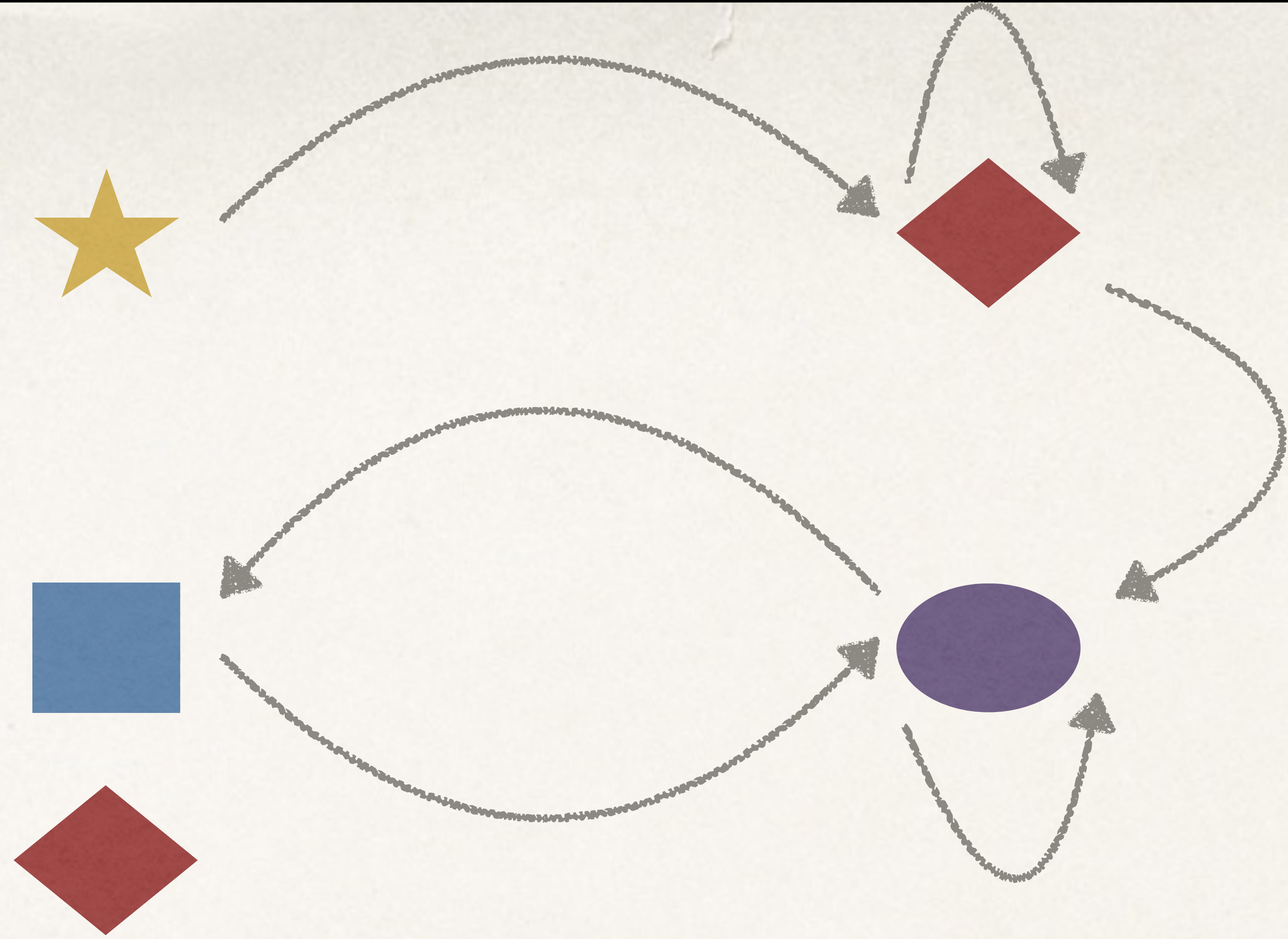


Abstractions

PHIL 50 - Introduction to Logic

Marcello Di Bello, Stanford University, Spring 2014

Week 6 — Wednesday Class - Predicate Logic

From Monday Class: Formulas We've Encountered So Far

$Dancing(babatunji)$

$\exists x(Dancing(x))$

$\forall x(Dancing(x))$

$Dancing(babatunji) \wedge Dancing(caroline)$

$\exists x(Dancing(x)) \wedge \exists x(\neg Dancing(x))$

And many more are possible…

From Monday Class:
The Square of Oppositions in Predicate Logic

ALL A are B	***ALL*** A are ***NOT*** B
SOME A are B	***SOME*** A are ***NOT*** B

In predicate logic:

$\forall x(A(x) \rightarrow B(x))$	$\forall x(A(x) \rightarrow \neg B(x))$
$\exists x(A(x) \wedge B(x))$	$\exists x(A(x) \wedge \neg B(x))$

What are examples of sentences you cannot express in Syllogistic Logic but that you can express in Predicate Logic?

"The wind blows my jacket away"

"Some CEOs bribe some members of congress"

"Every farmer owns a donkey"

"All houses in Santorini are painted"

The Wind Blows My Jacket Away

Blow-away (wind, my-jacket)

What's so special about this?

"***Blow-away***" is a **2-place predicate** that refers to the *blowing relation* between the wind and my jacket. Instead, "***wind***" and "***my-jacket***" are simply constant symbols.

*In syllogistic logic, you **only have 1-place predicates***

All houses in Santorini are painted

Step 1: Identify the quantifiers — *All* (∀x)

Step 2: Identify constants and predicates — ***santorini***, ***House(x), Painted(y), In(x, y)***

Step 3: Sentence's general form — $\forall x\ (\phi(x) \rightarrow \psi(y))$

$$\forall x((House(x) \wedge In(x, santorini)) \rightarrow Painted\ (x))$$

"***In***" is a 2-place predicate that refers to the relation of being in a location, while "***House***" and "***Painted***" are 1-place predicates. And **"santorini"** is a constant symbol.

Every Farmer Owns a Donkey

Step 1: Identify the quantifiers — *Every* (∀x), *A* (∃y)

Step 2: Identify predicates — *Farmer*(x), *Donkey(y)*, *Own*(x, y)

Step 3: Sentence's general form — $\forall x\ (\phi(x) \rightarrow \exists y\ \psi(x, y)$

$$\forall x(Farmer(x) \rightarrow \exists y(Donkey(y) \wedge Own(x,y)))$$

"***Own***" is a 2-place predicate that refers to the relation of owning, while "***Farmer***" and "***Donkey***" are 1-place predicates that refer to the attribute of being a farmer and of being a donkey respectively.

The Universal Quantifier and the Implication (1)

- All houses are painted

$$\forall x(House(x) \rightarrow Painted\ (x))$$

- Every farmer owns a donkey

$$\forall x(Farmer(x)\ \rightarrow \exists y(Donkey(y) \wedge Own(x,y))$$

- All the houses in Santorini are painted

$$\forall x((House(x) \wedge In(x,\ santorini)) \rightarrow Painted\ (x))$$

Expressions such as "*every*" and "*all*" should be translated in predicate logic with the universal quantifier $\forall x$.

Note also the use of the material implication → together with the universal quantifier $\forall x$.

The Universal Quantifier and the Implication (2)

Why this translation in predicate logic?

Every farmer owns a donkey

$\forall x(Farmer(x) \rightarrow \exists y(Donkey(y) \wedge Own(x,y)))$

Why not this translation?

Every farmer owns a donkey

$\forall x(Farmer(x) \wedge \exists y(Donkey(y) \wedge Own(x,y)))$

The *first translation* is true even if not everyone is a farmer, provided those who are famers own a donkey. The *second translation*, instead, requires that everybody is a farmer and that they own at least one donkey.

Some CEOs Bribe Some Members of Congress

Step 1: Identify the quantifiers — *Some* (∃x), *Some* (∃y)

Step 2: Identify predicates — ***CEO(x), MC(y), Bribe(x, y)***

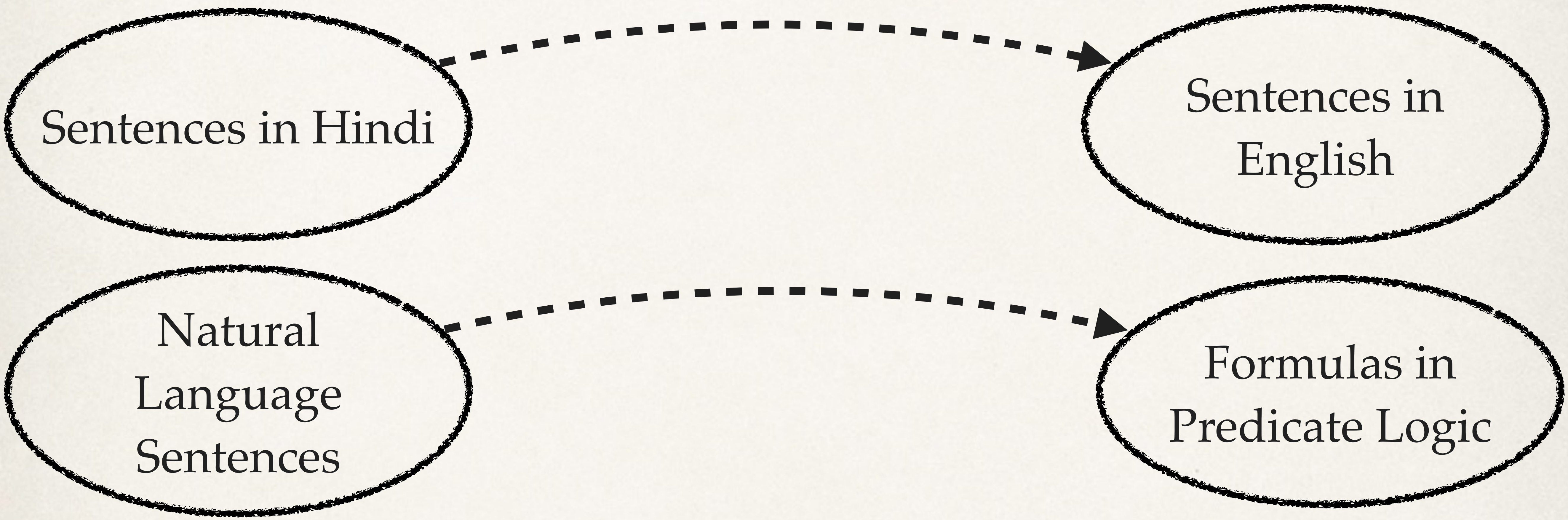
Step 3: Sentence's general form — $\exists x\,(\phi(x) \wedge \exists y\,\psi(x, y))$

$$\exists x(CEO(x) \wedge \exists y(MC(y) \wedge Bribe(x,y)))$$

"***Bribe***" is a 2-place predicate that refers to the relation of bribing, while "***CEO***" and "***MC***" are 1-place predicates that refer to the attribute of being a CEO and of being a member of congress.

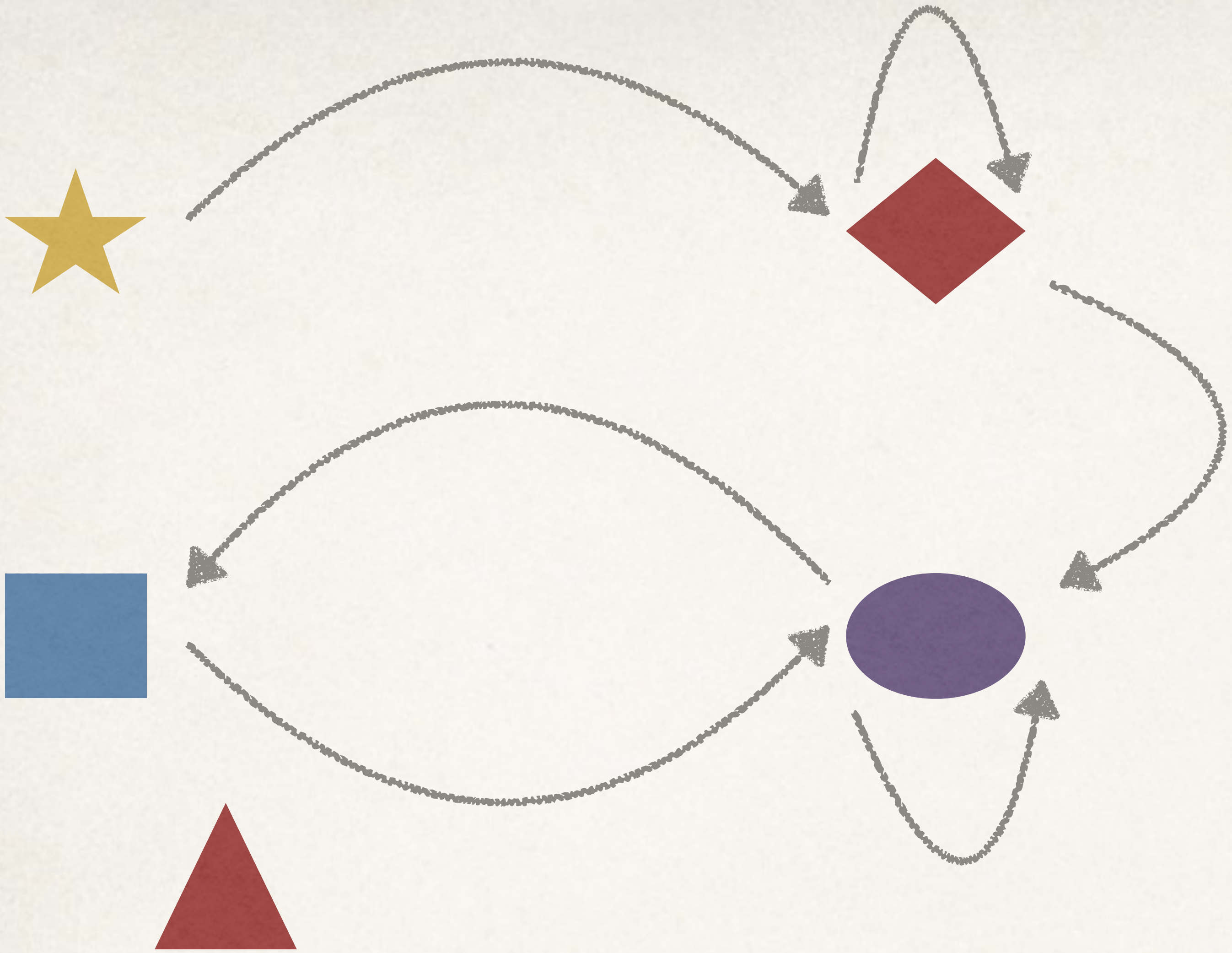
For a translation strategy, read the textbook, chapter 4, section 4.2

But Keep in Mind that No translation Is Perfect



You cannot get a perfect translation of Hindi sentences into English sentences, and similarly you cannot get a perfect translation of natural language sentences into formulas of predicate logic.

Let's Now Work More Abstractly (see textbook, section 4.4)



"star" refers to
"diamond" refers to
"square" refers to
"oval" refers to

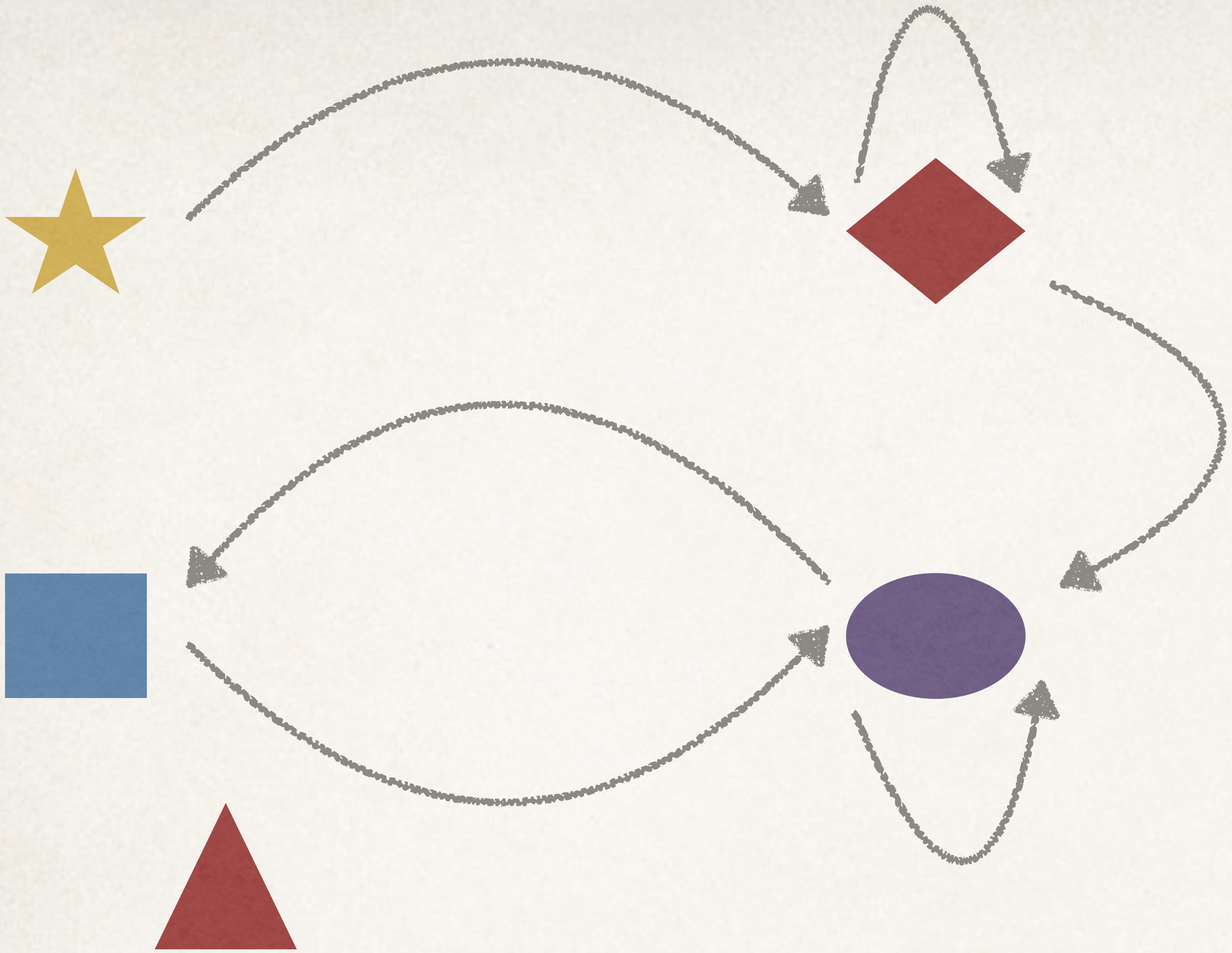
"Arrow(… , …)" refers to the arrow relation. "Yellow" "Red" "Blue" and "Purple" refers to the color attributes

True or false?

Arrow(star, diamond)
Arrow(diamond, star)
Arrow(star, star)
Arrow(diamond, diamond)
Arrow(oval, star)

Answers:

True
False
False
True
False



"star" refers to
"diamond" refers to
"square" refers to
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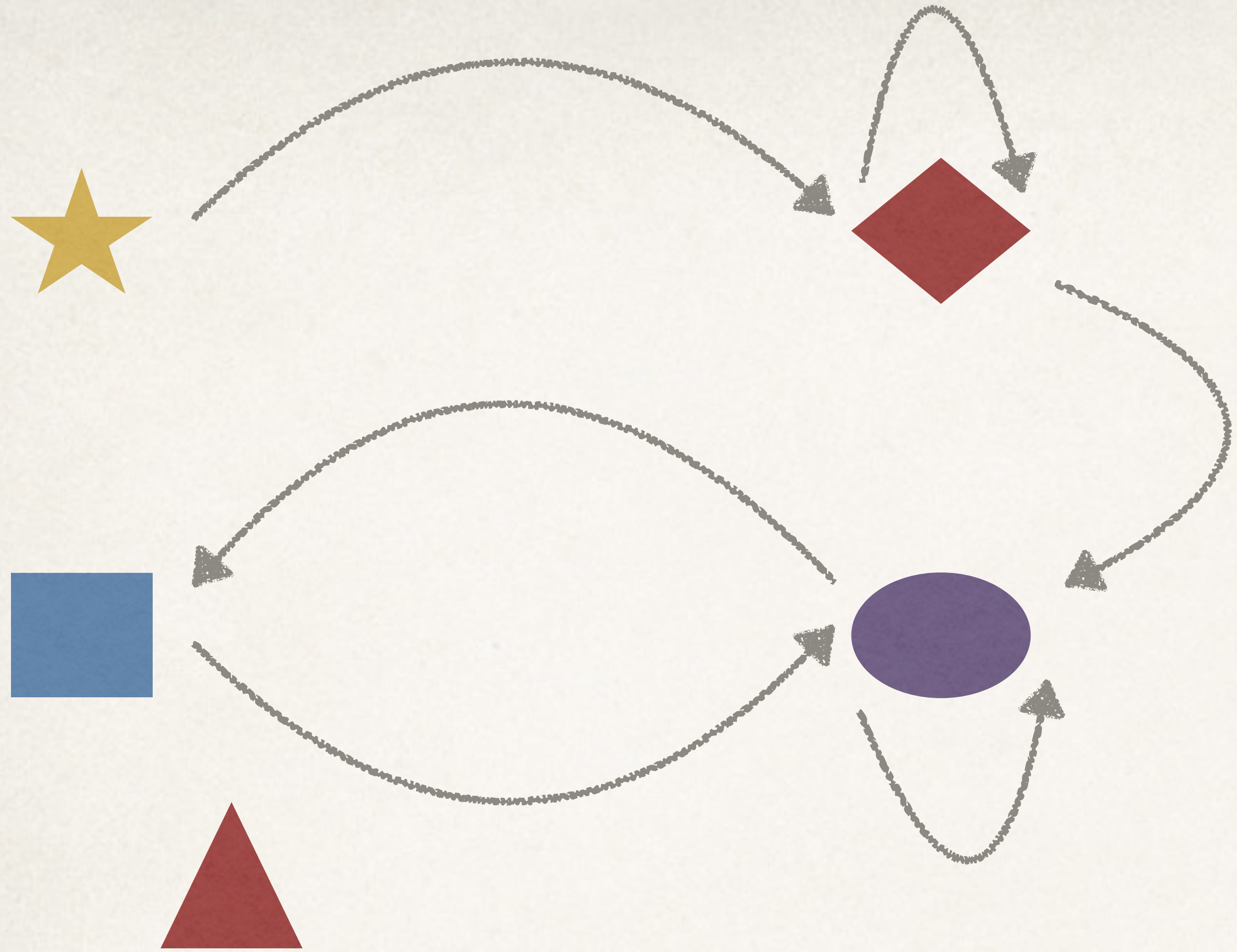
"Arrow(… , …)" refers to the arrow relation
"Yellow" "Red" "Blue" and "Purple" refers to the color attributes

True or false?

Yellow(diamond)
Blue(square)
Red(diamond)
Yellow(Red)
¬(Purple(oval) ∨ ¬Red(diamond))

Answers:

False
True
True
Not a formula!
False



"star" refers to
"diamond" refers to
"square" refers to
"oval" refers to

"Arrow(… , …)" refers to the arrow relation

"Yellow" "Red" "Blue" and "Purple" refers to the color attributes

True or false?

$\forall x \exists y(\text{Yellow}(x) \rightarrow (\text{Arrow}(x, y) \wedge \text{Red}(y)))$

$\forall x \exists y(\text{Yellow}(x) \wedge \text{Red}(x)) \rightarrow \text{Arrow}(x, y))$

$\forall x \exists y(\text{Arrow}(x, y))$

$\forall x(\text{Red}(x) \rightarrow \text{Arrow}(x, x))$

$\forall x(\text{Purple}(x) \rightarrow \text{Arrow}(x, x))$

Answers:

True

True

False

False

True