$$R(x,y)$$

2-place predicates
for relations

$$\forall\exists, \exists\forall$$
$$\forall\forall, \exists\exists$$

Nested quantifiers

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

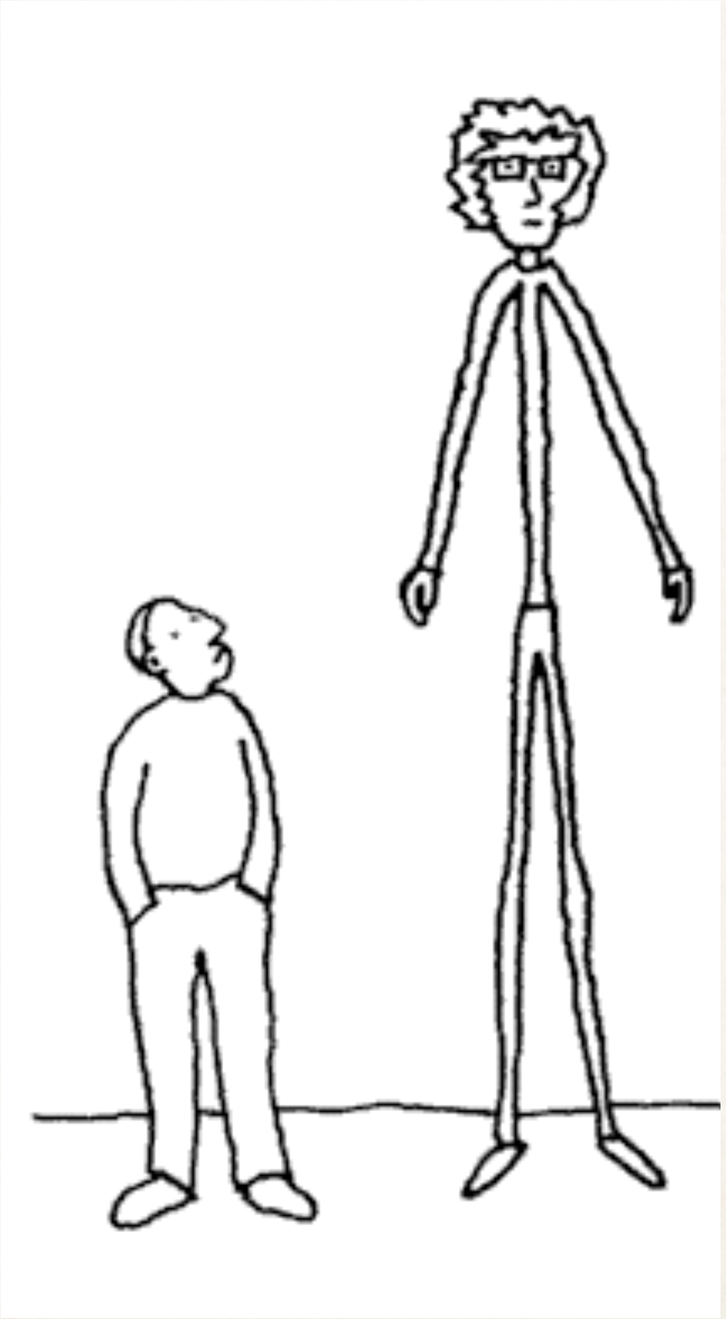
Week 6 — Friday Class - Predicate Logic

Two Innovations of Predicate Logic

1. Ability to express relations between objects

2. Ability to nest quantifiers

The Relation "Taller-than"



Mark is taller than John

In Predicate Logic:

Let "**Taller-than**"be a 2-place predicate for the relation *taller-than*. Let "**mark**" and "**john**" be two constant symbols for *Mark* and *John*.

The complete formula is

Taller-than(mark, john)

In Aristotle's Syllogistic Logic:

Instead of using a 2-place predicate, we would need to use a 1-place predicate "**Taller-than-john.**" In symbolic notation, we would have:

Taller-than-john(mark)

Object-centered conception:

Aristotle conceived of the world as consisting of individuals/objects each bearing certain attributes (or properties). For instance, Socrates is the individual bearing the attributes of being Greek, a philosopher, not very attractive, …

This is reflected in Aristotle's Syllogistic Logic whose statements are about individuals/objects and their attributes, e.g. *Socrates is Greek*, and about sets of individuals/objects having certain properties, e.g. *All men are mortal.*

Aristotele's syllogistic logic cannot express relations between two or more individuals/objects.

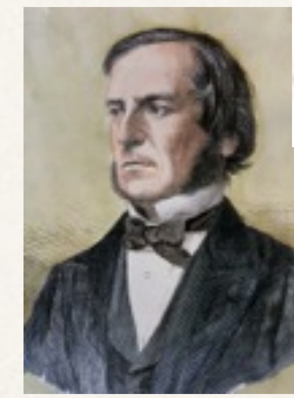
For Aristotle, relations between individuals/objects only make sense as attributes inhering in an single individual/object. For example, earlier we saw how the 2-place relation *taller-than* is reduced to the attribute *taller-than-john*.

Relational conception:
With Predicate Logic, relations among two or more individuals/objects can be expressed without reducing them to attributes of individuals/objects.

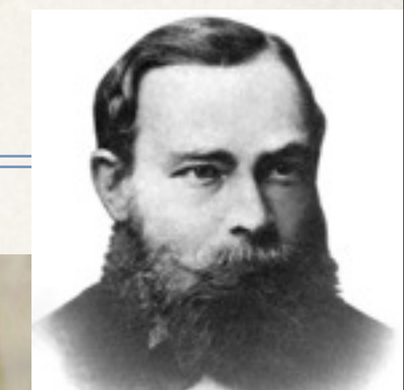
Toward Predicate Logic

Aristotle
384-322 BC
Syllogistic

From a conception of the world as made of *objects and their attributes* **toward** a conception of the world as made of *relations between objects.*

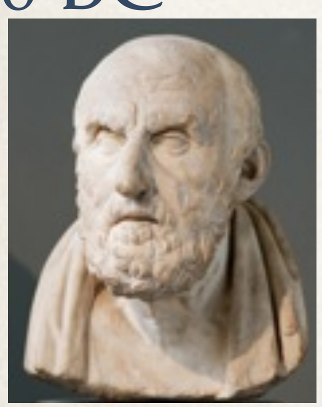


George Boole
1815-1864
Algebra of Logic



Gottlob Frege
1848-1925
Predicate Logic

500 BC | 0 | 500 | 1,000 | 1,500 | 2,000



Chrysippus
279-206 BC
Propositional Logic

This conceptual change *might* explain why it took almost 2,000 to get to predicate logic.

Charles Peirce
1839-1914
Predicate Logic

The Conceptual Change from Objects to Relations in Wittgenstein's Words

1.1 The world is the totality of facts, not of things.

2 What is the case — a fact— is the existence of states of affairs.

2.01 A state of affairs (a state of things) is a combination of objects (things).

2.0141 The possibility of its occurring in states of affairs is the form of an object.

Ludwig Wittgenstein,
Tractatus Logico-Philosophicus

The idea here is to **understand the world in terms of states of affairs, not in terms of objects**.

A state of affairs is a relation (or combination) of objects.

Objects, in turn, are understood in terms of their possibility of occurring in a state of affairs.

So, **the primary notion is that of a state of affairs, and not that of object.**

The Second Innovation of Predicate Logic: *Nested Quantifiers*

Using Nested Quantifiers

One occurrence of a quantifier:

Aristotle's Syllogistic Logic uses quantifiers. The existential **∃y** and the universal **∀x** quantifier are hidden behind the natural language expressions "SOME" and "ALL".

In Aristotle's Syllogistic Logic, **a quantifier can only occur once in a statement** (e.g. *all* men are mortal; *some* alligators are not swimmers).

Nested quantifiers:

In Predicate Logic, quantifiers can be nested. (A quantifier is **nested** when it **occurs within the scope of another quantifier**.)

Examples:

∀x∃y(Son-of(x, y))
∃x∀y(Father-of(x, y))
∀x∀y(Son-of(x, y))
∃x∃y(Son-of(x, y))

Examples of the Scope of a Quantifier

$\forall x \exists y (R(x, y))$

Scope of $\forall$**x** is **∃y(R(x, y))**

Scope of ∃y is **R(x, y)**

$\forall x \forall y (R(x, y))$

Scope of $\forall$**x** is **∀y(R(x, y))**

Scope of ∀y is **R(x, y)**

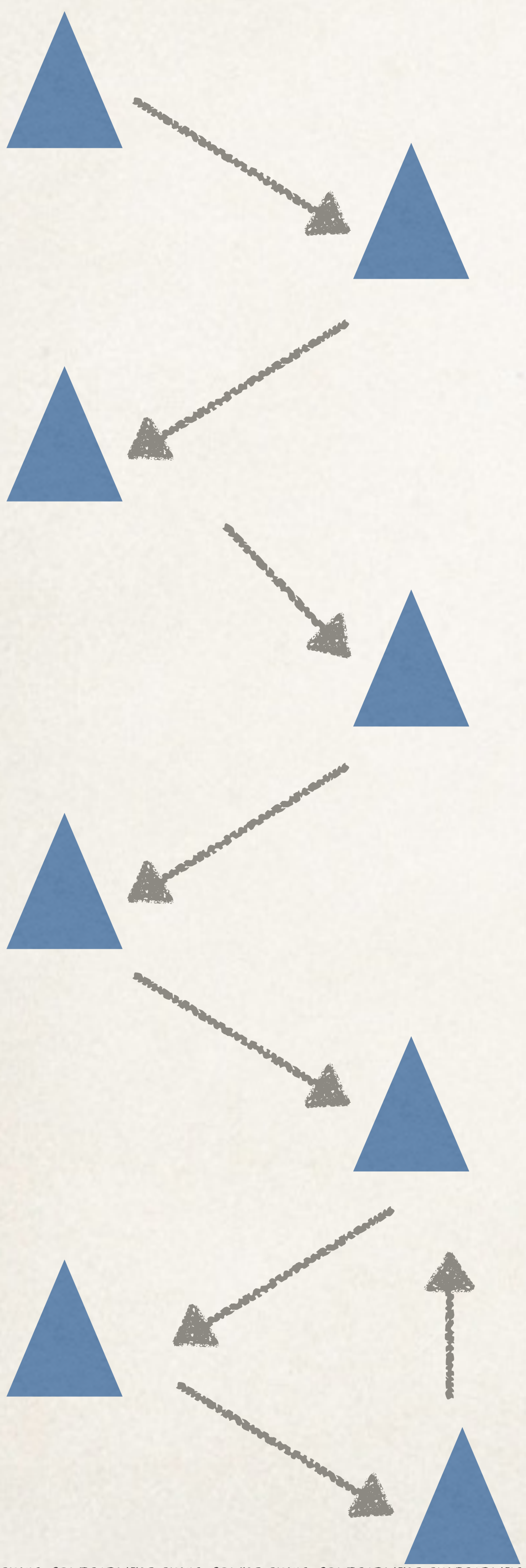
The Order Makes a Difference!

$\forall x \exists y(Son\text{-}of(x, y))$ means that for every ***x*** there is a ***y*** such that ***x*** is the son of ***y***.

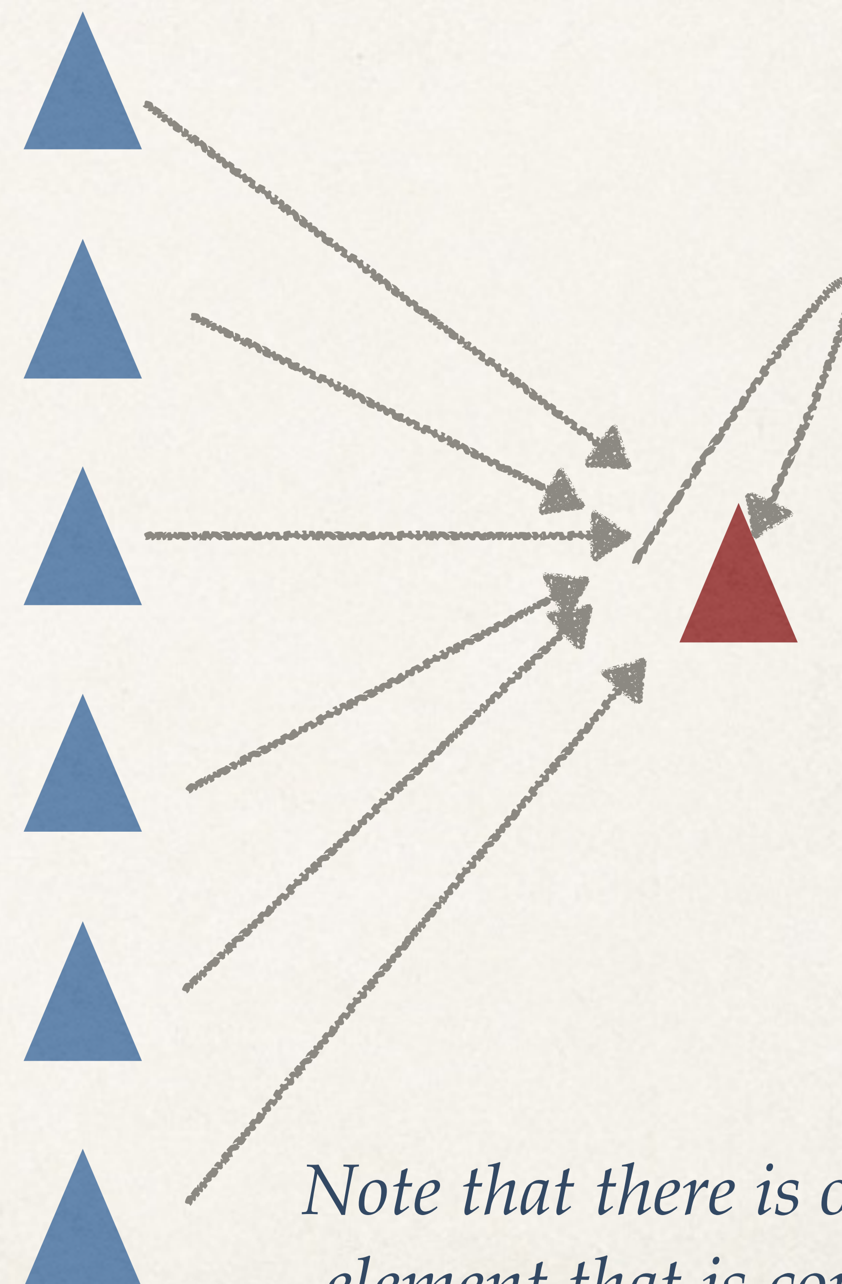
$\exists x \forall y(Son\text{-}of(x, y))$ means that there is an ***x*** such that ***x*** is the son of everybody (of every **y**).

Let ***R*** *stand for the arrow relation*

$\forall x \exists y (R(x, y))$



$\exists y \forall x (R(x, y))$



Note that there is one fixed element that is connected to every element.

To repeat, the two innovations of predicate logic are:

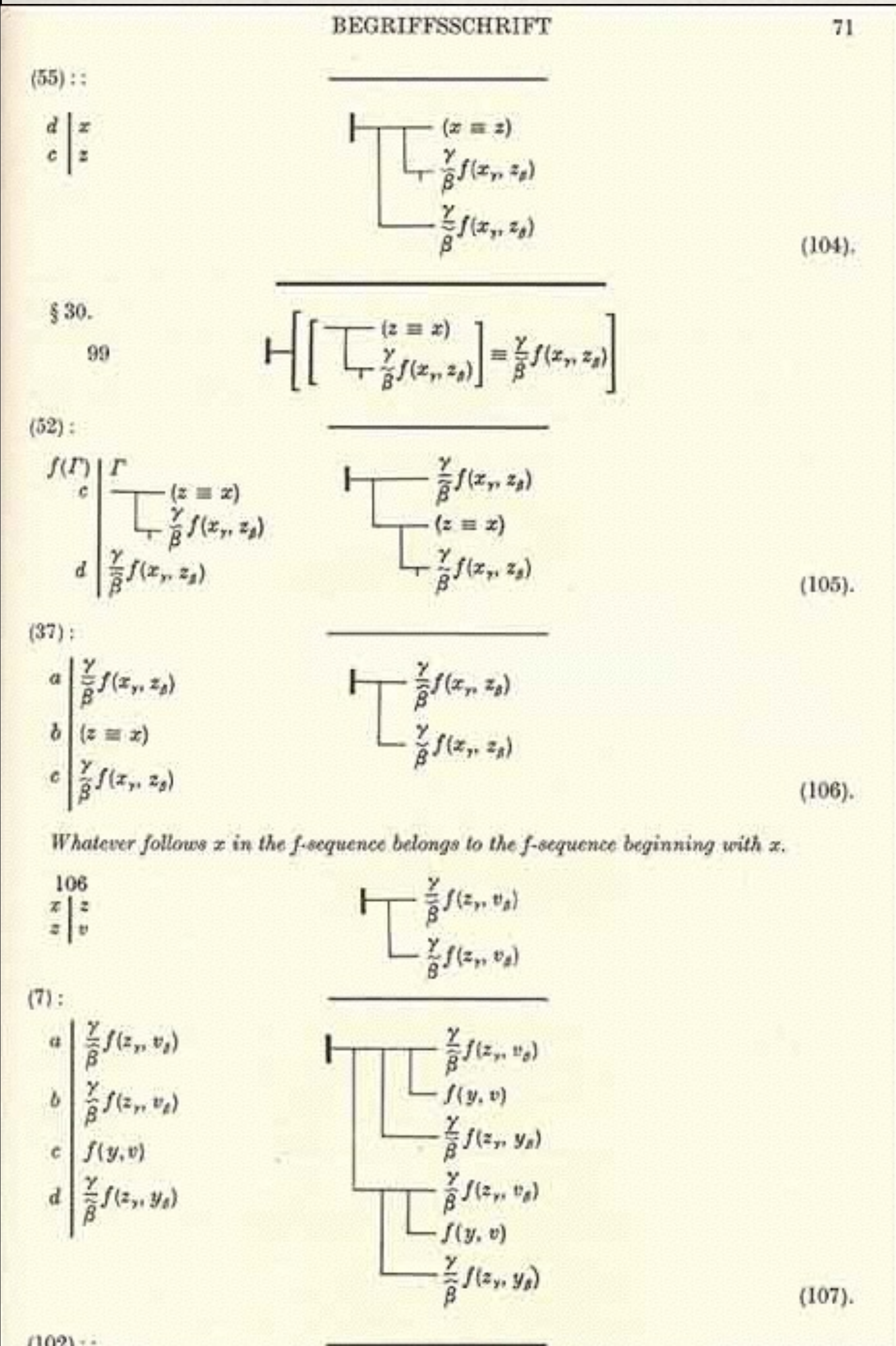
1. Ability to express relations between objects

2. Ability to nest quantifiers

What's the Point of Translating Sentences into Formulas of Predicate Logic? What Do We Gain from Doing That?

We can clarify the meaning of sentences in natural language and make them more precise and less ambiguous.

A formal language such as Predicate Logic can be the basis for programming and artificial intelligence.



I believe that I can best make the relation between my ***ideography*** *[i.e. essentially, Predicate Logic which you are now studying] to* ***ordinary language*** *clear if I compare it to that which the* ***microscope*** *has to the* ***eye****. Because the range of its possible uses and the versatility with which it can adapt to the most diverse circumstances,* ***the eye is far superior to the microscope****. Considered as an optical instrument, to be sure, it exhibits many imperfections, which ordinarily remain unnoticed only on account of its intimate connection with our mental life. But,* ***as soon as scientific goals demand great sharpness of resolution, the eye proves to be insufficient****. The microscope, on the other hand, is perfectly suited to precisely such goals, but that is just why it is useless for all others.*

Frege, *Ideography* (1879)

Ambiguos Sentences

- Everyone loves someone:

  $\forall x \exists y (L(x, y))$ *or* $\exists y \forall x (L(x, y))$?

- A professor talked to every student:

  $\forall x (Student(x) \rightarrow \exists y (Professor(y) \land Talk(y, x)))$

  or

  $\exists y (Professor(y) \land \forall x (Student(x) \rightarrow Talk(y, x)))$?

- A guard is standing in front of every gate

Suppose Someone is Telling You "Everybody (Should) Donate Money to the Poor!"

What does he really mean exactly?

“Everybody (Should) Donate Money to the Poor” (1)

Let’s assume our domain of quantification consist of **only people**, not inanimate things or objects.

$\forall x \forall y (Poor(x) \rightarrow Donate(y, x))$

I.e. for any poor person, everyone donates to him/her

This means that everybody donates to every poor person, even poor people donate to the poor. Too much?

"Everybody (Should) Donate Money to the Poor" (2)

$\forall y(Rich(y) \rightarrow \forall x(Poor(x) \rightarrow Donate(y, x)))$

I.e. all rich people donate to all poor people.

This might be inefficient because all rich people would donate to all poor people

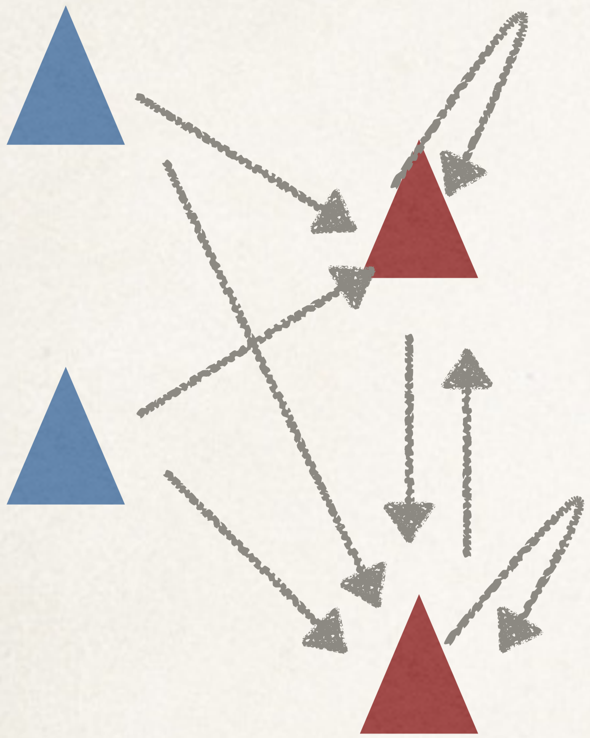
$\forall y(Rich(y) \rightarrow \exists x(Poor(x) \land Donate(y, x)))$

I.e. all rich people donate to some poor people.

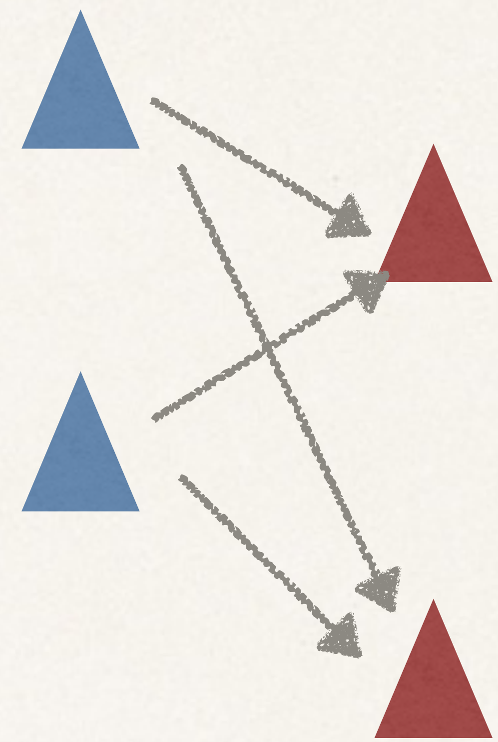
This is problematic since some poor person might receive no donation at all

*Let **"Donate"** stand for the arrow relation.*

*Let **red** objects represent poor people. Let **blue** objects represent rich people.*



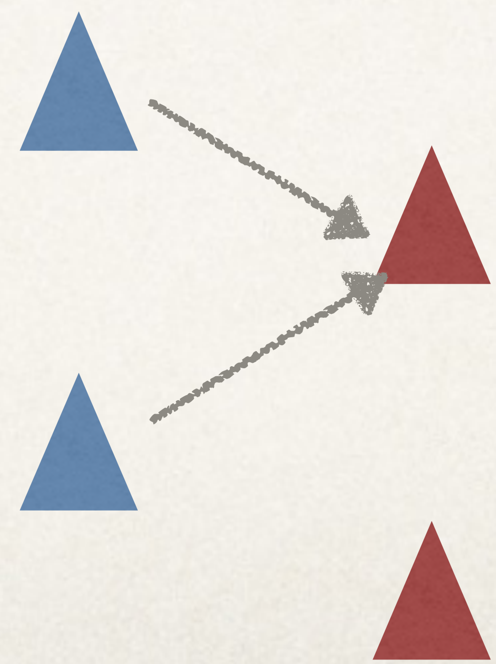
$\forall x \forall y(Poor(x) \rightarrow Donate(y, x))$



$\forall y(Rich(y) \rightarrow \forall x(Poor(x) \rightarrow Donate(y, x)))$

The picture on the right makes true the formula

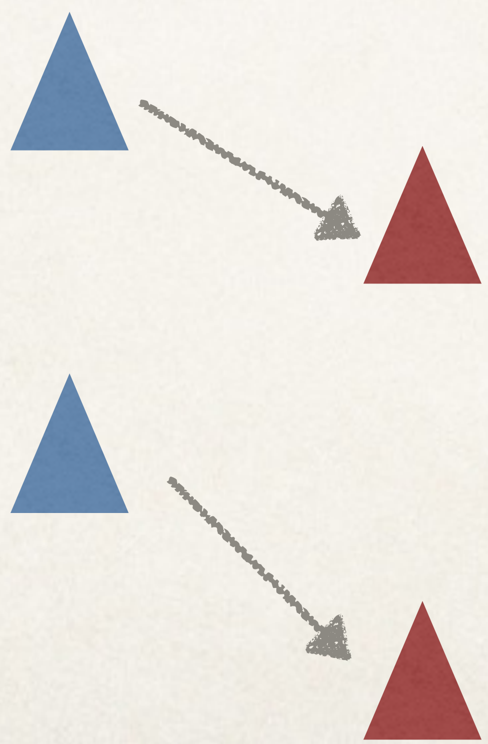
$\forall y(Rich(y) \rightarrow \exists x(Poor(x) \land Donate(y, x)))$



“Everybody (Should) Donate Money to the Poor” (3)

$\forall x(Poor(x) \rightarrow \exists y(Rich(y) \wedge Donate(y, x)))$

The picture below
makes true
the above formula



I.e. For every poor person, there is at least one rich person donating to him/her.