Alice

Descartes

Buddhist Monks Debating

PHIL 50 - Introduction to Logic

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Week 3 — Friday Class - Derivations in Propositional Logic (CONTINUED)

Rules From Wednesday

$$\frac{\bot}{\psi}\ \bot$$

$$\frac{\begin{array}{c}[\neg\phi]^{i}\\ \vdots\\ \bot\end{array}}{\phi}\ \mathbf{RAA}^{i}$$

RAA is a Powerful Derivation Rule

Admirable Consequence (*consequentia mirabilis*)

Consider the proposition "I exist"

Let's assume for the sake of argument that its negation holds, i.e. "I do not exist."

If I do not exist, in order to entertain the proposition "I do not exist" I need to exist, whence "I exist."

$(\neg\phi \rightarrow \phi) \rightarrow \phi$

Descartes (sort of…)

Establishing $\vdash (\neg\phi \rightarrow \phi) \rightarrow \phi$

$[\neg\phi \rightarrow \phi]^1 \quad [\neg\phi]^2$

—————————— $\rightarrow$**E**

$\phi \quad [\neg\phi]^2$

—————————— $\rightarrow$**E**

$\bot$

—————— **RAA**^{2}

ϕ

—————————— $\rightarrow$**I**^{1}

$(\neg\phi \rightarrow \phi) \rightarrow \phi$

The derivation of $(\neg\phi \rightarrow \phi) \rightarrow \phi$ crucially rests upon **RAA**

And Now the Rules for ∨

Rules for ∨

$$\frac{\phi}{\phi \vee \psi}\ \vee\mathbf{I}$$

$$\frac{\psi}{\phi \vee \psi}\ \vee\mathbf{I}$$

$$\frac{\phi \vee \psi \qquad \begin{matrix}[\phi]^i \\ \vdots \\ \sigma\end{matrix} \qquad \begin{matrix}[\psi]^i \\ \vdots \\ \sigma\end{matrix}}{\sigma}\ \vee\mathbf{E}^i$$

If you derive a formula, you can always add a disjunct to it.

This formalizes **proof by cases**

Proof by Cases: *Alice in Wonderland*

Soon her eye fell on a little glass box that was lying under the table: she opened it, and a found a very small cake, on which the words "EAT ME" were beautifully marked in currants.

*"Well, I'll eat it, " said Alice, "and **if it makes me larger**, I can reach the key; and **if it makes me smaller**, I can creep under the door; so **either way I'll get into the garden**.*

Proof by Cases: *Buddhist Logic*

If something is known, *giving a definition of it is useless.*

If something is not known, *giving a definition of it is impossible, and hence useless.*

Either way giving a definition of something is useless.

Theodore Stcherbatsky, *Buddhist Logic*

On Rule ∨E

$$\dfrac{\phi \vee \psi \qquad \begin{matrix}[\phi]^{i} \\ \vdots \\ \sigma\end{matrix} \qquad \begin{matrix}[\psi]^{i} \\ \vdots \\ \sigma\end{matrix}}{\sigma}\ \vee\mathbf{E}^{i}$$

If by assuming **ϕ**, one can derive **σ**, and by assuming **ψ**, one can also derive **σ**, then one can derive **σ** from **$\phi \vee \psi$.**

The formula **$\phi \vee \psi$** will become a new assumption unless it is the result of another independent derivation.

Establishing $\vdash (\phi \lor \psi) \to (\psi \lor \phi)$

$$\dfrac{[\phi \lor \psi]^2 \qquad \dfrac{[\phi]^1}{\psi \lor \phi}\,\lor\mathbf{I} \qquad \dfrac{[\psi]^1}{\psi \lor \phi}\,\lor\mathbf{I}}{\dfrac{\psi \lor \phi}{(\phi \lor \psi) \to (\psi \lor \phi)}\,{\to}\mathbf{I}^2}\,\lor\mathbf{E}^1$$

Summary: Second Batch of Rules

$$\frac{\bot}{\psi}\ \bot$$

$$\frac{\begin{array}{c}[\neg\phi]^{i}\\ \vdots\\ \bot\end{array}}{\phi}\ \mathbf{RAA}^{i}$$

$$\frac{\phi}{\phi \vee \psi}\ \vee\mathbf{I}$$

$$\frac{\psi}{\phi \vee \psi}\ \vee\mathbf{I}$$

$$\frac{\phi \vee \psi \qquad \begin{array}{c}[\phi]^{i}\\ \vdots\\ \sigma\end{array} \qquad \begin{array}{c}[\psi]^{i}\\ \vdots\\ \sigma\end{array}}{\sigma}\ \vee\mathbf{E}^{i}$$

Summary: First Batch of Rules (Monday)

$$\frac{\phi}{\phi}\,\mathbf{R}$$

$$\frac{\phi \quad \psi}{\phi \wedge \psi}\,\mathbf{\wedge I}$$

$$\frac{\phi \wedge \psi}{\phi}\,\mathbf{\wedge E}$$

$$\frac{\phi \wedge \psi}{\psi}\,\mathbf{\wedge E}$$

$$\frac{\begin{array}{c}[\phi]^{i}\\ \vdots \\ \psi\end{array}}{\phi \to \psi}\,\mathbf{\to I^{i}}$$

$$\frac{\phi \quad \phi \to \psi}{\psi}\,\mathbf{\to E}$$

Derivability: $\vdash$

$\vdash \psi$ *iff*

there is a derivation of ψ in which all assumptions are canceled.

$\phi_1, \phi_2, \ldots, \phi_k \vdash \psi$ *iff*

there is a derivation of ψ from assumptions $\phi_1, \phi_2, \ldots, \phi_k$

The Equivalence of $\vdash$ and $\models$

$$\phi_1, \phi_2, \ldots, \phi_k \vdash \psi$$

SOUNDNESS $\Downarrow$

$\Uparrow$ **COMPLETENESS**

$$\phi_1, \phi_2, \ldots, \phi_k \models \psi$$

More on this next week….