PHIL 50 – INTRODUCTION TO LOGIC

MARCELLO DI BELLO – STANFORD UNIVERSITY

HOMEWORK – WEEK #9 - DUE WEDNESDAY JUNE 4TH

1 IS IT VALID OR NOT? [50 POINTS]

(a) Construct a derivation showing that $\forall x \varphi(x) \vdash \exists x \varphi(x)$. That is, construct a derivation with uncanceled assumption $\forall x \varphi(x)$ and ending with the formula $\exists x \varphi(x)$. [*Hint*: have a look at the derivations in the derivation guide for predicate logic.]

(b) Show that $\forall x \varphi(x) \not\models \exists x \varphi(x)$ by constructing a model M consisting of a (possibly empty) domain D of objects, an interpretation function I for constant and predicate symbols, and an assignment function g for variable symbols.

(c) What do (a) and (b) show? Do they contradict soundness or completeness of predicate logic? In order to recover soundness or completeness of predicate logic, decide how the definition of model M given in part (b) should be amended.

2 RUTH BARCAN MARCUS [20 POINTS]

Read the *New York Times* obituary about Ruth Barcan Marcus. The weblik is posted on the course website. How does the article explain the difference between $\Box\forall x \varphi(x)$ and $\forall x \Box\varphi(x)$?

3 SAUL KRIPKE [30 POINTS]

The American philosopher Saul Kripke argued against dualism in philosophy of mind using modal logic. A central question in philosophy of mind is whether mental states can be identified with (or are the same thing as) physical or brain states. In other words, is the mind the same thing as the brain or not? If mental states are the same as physical or brain states, then everything would be reducible to the physical level, and dualism would thus be false. By contrast, if mental states are not identical to physical states, then not everything would be reducible to the physical level, and dualism would thus be true.

Kripke wanted to offer an argument in defense of dualism—i.e., an argument for the claim that mental states cannot be identified with brain states. For more background on this, read the lecture notes on modal logic (and in particular, the last section). Kripke's argument rests on the claim that

$$\models (a = b) \rightarrow \Box(a = b)$$

Show that if RIGIDITY holds, the above claim holds, i.e. show that if RIGIDITY holds, the formula $(a = b) \rightarrow \Box(a = b)$ is valid. (See the lecture notes on modal logic, last section, for a statement of RIGIDITY). At some point in the proof, you will need to appeal to an *ad hoc* assumption, namely that $I_w(=)$ is the same as $I_v(=)$ for any worlds w and v.