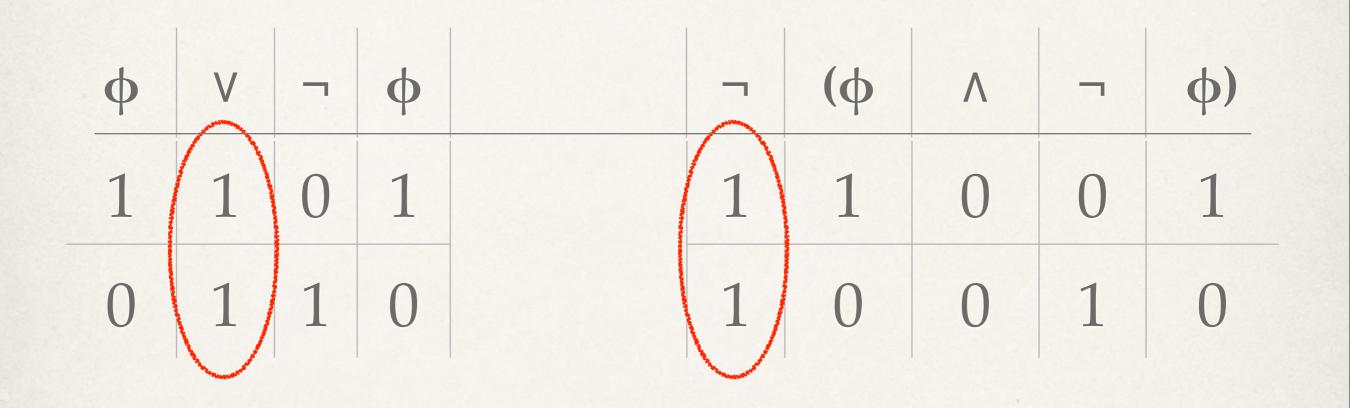
• Those that are always true (valid, tautology):

 $(p \land (p 
ightarrow q)) 
ightarrow q, \ldots$ 

If the formula  $\varphi$  is valid, we write  $\models \varphi$ 

The expression  $V \vDash \phi$ means that  $\phi$  is true relative to ONE valuation. Instead, the expression  $\vDash \phi$ means that  $\phi$  is true relative to ALL valuations.

#### Validity of PEM and PNC



If we assume that formulas can take value 0 or 1 (i.e. **principle of bivalence**), then PEM and PNC are both **valid**.

We can write:  

$$\models \phi \lor \neg \phi$$
and  

$$\models \neg(\phi \land \neg \phi)$$

# What Happens to PEM and PNC if we Drop Bivalence?

For you to discover in the homework

#### Establishing the Equivalence $(\phi \rightarrow \psi) \equiv \neg \phi \lor \psi$



#### Establishing the Equivalence $(\phi \land \psi) \equiv \neg(\neg \phi \lor \neg \psi)$

(φ	٨	ψ)	-	(¬		V	7	ψ)
1	$\begin{pmatrix} 1 \end{pmatrix}$	1	1	0	1		0	1
1		0	0	0	1	1	1	0
0		1	0	1	0	1	0	1
0	0	0	0	1	0 0	1	1	0

#### So...Connectives Can Be Inter-defined!

Useful equivalences

$$(\phi \rightarrow \psi) \equiv \neg \phi \lor \psi$$

$$(\phi \land \psi) \equiv \neg (\neg \phi \lor \neg \psi)$$

$$(\phi \leftrightarrow \psi) \equiv (\phi \rightarrow \psi) \land (\psi \rightarrow \phi)$$

These equivalences show that we only need V and  $\neg$  to express all other connectives such as  $\Lambda$ ,  $\rightarrow$  and  $\leftrightarrow$ 

## What We Have Learned So Far about the **SEMANTICS** of Propositional Logic

How to evaluate a formula **relative to ONE Valuation** 

How to evaluate a formula **relative to ALL valuations** 

Can we get an account of (deductively) valid argument?

#### Deductively Valid Arguments

<u>Informally speaking</u>, an *argument* is said to be **deductively valid** 

if and only if

whenever the premises are true, the conclusion is always true. <u>Given the semantics of</u> <u>propositional logic</u>, an *argument* is said to be **deductively valid** 

if and only if

whenever **all** valuations that make **true** the **premises** make **true** the **conclusion**.

This definition is **systemrelative**; it applies within the system of propositional logic.

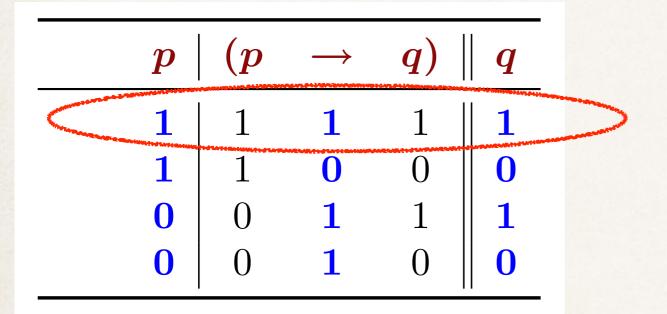
#### **Recall Modus Ponens**

*Premise* 1: If you take the medication, then you will get better *Premise* 2: You are taking the medication

*Conclusion*: You will get better

Modus Ponens:	Modus Ponens:
If <i>p,</i> then <i>q</i> <i>p</i>	$\begin{array}{l} p \rightarrow q \\ q \end{array}$
<i>q</i>	$\overline{q}$

#### Is Modus Ponens Valid?



We only need to check the first line of the table because this is where the premises are all true.

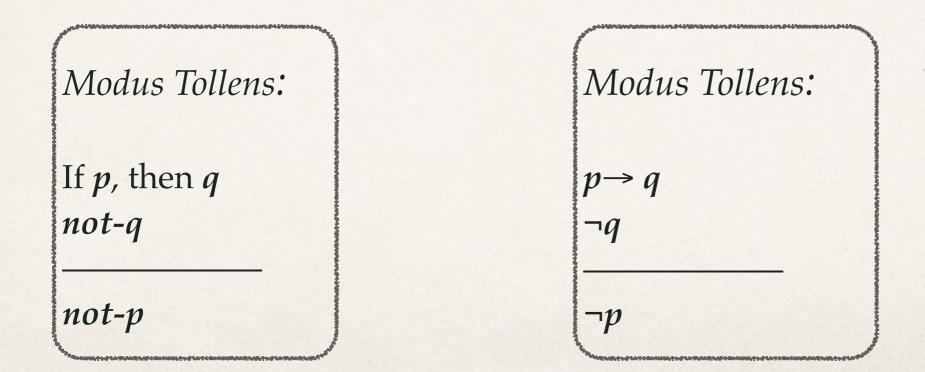
We can write

 $p, p \rightarrow q \vDash q$ 

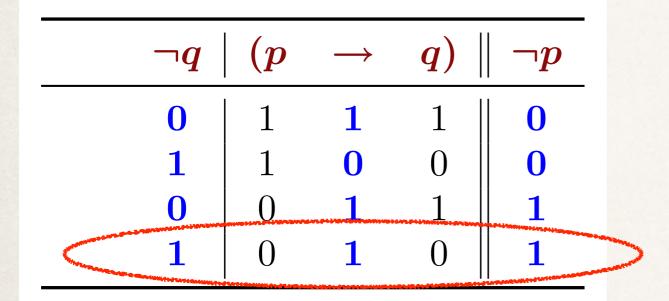
#### **Recall Modus Tollens**

*Premise* 1: If you take the medication, then you will get better *Premise* 2: You are NOT getting better

Conclusion: You are NOT taking the medication



#### Is Modus Tollens Valid?



We only need to check the last line of the table because this is where the premises are all true.

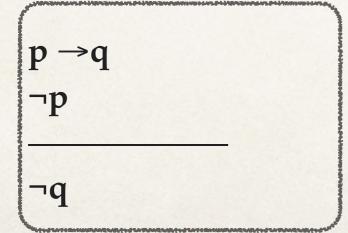
We can write

 $\neg q, p \rightarrow q \vDash \neg p$ 

#### Is "Denying the Consequent" a Valid Argument Pattern?

*Premise* 1: If the money supply increases by less than 5%, inflation will decrease *Premise* 2: The money supply does NOT increase by less than 5%

Conclusion: Inflation will NOT decrease



If you construct the appropriate truth table, you see that this argument patters is NOT valid.

#### Truth Table Method to Check "Denying the Consequent"

¬p	(p	$\rightarrow$	q)	٦q	
0	1	1	1	0	
0	1	0	0	1	
 1	0	1	1	0	
1	0	1	0	1	

Not all valuations that make true the premises  $p \rightarrow q$ and  $\neg p$  make true the conclusion  $\neg q$ . So "Denying the Consequent" is not valid in propositional logic.

# Validity is Relative to the Logical System

We could formally establish - **within the system of propositional logic -** that *Modus Ponens* and *Modus Tollens* are valid argument patterns, while *Denying the Consequent* is not.

But how significant is this result? Should we be convinced by it?

We should always bear in mind that **formal proofs of validity are relative to a logical system**. But is our logical system adequate for what we want it to do, e.g. accounting for good reasoning?

#### A Clarification on Truth-Functional Connectives

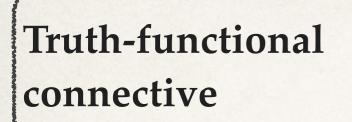
#### **Truth-Functional Connectives**

A one-place connective C is used truth-functionally whenever the truth value of the formula  $C\phi$  is a function of (is completely determined by) the truth value of the constituent formula  $\phi$ . An example of a one-place truth functional connective is ¬.

A two-place connective C is used truth-functionally whenever the truth value of the formula ( $\phi C \psi$ ) is a function of (is completely determined by) the truth values of the constituent formulas  $\phi$  and  $\psi$ . An example of a two-place truth functional connective is  $\Lambda$ .

And similarly for any **n-ary** connective...

#### **AND-THEN** Is Not a Truth-Functional Connective



Truth-functional
connective

Not a truth-functional connective

arphi	$\wedge$	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0

arphi	$\rightarrow$	$\psi$
1	1	1
1	0	0
0	1	1
0	1	0

	φ	AND-THEN	ψ
<	1	????	1
	1	0	0
	0	0	1
	0	0	0

In one case, assigning truth values to  $\phi$  and  $\psi$  does not determine the truth value of " $\phi$ **AND-THEN**  $\psi$ ". The temporal order of  $\phi$  and  $\psi$  matters, not merely their truth values.

#### Other Examples of Non-Truth Functional Connectives

		φ	BECAUSE	ψ
"I avoid the lecture"	<	1	????	1
BECAUSE		1	0	0
"the instructor is confusing"		0	0	1
		0	0	0

"Wittgenstein wrote his thesis" WHILE "he was fighting in the Great War"

	φ	WHILE	ψ
<	1	????	1
	1	0	0
	0	0	1
	0	0	0