PHIL 50 – INTRODUCTION TO LOGIC

MARCELLO DI BELLO – STANFORD UNIVERSITY

HOMEWORK – WEEK #10 – DUE MONDAY JUNE 9TH BY 12 NOON IN MARCELLO'S MAILBOX IN BUILDING 90 – THIS LAST HOMEWORK IS OPTIONAL

1 PROVING THINGS WITH THE PROBABILITY AXIOMS [40 POINTS]

Using the probability axioms, the definition of conditional probability, and the theorems NEGATION, EQUIVALENCE, and TOTAL PROBABILITY, show the following:

(a) If $\varphi \models \psi$, then $P(\varphi) \leq P(\psi)$

(b) $P(\varphi \vee \psi) = P(\varphi) + P(\psi) - P(\varphi \wedge \psi)$

2 BAYES [30 POINTS]

Imagine that, in a small town, there are two bus companies, GREEN and BLUE, whose buses are respectively painted green and blue. GREEN company covers 85 percent of the market and BLUE company covers the rest. There are no other companies around. On a misty day, a bus hits and injures a passerby, but it drives off.

A witness reports that it was a blue bus. The witness is right only 80 percent of the time. This means that he gets the color right 80 percent of the time. More formally, this means

$0.8 = Pr($*witness says the bus is blue*$|$*the bus is blue*$)$, and also

$0.8 = Pr($*witness says the bus is green*$|$*the bus is green*$)$.

Given the witness report, what is the probability that the taxi cab involved in the accident was in fact blue? Hint: Use Bayes' rule and the rule of total probability.

3 IS PROBABILITY TRUTH-FUNCTIONAL? [30 POINTS]

In propositional logic, once you know the truth values of the atomic propositions, you can know the truth values of any arbitrarily complex formula containing such atomic propositions. So, if you know the truth values of p, q and r, it is straightforward to determine the truth value of, say, $p \vee (\neg q \wedge \neg(q \wedge r))$. Does something similar hold for probability assignments? In other words, is it the case that if you know the probability of p and q, then you'll know the probability of $p \vee q$, or the probability of $p \wedge q$, or the probability of any arbitrarily complex formula built out of p and q?

(a) Check whether if you know the probability of p and q, then you know the probability of $p \wedge q$. If not, please provide a counterexample. The counterexample will be such that, given a certain assignment of probability for p and q, there are multiple probability assignments for $p \wedge q$ that are all compatible with the probability axioms.

(b) Do the same as with part (a), but use $p \vee q$ instead of $p \wedge q$.

(c) Do the same as with parts (a) and (b), but use $\neg p$.