PHIL 50 – INTRODUCTION TO LOGIC

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HOMEWORK – WEEK #5

1 A DERIVATION FOR RUSSELL'S PARADOX [20 POINTS]

We have seen that Russell's paradox amounts to the claim that if $R \in R$, then $R \notin R$, and if $R \notin R$, then $R \in R$. This looks like a contradiction. But is it really? Let's prove it! We can denote the statement $R \in R$ by φ and the statement $R \notin R$ by $\neg\varphi$. Now Russell's paradox boils down to the pair of statements $\varphi \to \neg\varphi$ and $\neg\varphi \to \varphi$. This exercise asks you to derive a contradiction from these two statements. In other words, please show (syntactically) that

$$\varphi \to \neg\varphi, \neg\varphi \to \varphi \vdash \bot$$

2 DEVIL'S LOGIC [20 POINTS]

Suppose God wants to pardon you and let you enter the gates of Heaven. And suppose that you—just like all of us—have willfully sinned at some point in your life. So, here is Devil's argument against you being pardoned.

> All pardoned people are penitent.
> No willful sinner is penitent.
> ———————————————
> No willful sinner is pardoned.[1]

(a) Translate the syllogism in set theoretic terms.

(b) Check whether the syllogism is valid or invalid. If the syllogism is valid, give a set theoretic argument (i.e. the type of argument you find in Monday's or Wednesday's slides). If the syllogism is invalid, give a counterexample.

[1] This is a syllogistic reading of a passage from Dante's *Divine Comedy*, *Inferno*, XXVII, 118-120:
No power can the impenitent absolve;
Nor to repent and will at once consist,
By contradiction absolute forbid.

3 THE EMPTY SET [30 POINTS]

Consider the following syllogism

> All ideas are old ideas.
> All old ideas are plagiarized.
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> Some ideas are plagiarized.

Is this syllogism valid? Well, Aristotle would say yes, and modern logicians would say no. Why? They disagree on whether sets can be empty or not. With this in mind, please do the following:

(a) Write the syllogism above in set theoretical terms.

(b) Assume that every set has to be non-empty, so there should be at least one element in every set. Show that the syllogism in question is valid using a set theoretic argument just like the argument that you find in Monday's or Wednesday's slides.

(c) Now drop the assumption that every set has to be non-empty, and construct a counterexample to the above syllogism.

4 VALUATIONS AND FORMULAS [30 POINTS]

Let Γ and Δ be sets of formulas. Let $Val(\Gamma)$ be the set of valuations which make true all the formulas in Γ. More precisely, $Val(\Gamma) = \{V| \textit{ for all } \varphi, \textit{ if } \varphi \in \Gamma, \textit{ then } V(\varphi) = 1\}$. And similarly for $Val(\Delta)$, so that $Val(\Delta) = \{V| \textit{ for all } \varphi, \textit{ if } \varphi \in \Delta, \textit{ then } V(\varphi) = 1\}$. In other words, Val gives us all the valuations that make true all all the formulas in a given set of formulas, be this set Γ or Δ. Now, please do following:

(b) Construct a counterexample to the claim that if $\Gamma \subseteq \Delta$, then $Val(\Gamma) \subseteq Val(\Delta)$.

(a) Suppose $\Gamma \subseteq \Delta$. Show that $Val(\Delta) \subseteq Val(\Gamma)$.