PHIL 50 – INTRODUCTION TO LOGIC

MARCELLO DI BELLO – STANFORD UNIVERSITY

HOMEWORK – WEEK #3 – DUE MONDAY APRIL 21ST, 2014

1 IMPLICATIONS [25 POINTS]

Construct derivations for the following formulas:

(a) $\varphi \to (\psi \to (\varphi \wedge \psi))$

(b) $(\varphi \to (\psi \to \sigma)) \to (\psi \to (\varphi \to \sigma))$

(c) $((\varphi \to \psi) \to (\varphi \to \sigma)) \to (\varphi \to (\psi \to \sigma))$

This group of derivation should make you familiar with the rules for $\wedge$ and $\to$. You will use rules '$\to I$' and '$\to E$' a lot.

2 DOUBLE IMPLICATION [20 POINTS]

Consider the formula $((p \to q) \leftrightarrow p) \to q$.

(a) Come up with an informal argument that motivates why the formula is true.

(b) Construct a derivation for the given formula. [*Note that there is no derivation rule for the symbol $\leftrightarrow$, so when you encounter a formula containing that symbol just unpack it. Your derivation at some point will look like this:*

$$\dfrac{\begin{array}{c}\vdots \\ (p \to q) \leftrightarrow p\end{array}}{\begin{array}{c}((p \to q) \to p) \wedge (p \to (p \to q)) \\ \vdots\end{array}}\ \text{unpack} \leftrightarrow$$

3 MORE DERIVATIONS [35 POINTS]

Construct derivations for the following formulas:

(a) $(\varphi \to \psi) \to \neg(\varphi \wedge \neg\psi)$

(b) $\neg(\varphi \wedge \neg\psi) \to (\varphi \to \psi)$

(c) $\neg(\varphi \vee \psi) \rightarrow (\neg\varphi \wedge \neg\psi)$

These derivation should make you familiar with with the other derivation rules.

(d) Which one among the derivations you have offered in exercise 3 is the intuitionistic logician unlikely to accept? What does this tell you about the inter-definability of the connectives in intuitionistic logic? Explain.

4 DISJUNCTIVE SYLLOGISM [20 POINTS]

Disjunctive syllogism is a derivation rule that looks like this:

$$\frac{\varphi \vee \psi \quad \neg\varphi}{\psi}\ DS$$

(a) There is no need to add rule DS to our derivation rules, however. For it is possible to derive DS from the rules we have. To this end, construct a derivation establishing that $\varphi \vee \psi, \neg\varphi \vdash \psi$.

(b) Consider the following formulas associated with statements in natural language about a murder case:

w : Mrs White is guilty

s : Miss Scarlet is guilty

m : Colonel Mustard is guilty

$s \vee (w \vee m)$: At least one of them is guilty

$w \rightarrow m$: If Mrs White si guilty, so is Colonel Mustard

$\neg s \rightarrow \neg m$: If Miss Scarlet is innocent, then so is Colonel Mustard

Using DS and some of the other derivation rules you've learned, construct a derivation establishing that

$$s \vee (w \vee m), w \rightarrow m, \neg s \rightarrow \neg m \vdash s$$