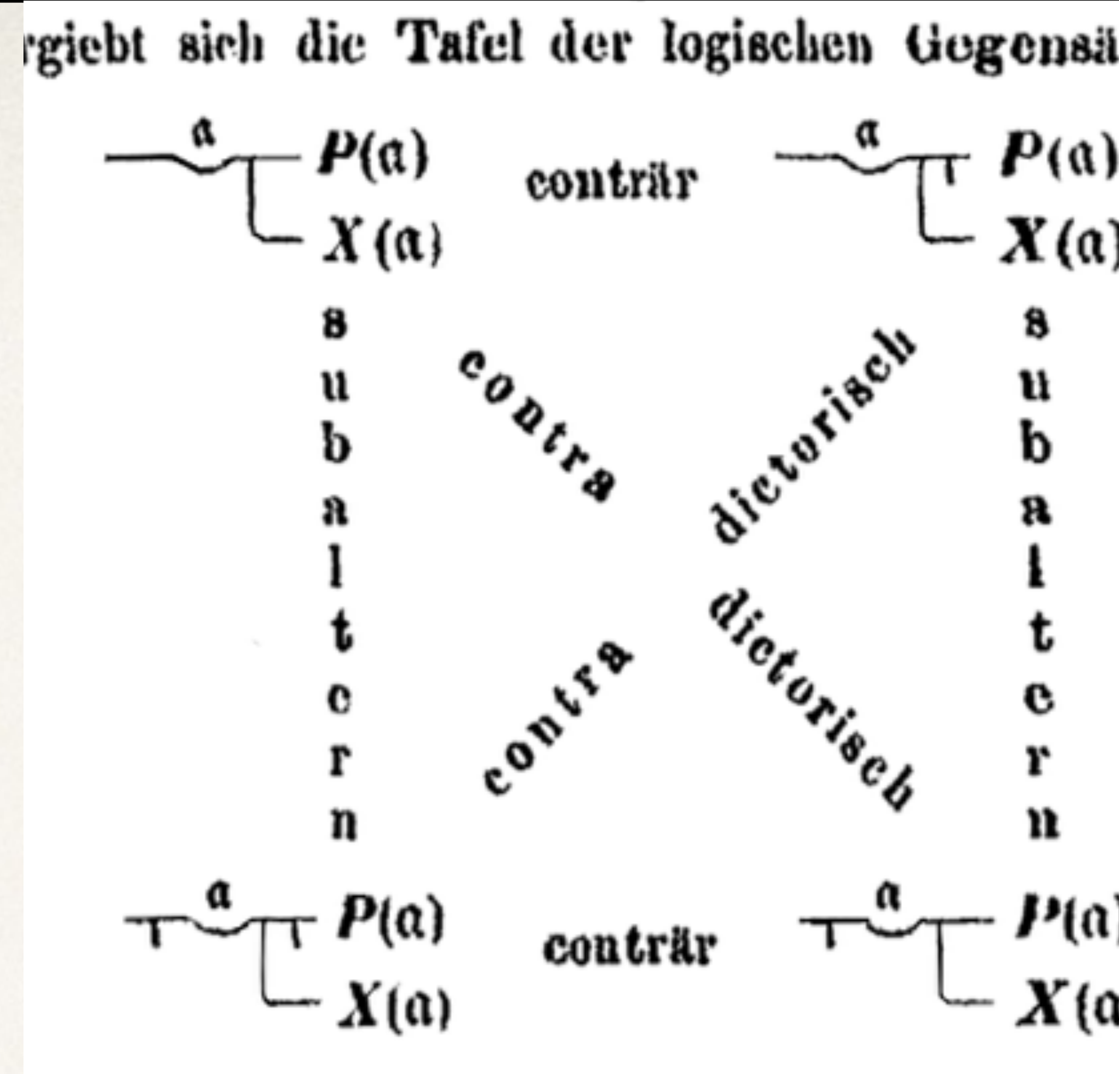


Squares of Oppositions

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

Week 5 — Friday Class - The Square of Oppositions

Monday Class: Checking Validity Using Set Theory

Syllogism

All A are B
All B are C

All A are C

Set-theoretic translation

$\boldsymbol{A \subseteq B}$
$\boldsymbol{B \subseteq C}$

$\boldsymbol{A \subseteq C}$

We showed the validity of the syllogism by **relying on reasoning about** $\subseteq$

Wednesday Class: Checking Validity Using Set Theory

Syllogism	**Set-theoretic translation**
No A is B	$A \cap B = \varnothing$
All C are A	$C \subseteq A$
No C is B	$C \cap B = \varnothing$

We showed the validity of the syllogism by **relying on reasoning about the subset relation $\subseteq$ and the intersection operation $\cap$**

Wednesday Class: Counterexample to Validity

All tomatoes are rotten
Some chickpeas are **not** rotten

No chickpeas are tomatoes

All *A* is *B*
Some *C* are **not** *B*

No *C* is *A*

$A \subseteq B$
$C \nsubseteq B$

$C \cap A = \varnothing$

Counterexample:

Tomatoes = {a}
Rotten = {a, b}
Chickpeas = {a, b, c}

Be Aware of Russell's Paradox

We should define our sets carefully to avoid the contradiction

Instead of $\{ \mathbf{x} \mid \mathbf{x} \textbf{ is } \mathbf{P} \}$, it's better to write $\{ \mathbf{x} \in \mathbf{U} \mid \mathbf{x} \textbf{ is } \mathbf{P} \}$ where **U** is the universe of discourse

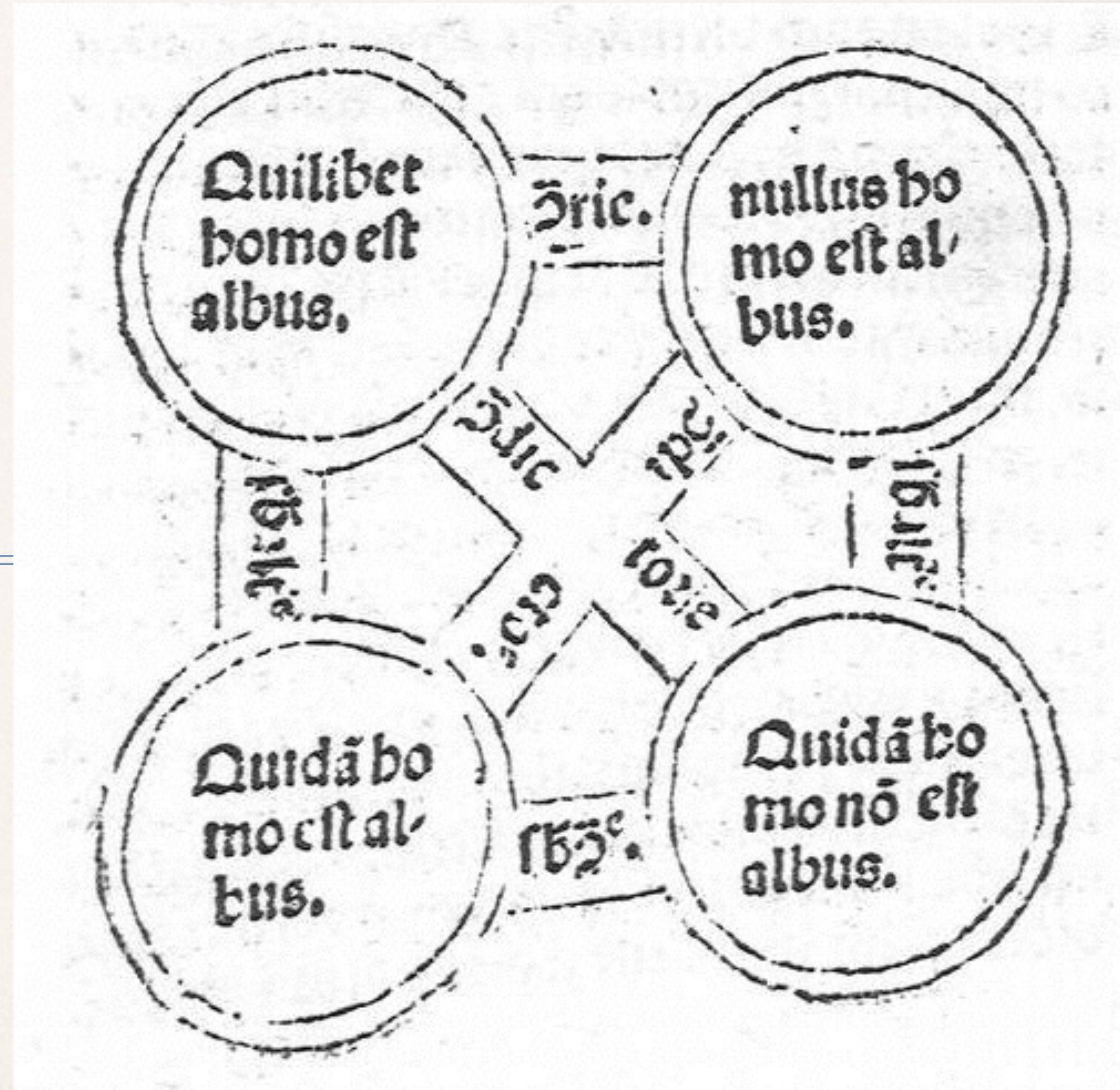
The set of all sets cannot itself be a set

Let's Now Turn to

The Square of Oppositions

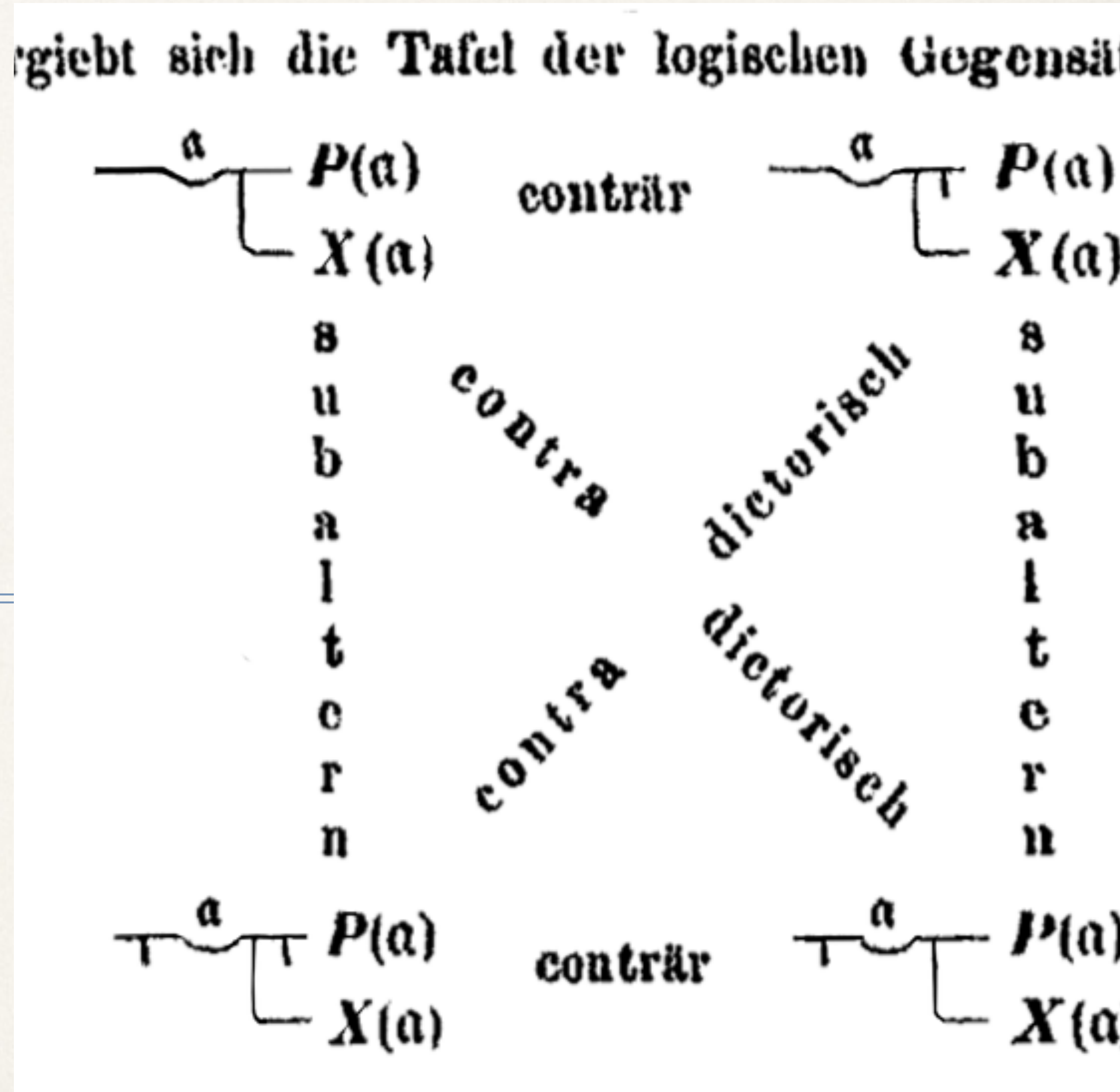
Medieval Square of Oppositions

From Latin…



Frege's Square of Oppositions

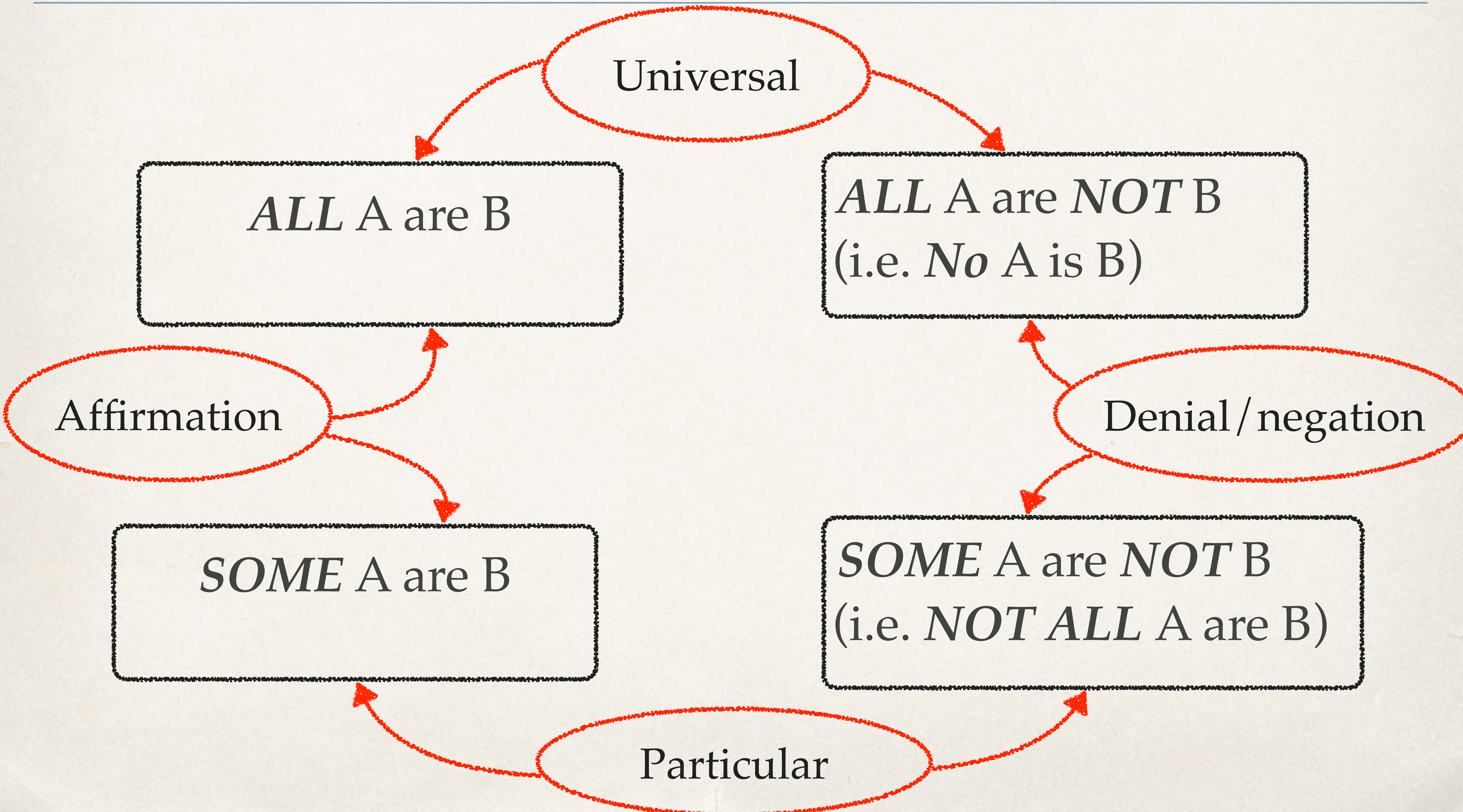
…to German and symbolic notation



The Four Types of Statements That Can Occur as Premises or Conclusion in a Syllogism

All A are B
Some A are B
All A are **not** B (i.e. **No** A is B)
Some A are **not** B (i.e. **Not all** A are B)

Aristotle's Classification of Statements



Four Types of Statements

Universal affirmative	**All** lizards are bald creatures
Universal negative	**All** lizards are **not** bald creatures (i.e. **No** lizard is a bald creature)
Particular affirmative	**Some** lizards are bald creatures
Particular negative	**Some** lizards are **not** bald creatures (i.e. **Not all** lizards are bald creatures)

Aristotle's Square of Oppositions

ALL A are B

ALL A are ***NOT*** B
(i.e. ***No*** A is B)

SOME A are B

SOME A are ***NOT*** B
(i.e. ***NOT ALL*** A are B)

And the set-theoretic translation:

$A \subseteq B$

$A \cap B = \emptyset$

$A \cap B \neq \emptyset$

$A \nsubseteq B$

Contradictory Statements

ALL A are B

ALL A are ***NOT*** B
(i.e. ***No*** A is B)

Contradictories

Contradictories

SOME A are B

SOME A are ***NOT*** B
(i.e. ***NOT ALL*** A are B)

Contradictory statements are such that **they cannot be both true, nor can they be both false.**

…And in the Language of Set Theory

$A \subseteq B$

$A \cap B = \varnothing$

Contradictories

Contradictories

$A \cap B \neq \varnothing$

$A \not\subseteq B$

In the language of set theory, it is easier to see the relation of contradiction

Contrary Statements

ALL A are B

Contraries

ALL A are ***NOT*** B
(i.e. ***No*** A is B)

SOME A are B

SOME A are ***NOT*** B
(i.e. ***NOT ALL*** A are B)

Contraries are such that **they cannot be both true, but they can be both false** (*provided A is non-empty*).

Contraries Cannot Be Both True

We need to show that "**All A are B**" and "**All A are NOT B**" cannot be both true. In terms of set theory, we need to show that from (*) $\mathbf{A} \subseteq \mathbf{B}$ and (**) $\mathbf{A} \cap \mathbf{B} = \varnothing$, we arrive at a contradiction.

Suppose (***) there is an element **a** such that $\mathbf{a} \in \mathbf{A}$

Since (**) $\mathbf{A} \subseteq \mathbf{B}$, it follows that $\mathbf{a} \in \mathbf{B}$.

So, $\mathbf{a} \in \mathbf{A}$ and $\mathbf{a} \in \mathbf{B}$, which means that $\mathbf{A} \cap \mathbf{B} \neq \varnothing$.

This contradicts (**) $\mathbf{A} \cap \mathbf{B} = \varnothing$.

***NB:** If **A** is empty, then the argument does not work!*

Subcontrary Statements

ALL A are B

ALL A are ***NOT*** B
(i.e. ***No*** A is B)

SOME A are B

Subcontraries

SOME A are ***NOT*** B
(i.e. ***NOT ALL*** A are B)

Subcontrary statements are such that **they cannot be both false, but they can be both true.**

Subcontraries Cannot Be Both False

We need to show that "**Some A are B**" (think of $\mathbf{A} \cap \mathbf{B} \neq \varnothing$) and "**Some A are NOT B**" (think of $\mathbf{A} \not\subseteq \mathbf{B}$) cannot be both false.

So, we need to show *if* $\mathbf{A} \cap \mathbf{B} \neq \varnothing$ does not hold *and* $\mathbf{A} \not\subseteq \mathbf{B}$ do not hold either, *then* we arrive at a contradiction. And this amounts to showing that from $\mathbf{A} \cap \mathbf{B} = \varnothing$ and $\mathbf{A} \subseteq \mathbf{B}$ we arrive at a contradiction.

We have proven this earlier while reasoning about contrary statements (*assuming* **A** *is non-empty*).

Subaltern Statements

ALL A are B

Subalterns

SOME A are B

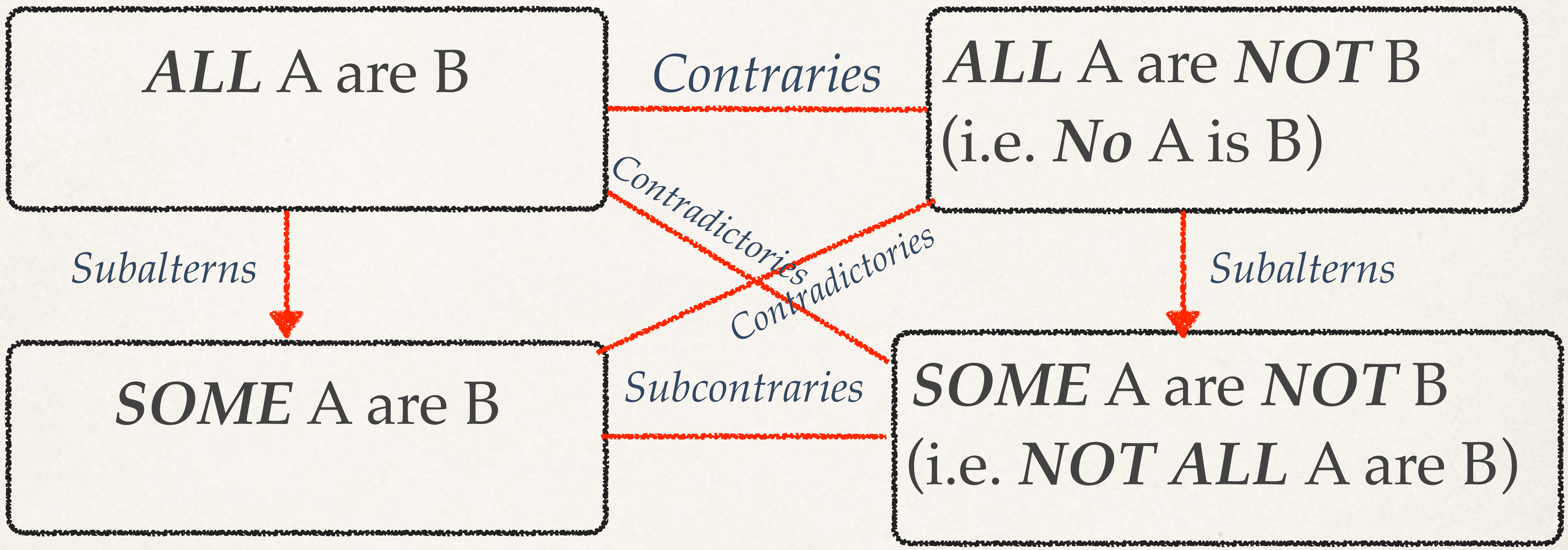
ALL A are ***NOT*** B
(i.e. ***No*** A is B)

Subalterns

SOME A are ***NOT*** B
(i.e. ***NOT ALL*** A are B)

Aristotle believed that "**All A are B**" implies "**Some A are B**" and that "**All A are not B**" implies "**Some A are not B**". *This, once again, holds provided the set A is non-empty.*

The Square of Oppositions at a Glance



The square of oppositions captures, in a general form, the logical relations between four types of statements. **It is an example of one of the first logical systems.**

Medieval Naming Conventions

Medieval Logicians Called this Syllogistic Pattern BARBARA…

All *B* are *C*
All *A* are *B*

All *A* are *C*

Why such a name?

Some Latin

"**adfirmo**" is the Latin for "I assert"

"**nego**" is the Latin for "I deny"

The *universal affirmative* was referred to as **A**

The *particular affirmative* was referred as **I**

The *universal negative* was referred to as **E**

The *particular negative* was referred to as **O**

Adf**I**rmo

n**E**g**O**

Back to BARBARA

All *B* are *C*
All *A* are *B*

All *A* are *C*

The syllogism contains *three universal affirmative statements*, each referred to as **A**, hence the mnemonic name b**A**rb**A**r**A**

Example of CELARENT (c**E**l**A**r**E**nt) Syllogistic Pattern

No *A* is *B*
All *C* are *A*

No *C* is *B*

E
A

E

A comparison Between Propositional Logic and Syllogistic Logic

Syllogistic logic allows us to reason with statements (or formulas) of the form

All A are B

Some A are B

All A are Not B
(i.e. No A is B)

Some A are Not B
(i.e. Not all A are B)

Propositional logic allows us to reason with statements (or formulas) of the form

p, q, r …

$\neg \psi$

$\phi \wedge \psi$

$\phi \vee \phi$

$\phi \rightarrow \psi$

Can we combine the two logics into a more powerful logic?

Combining **Propositional Logic** with **Syllogistic Logic** is Possible through **Predicate Logic**

This is for next week….