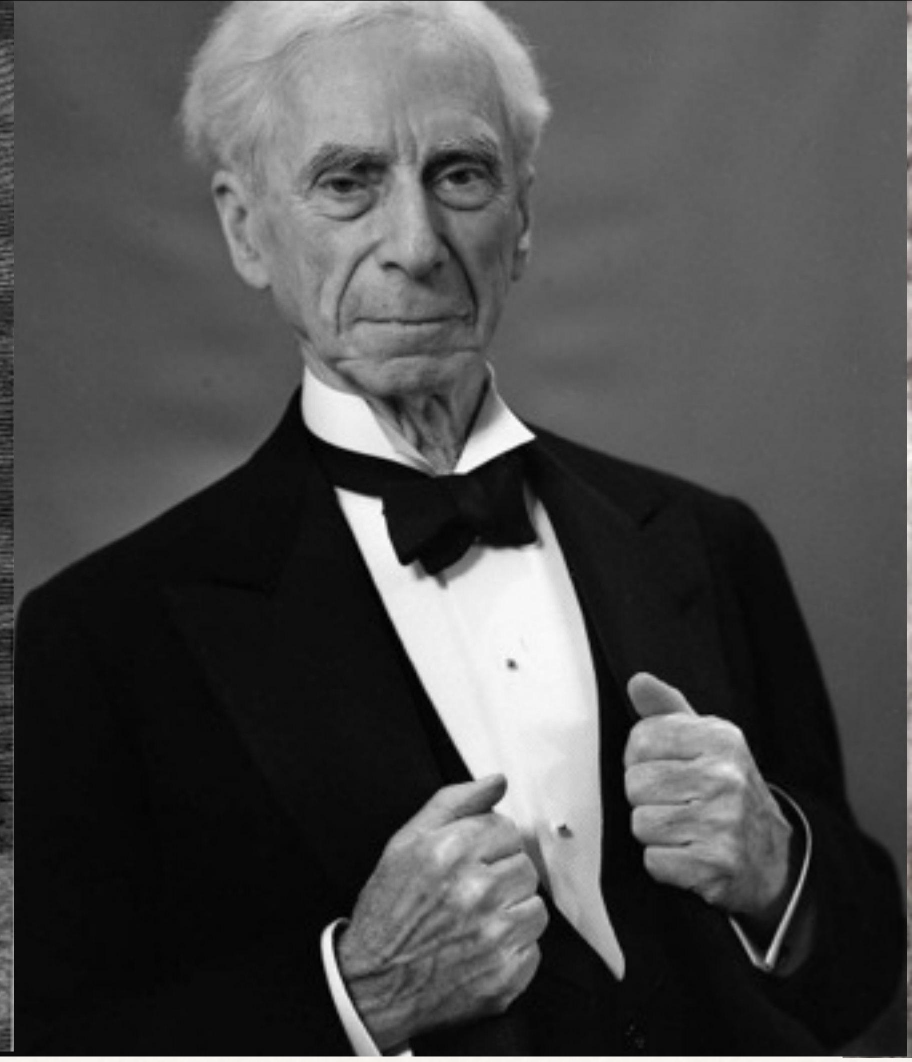
Let There be Light!



Betrand Russell

Ludwig Wittgenstein

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

Week 2 — Wednesday Class

Recall — Truth Tables...

For **conjunction** $\wedge$

φ	$\wedge$	ψ
1	**1**	1
1	**0**	0
0	**0**	1
0	**0**	0

For **disjunction** $\vee$

φ	$\vee$	ψ
1	**1**	1
1	**1**	0
0	**1**	1
0	**0**	0

Conjunction, Disjunction, and Negation…What About Implication?

Extending V for Implication

For **equivalence** $\leftrightarrow$

φ	$\leftrightarrow$	ψ
1	**1**	1
1	**0**	0
0	**0**	1
0	**1**	0

For **implication** $\rightarrow$

φ	$\rightarrow$	ψ
1	**1**	1
1	**0**	0
0	**1**	1
0	**1**	0

Formalizing a Murder Case

Mrs White is guilty.	w
Miss Scarlet is guilty.	s
Colonel Mustard is guilty.	m

- At least one of them is guilty. $w \vee s \vee m$
- Not all of them are guilty. $\neg(w \wedge s \wedge m)$
- If Mrs White is guilty, then Colonel Mustard helped her. $w \rightarrow m$
- If Miss Scarlet is innocent then so is Colonel Mustard. $\neg s \rightarrow \neg m$

Let's Look at the Table for Implication More Closely

φ	$\rightarrow$	ψ
1	1	1
1	0	0
0	1	1
0	1	0

Why is it that whenever the antecedent is false, the **if…then** statement is always true *regardless of the truth value of the consequent*?

We are working with a particular kind of implication, called **material implication**

Does the *if…then* statement

"If it rains, the soil gets wet"

contain a material implication?

Evaluating One Formula Relative to One Valuation

$$\begin{array}{lcccc} & (\neg & p) & \wedge & q \\ V: & 1 & 0 & \mathbf{1} & 1 \end{array} \qquad \boxed{V \models (\neg p) \wedge q}$$

$$\begin{array}{lccccccc} & (p & \wedge & (p & \to & q)) & \to & q \\ V: & 1 & 0 & 1 & 0 & 0 & \mathbf{1} & 0 \end{array} \qquad \boxed{V \models (p \wedge (p \to q)) \to q}$$

$$\begin{array}{lccc} & \neg & \neg & p \\ V: & \mathbf{0} & 1 & 0 \end{array} \qquad \boxed{V \not\models \neg\neg p}$$

$$\begin{array}{lccccccc} & (p & \to & q) & \vee & (q & \to & p) \\ V: & 0 & 1 & 1 & \mathbf{1} & 1 & 0 & 0 \end{array} \qquad \boxed{V \models (p \to q) \vee (q \to p)}$$

Two Side Remarks:

1. Truth-Functional Connectives
2. Logical Atomism

Truth-Functional Connectives

A **one-place** connective **C** is used truth-functionally whenever the truth value of the formula **C**$\boldsymbol{\phi}$ is **a function of** (is completely determined by) the truth value of the constituent formula $\boldsymbol{\phi}$**.** *An example of a one-place truth functional connective is* ¬.

A **two-place** connective **C** is used truth-functionally whenever the truth value of the formula **(**$\boldsymbol{\phi}$ **C** $\boldsymbol{\psi}$**)** is **a function of** (is completely determined by) the truth values of the constituent formulas $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$**.** *An example of a two-place truth functional connective is* ∧.

And similarly for any **n-ary** connective…

Which Connectives are (Used) Truth-Functional(ly) and Which Aren't?

In the beginning God created the heaven **and** the earth.

And the earth was without form, **and** void; **and** darkness was upon the face of the deep. **And** the Spirit of God moved upon the face of the waters.

And God said, Let there be light: **and** there was light.

And God saw the light, that it was good: **and** God divided the light from the darkness.

And God called the light Day, **and** the darkness he called Night.

The "and" within the blue rectangle are used truth-functionally, because they have no temporal meaning.

Not a connective!

The "and" within the red rectangle is not used truth-functionally because it has a temporal meaning.

The Creation of Light

Gustave Doré

Logical Atomism

The truth value of atomic formulas **does not depend** on the truth value of other atomic formulas

So, **no** atomic formula can **contradict** another atomic formula

The truth value of a complex formulas **depends** on the truth value of its atomic formulas

If two complex formulas **share any atomic formula**, the truth values of the two complex formulas aren't independent.