Alonzo King's Ballet

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

Week 6 — Monday Class - Predicate Logic

What We Have Seen So Far

Propositional logic allows us to reason with statements (or formulas) of the form

p, q, r …

$\neg \psi$

$\phi \wedge \psi$

$\phi \vee \phi$

$\phi \rightarrow \psi$

Syllogistic logic allows us to reason with statements (or formulas) of the form

All A are B

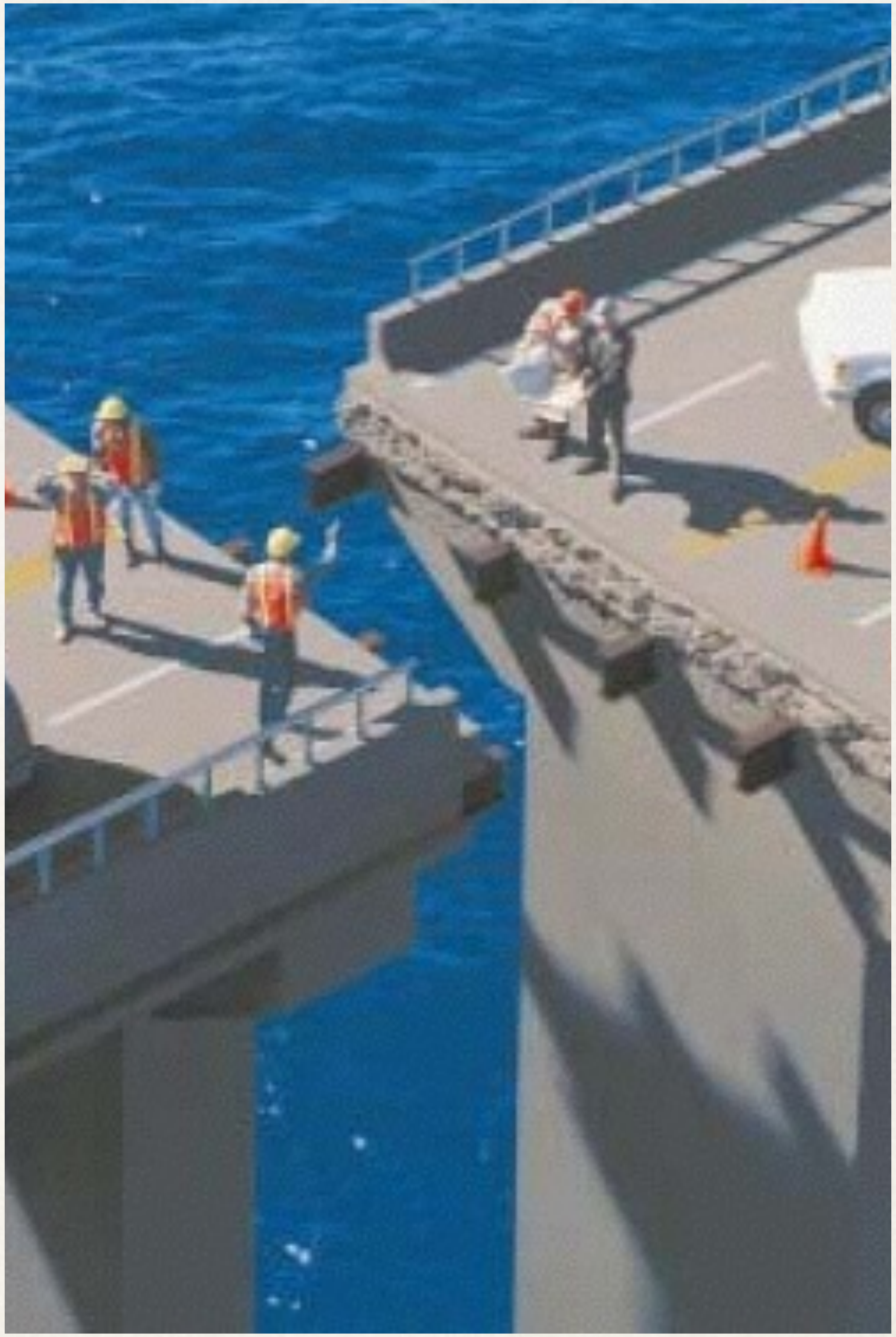
Some A are B

All A are Not B
(i.e. No A is B)

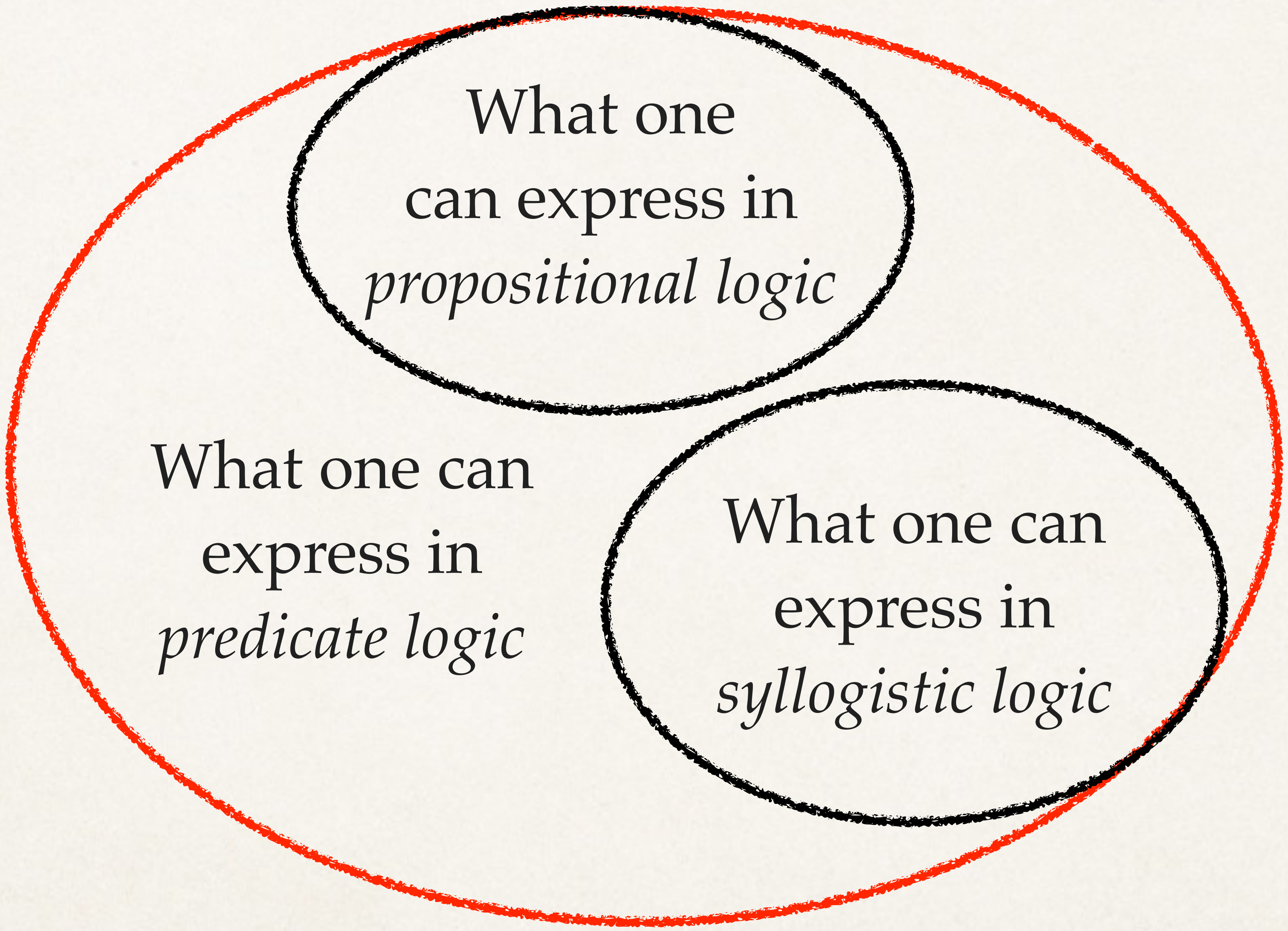
Some A are Not B
(i.e. Not all A are B)

Can we combine the two logics into a more powerful logic?

Predicate Logic Combines **Propositional Logic** and **Syllogistic Logic**



Predicate Logic Encompasses both Propositional Logic and Syllogistic Logic

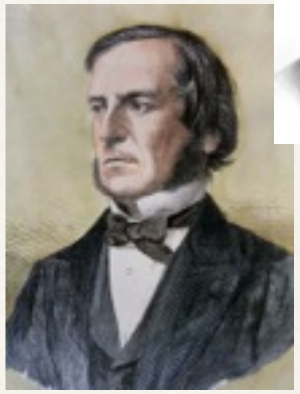


Predicate Logic is superior to syllogistic logic and propositional logic combined.

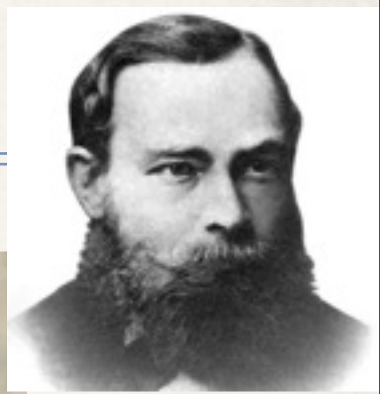
Towards Predicate Logic

Aristotle
384-322 BC
Syllogistic

It took almost 2,000 years to arrive at predicate logic.
It is unclear why.

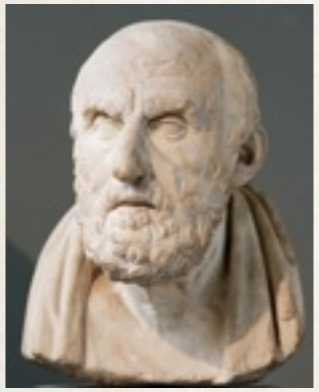


George Boole
1815-1864
Algebra of Logic



Gottlob Frege
1848-1925
Predicate Logic

500 BC | 0 | 500 | 1,000 | 1,500 | 2,000



Chrysippus
279-206 BC
Propositional Logic

Charles Peirce
1839-1914
Predicate Logic

The Plan for the Next Three Weeks

Week 6:

Gain some familiarity with predicate logic, its language and semantics

(Grasp the motivations behind predicate logic)

Book: *sections* **4.1—4.4**

Week 7:

Formal syntax and semantics

Book: *sections* **4.5—4.7**

Week 8:

Derivations

Lecture notes only; no book.

The Language of Predicate Logic

The Ingredients of the Language of Predicate Logic

Constant symbols: ***a, b, c, …***

Variable symbols: ***x, y, z, …***

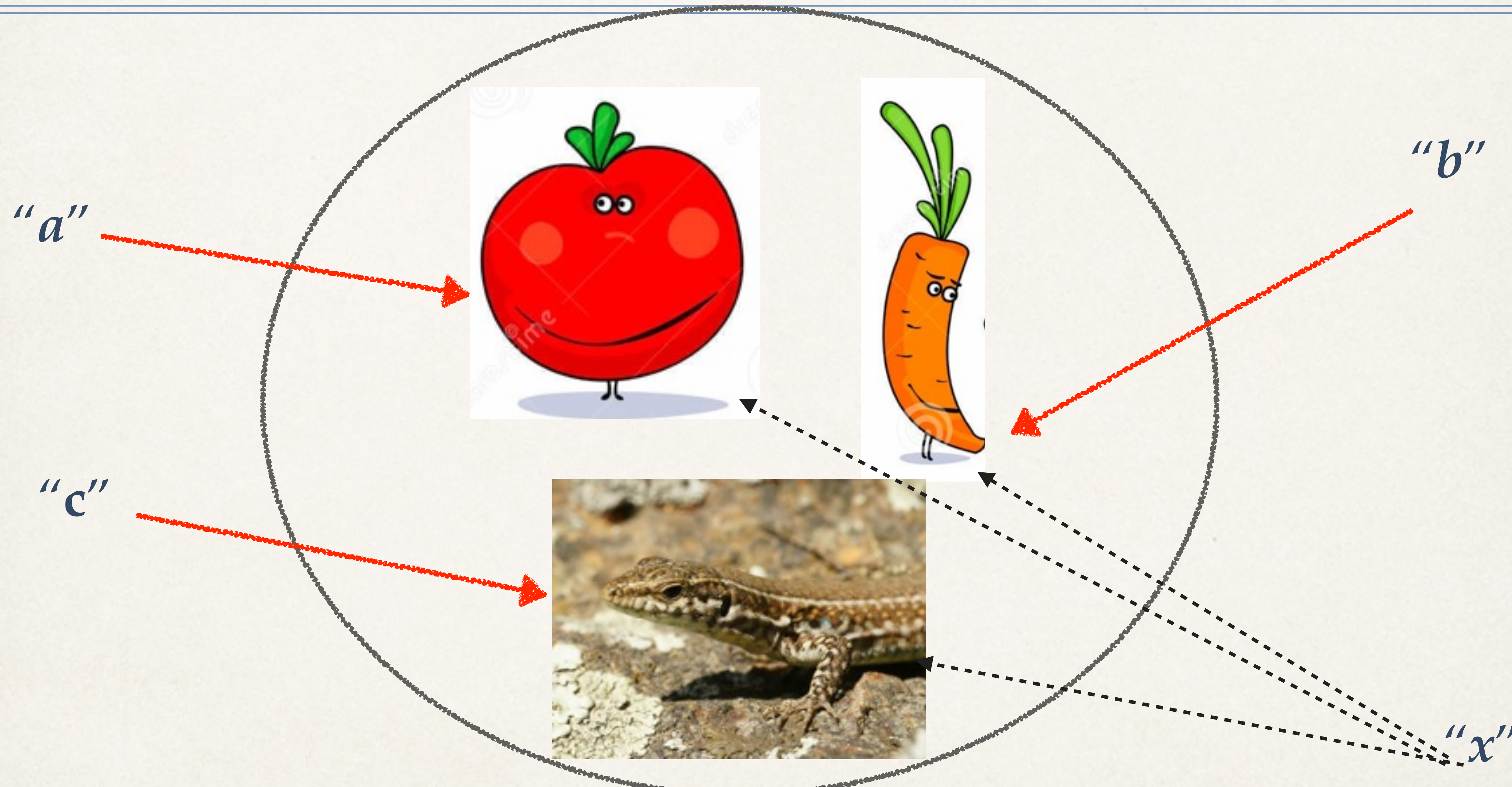
Predicate symbols: ***A, B, C, …***

Logical connectives or operators: $\neg$, $\wedge$, $\vee$, $\rightarrow$

Existential quantifier: $\exists$***x*** (*"there is an x"*)

Universal quantifier: $\forall$***x*** (*"for all x"*)

Constants Symbols *versus* Variable Symbols (1)



The constant symbols ***a, b, c*** refer to a *specific object,* while the variable ***x*** can range over *different objects*.

Constants Symbols *versus* Variable Symbols (2)

You should think of **constant symbols** (*a, b, c, …*) as having the same role that *proper names* play in natural language or as having the same role that *numerals* play in mathematics.

Proper names are meant to refer to a <u>specific</u> individual. For example, "John" refers to the specific individual John.

Numerals are meant to refer to a <u>specific</u> number. For example, the numeral "145" refers to the number 145.

Unlike constant symbols, **variable symbols** can range over <u>more than one</u> object or individual. That's why they are *variable* symbols.

An Aside: *Using* versus *Mentioning*

When we are *mentioning* an expression of the language, as opposed to *using* it, we typically put it between inverted commas.

Examples of *using:*

Lisa got the job!

2+2=4

Examples of *mentioning*:

The name “Lisa” contains two vowels.

The symbol “+” is the symbol for addition.

Notation for Constant Symbols

Although constant symbols are — *strictly speaking* — only lower case letters (i.e. ***a, b, c, …***), you should feel free to use any string of lower case letters.

For example:

- the string of lower case letters "***mark***" can be a constant symbol referring to the individual Mark; or
- the string of lower case letters "***apple***" can be a constant symbol referring to a specific apple; etc.

"bilbao-museum" *"louise-bourgeois-sculpture"*

Predicate Symbols

Predicate symbols can refer to
- *attributes* of individual or objects; and
- *relations* between individuals or objects.

Relations
hold between more
than one individual
or object

Example of an *attribute*:

Babatunji **is dancing**

Example of a *relation*:

Mark **is taller than** John

Dancing is an attribute
of Babatunji

Being taller than is a relation
between Mark and John.

Notation for Predicate Symbols

Although predicate symbols are — *strictly speaking* — only upper case letters (i.e. ***A, B, C, …***), you should feel free to use any string of letters beginning with an upper case letter.

For example:

- the string of letters "***Dancing***" can be a predicate symbol used to refer to the attribute of dancing; or
- the string of letters "***Taller-than***" can be a predicate symbol used to refer to the relation of someone being taller than someone else; etc.

Quantifiers, Variables, Constants

The following are formulas of predicate logic:

Dancing(a)

(this means that the object/individual referred to by the constant symbol "**a**" has the attribute referred to by the predicate symbol "**Dancing**")

∃*x*(*Dancing*(*x*))

(this means that <u>an</u> object, generically referred to by ***x***, has the attribute referred to by "***Dancing***")

∀*x*(*Dancing*(*x*))

(this means that <u>all</u> objects, each generically referred to by ***x***, have the attribute referred to by "***Dancing***")

We Will Now Start Writing Formulas in Predicate Logic in Four Stages

I. Simple formulas with constant and predicate symbols

II. Formulas in predicate logic with propositional connectives

III. Formulas with existential and universal quantifiers

IV. Formulas mixing connectives and quantifiers

Stage 1:
Simple Sentences in Predicate Logic

Example (1)

"***Dancing***" is the predicate symbol

"***babatunji***" is the constant symbol

In English:

Babatunji is dancing

In Predicate Logic:

Dancing(babatunji)

Natural Language *versus* Predicate Logic

In Natural Language:

Babatunji is dancing

proper name with the function of subject

verb phrase

In Predicate Logic:

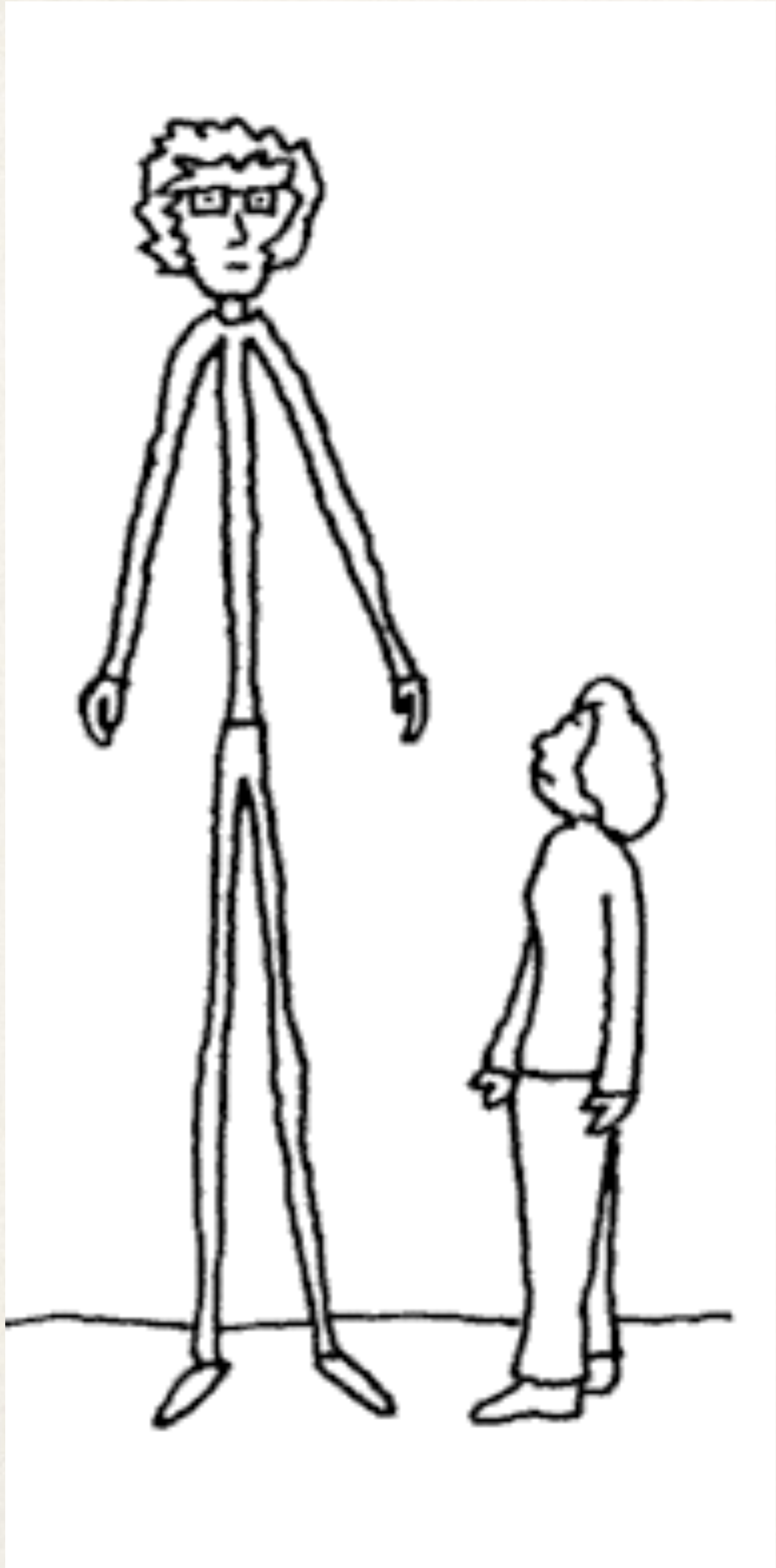
Dancing(babatunji)

predicate symbol

constant symbol

Predicate logic is a new language that has some similarities with natural language, but it is also different from it. ***Now you are learning the new language of predicate logic.***

Example (2)



In English:

Mark is taller than John

In Predicate Logic:

“***Taller-than***” is the predicate symbol

“***mark***” and “***john***” are the constant symbols

The complete formula is
Taller-than(mark, john)

Stage2:
Formulas in Predicate Logic with the Propositional Connectives

Using the Logical Connectives

Dancing(babatunji)

Dancing(caroline)

Dancing(babatunji) ∧ *Dancing(caroline)*

Predicate Logic Can Say More than Propositional Logic

Predicate logic: $\boldsymbol{Dancing(babatunji) \land Dancing(caroline)}$

The sentence above, *in propositional logic*, would simply be:

$$\boldsymbol{p \land q}$$

The language of propositional logic is blind to the internal structure of $\boldsymbol{p}$ and $\boldsymbol{q}$.

The language of propositional logic cannot refer to specific individuals or to their attributes and relations.

Stage 3:
Formulas in Predicate Logic with Existential and Universal Quantifiers

Introducing the Existential Quantifier

In predicate logic, we need not always refer to a specific individual who is dancing.

Dancing(babatunji)

In predicate logic, we can also express that there is an individual who is dancing, as follows:

$$\exists x(Dancing(x))$$

What's the Difference Between **∃x(Dancing(x))** and **Dancing(babatunji)**?

Dancing (babatunji) is true provided Babatunji is dancing.

∃x(Dancing(x)) is true no matter who is dancing, provided *at least someone is dancing*.

Introducing the Universal Quantifier

In predicate logic, we can also express the fact that <u>everybody</u> is dancing, as follows:

$$\forall x(Dancing(x))$$

Stage 4:
Mixing Connectives and Quantifiers

Some are…AND some are *NOT*…

In predicate logic, we can express the fact that <u>some</u> are dancing, and that <u>some</u> are <u>not</u> dancing, as follows:

$$\exists x(Dancing(x)) \land \exists x(\neg Dancing(x))$$

Formulas Encountered So Far

$Dancing(babatunji)$

$Dancing(babatunji) \wedge Dancing(caroline)$

$\exists x(Dancing(x))$

$\forall x(Dancing(x))$

$\exists x(Dancing(x)) \wedge \exists x(\neg Dancing(x))$

And many more are possible…

How Can We Express the Statements Used in Syllogistic Logic with the Language of Predicate Logic?

All A are B
Some A are B
All A are Not B
Some A are Not B

Some houses are white

$\exists x(House(x) \wedge White(x))$

All houses are painted

$\forall x\,(House(x) \rightarrow Painted(x))$

N.B: The above sentences are just like "**Some A are B**" and "**All A are B**" from syllogistic logic.

Aristotle's Square of Oppositions in the Language of Predicate Logic

ALL A are B

ALL A are ***NOT*** B

SOME A are B

SOME A are ***NOT*** B

In predicate logic:

$\forall x(A(x) \rightarrow B(x))$

$\forall x(A(x) \rightarrow \neg B(x))$

$\exists x(A(x) \wedge B(x))$

$\exists x(A(x) \wedge \neg B(x))$

What is an example of a sentence you cannot express in Syllogistic Logic but that you can express in Predicate Logic?

"The wind blows my jacket away"

The Wind Blows My Jacket Away

Blow-away (wind, my-jacket)

*"**Blow-away**"* is a **2-place predicate** that refers to the *blowing relation* between the wind and my jacket. Instead, *"**wind**"* and *"**my-jacket**"* are simply constant symbols.

To be continued on Wednesday…

What's so special about this?

*In syllogistic logic, you **only have 1-place predicates***