PHIL 50 – INTRODUCTION TO LOGIC

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HOMEWORK – WEEK #2 – DUE MONDAY APRIL 14TH, 2014

1 FORMULAS [10 POINTS]

Check whether the following are formulas of our propositional language:
$(\varphi \wedge \psi) \rightarrow \neg(\varphi \vee (\varphi \vee \varphi))$
$\neg\neg\neg \rightarrow \varphi$

2 INDUCTIVE DEFINITIONS [20 POINTS]

(a) Give an inductive definition of the number of brackets for the formulas in the propositional language. As inductive cases, consider only $\neg$ and $\wedge$.

(b) Give an inductive definition of the function that assigns truth values (1 and 0) to the formulas in the propositional language. As inductive cases, consider only $\neg$ and $\wedge$.

3 IF....THEN [20 POINTS]

You have learned that statements of the form $\varphi \rightarrow \psi$ are (vacuously) true whenever φ is false. But this might be different from the ordinary meaning we attribute to statements of the form *if...then*.

(a) Collect two examples of natural language statements of the form *if...then* which do not (seem to) conform to the material conditional $\rightarrow$.[1]

(b) Does the connective *if...then* which occurs in the examples you have collected in point (a) above behave truth-functionally or not?

Explain your answers.

4 TRUTH-FUNCTIONS AND LOGICAL CONSEQUENCE [15 POINTS]

(a) Write a mathematical function corresponding to the meaning of the connective $\rightarrow$.

(b) Let $\{\varphi_1, \varphi_2, \varphi_3, \dots\}$ be an infinite set of formulas. Can you use truth-tables to check whether the following holds?

$$\{\varphi_1, \varphi_2, \varphi_3, \dots\} \models \psi$$

If not, why not?

[1] Here is an example: *If the US economy collapses, the world economy collapses.* Now, as of now the US economy has not (yet) collapsed, so the antecedent of this *if...then* statement is false. But the mere fact that the antecedent is false does not make the entire statement true in so far as ordinary language is concerned. Since the statement in question does not seem (vacuously) true, then we may conclude that the statement does not contain a material conditional, but some other connective.

(c) Suppose you know that the following holds (with p_1 and p_2 atomic formulas):

$$\{p_1, p_2\} \models \psi$$

Does it follow that the following also holds?

$$\{p_1, p_2, \neg\psi\} \models \psi$$

(d) Let $\{p_1, p_2, p_3, \dots\}$ be an infinite set of atomic formulas. Can you use truth-tables to check whether the following holds?

$$\{p_1, p_2, p_3, \dots\} \models p_1 \to p_3$$

Explain your answers.

5 BIVALENCE SURRENDERED[15 POINTS]

We have seen that the Principle of Non-Contradiction $\neg(\varphi \wedge \neg\varphi)$ and the Principle of Excluded Middle $(\varphi \vee \neg\varphi)$ are valid provided we maintain the Principal of Bivalence. What happens if we do away with it? Let's suppose that formulas can be assigned three truth values, namely 1, 0, and 0.5. And let's suppose that negation, conjunction and disjunction behave as follows:

$\neg$	φ
0	1
1	0
0.5	0.5

φ	$\wedge$	ψ
1	1	1
1	0.5	0.5
1	0	0
0.5	0.5	1
0.5	0.5	0.5
0.5	0	0
0	0	1
0	0	0.5
0	0	0

φ	$\vee$	ψ
1	1	1
1	1	0.5
1	1	0
0.5	1	1
0.5	0.5	0.5
0.5	0.5	0
0	1	1
0	0.5	0.5
0	0	0

(a) Write three mathematical functions, each corresponding to the behavior of the three connectives as defined by three truth tables above.

(b) Use the truth table method to check whether or not the Principle of Non-Contradiction and the Principle of Excluded Middle are still valid.

(c) Use the truth table method to check whether or not the Principle of Non-Contradiction and the Principle of Excluded Middle are still equivalent.

6 ODD WAYS TO SAY SIMPLE THINGS [20 POINTS]

Suppose that $\bot$ is the connective that is always false, i.e it always gets assigned the value 0. (Strictly speaking $\bot$ is not a connective because it does not connect anything; it just is always false.) Now, consider a propositional language whose connectives are simply $\to$ and $\bot$. Find ways to write formulas equivalent to $\varphi \wedge \psi$, $\varphi \vee \psi$ and $\neg\varphi$ just by using the connectives $\to$ and $\bot$. Check your answers using truth tables.