PHIL 50 – INTRODUCTION TO LOGIC – SPRING 2014

FINAL EXAM – FRIDAY JUNE 6TH – 9:30-11:30 AM

The final exam consists of PART A and PART B. The former consists of ten questions which you are expected to answer as clearly and as concisely as possible. The latter part consists of five problems. The time allotted for the final exam is 2 hours, from 9:30 AM until 11:30 AM. ***You may not consult textbooks, notes and other study materials during the exam****.*

PART A

1. Write the square of oppositions in the language of set theory.
2. Translate into predicate logic the statement *All houses in Santorini are colorful*.
3. What is a model in predicate logic?
4. What is the truth condition for a universally quantified formula? State the condition and give an example.
5. Why do we need a modified variable assignment of the form $g_{[x:=d]}(x)$?
6. What restriction governs $\forall I$? What is its rationale? Please give an example.
7. What is the transformative power of negation with respect to the quantifiers? Illustrate this with an example.
8. How can you show that $\varphi \nvdash \psi$?
9. What is the difference between $\forall x \Box \varphi(x)$ and $\Box \forall x \varphi(x)$?
10. What does it mean to say that probability theory has an underlying logic? Give a couple of examples.

PART B

1. By constructing the appropriate derivation, show that:

   (a) $\vdash ((\varphi \to \psi) \to \psi) \to (\neg\varphi \to \psi)$

   (b) $\varphi \vee \psi, \neg\varphi \vdash \psi$

   (c) $\vdash \forall x(A(x) \wedge B(x)) \to (\forall x A(x) \wedge \forall x B(x))$

   (d) $\vdash \forall x \exists y R(x, y) \to \forall x' \exists y' R(x', y')$

   (e) $\vdash \exists x \forall y R(x, y) \to \forall y \exists x R(x, y)$

2. Let $Val(\Gamma) = \{V | \textit{for all } \varphi, \textit{ if } \varphi \in \Gamma, \textit{ then } V(\varphi) = 1\ \}$; and similarly for $Val(\Delta)$.

   Show that the claim $Val(\Gamma \cup \Delta) = Val(\Gamma) \cup Val(\Delta)$ is false.

3. Consider a model whose domain consists of the positive natural numbers, including 0. You should picture such a model as having numbers 0, 1, 2, 3, 4, 5, etc. The numbers are ordered in the standard way. Let *Greater-than* be a two-place predicate that is interpreted as the relation $>$, or more precisely, $I(\textit{Greater-than}) = \{\langle n, m\rangle | n > m\}$. Call this model N. Check whether the following hold or not:

   (a) $N \models \exists x \exists y \neg(x = y)$

   (b) $N \models \forall x \exists y \textit{Greater-than}(y, x)$

   (c) $N \models \exists x \forall y ((\neg(x = y)) \to (\textit{Greater-than}(y, x))$

   (d) $N \models \exists x \forall y \textit{Greater-than}(x, y)$

   Please explain your answers as carefully as you can.

4. Show that the following hold or give a counterexample:

   (a) $\forall x(P(x) \vee Q(x)) \models \forall x P(x) \vee \forall x Q(x)$

   (b) $\exists x(P(x) \vee Q(x)) \models \exists x P(x) \vee \exists x Q(x)$

   (c) $\exists x P(x) \wedge \exists x Q(x) \models \exists x(P(x) \wedge Q(x))$

5. Show with a semantic argument that $\forall x \Box P(x) \to \Box \forall x P(x)$ is valid provided $D_w = D_v$ for all possible worlds w and v.