PHIL 50 – INTRODUCTION TO LOGIC

PREPARATION FOR THE SECOND PART OF THE FINAL

Below are examples of the type of questions you will find in the second part of the PHIL50 final exam. Please note that the questions you will find in the final exam need not be identical to those that you see below. The questions are divided by topic and some of them are from past homework assignments. Please review the homework assignments and their solutions.

SKILLS AND TOPICS YOU SHOULD HAVE MASTERED

- S1 Construct derivations using the rules for propositional and predicate logic.
- S2 Familiarity with elementary set theory and its operations, such as intersection, union, and complementation. Ability to prove claims involving set theoretical notions, see e.g. exercise 4 in homework 5.
- S3 Ability to translate statements from natural language into formulas of predicate logic.
- S4 Familiarity with the notion of a valuation (in propositional logic), the notion of a model (in predicate logic), the notion of possible world and model (in modal logic). Ability to check whether formulas are true or false relative to a given model by means of applying the appropriate truth conditions.
- S5 Familiarity with the notion of truth, validity and logical consequence, soundness, and completeness in propositional, predicate and modal logic. Ability to check whether a formula is valid or not by means of semantic reasoning about arbitrary models.

DERIVATIONS

1. $\vdash (\varphi \wedge \neg\psi) \to ((\varphi \vee \psi) \wedge \neg\psi)$
2. $\vdash \varphi \to (\psi \to (\sigma \to \theta)) \to (((\varphi \wedge \psi) \wedge \sigma) \to \theta)$
3. $\vdash ((\varphi \to \psi) \to \psi) \to (\neg\varphi \to \psi)$
4. $\vdash ((\varphi \to \psi) \to (\varphi \to \sigma)) \to (\varphi \to (\psi \to \sigma))$
5. $\vdash (\varphi \to \psi) \to \neg(\varphi \wedge \neg\psi)$

6. $\varphi \vee \psi, \neg\psi \vdash \varphi$
7. $\vdash \forall x(\varphi(x) \wedge \psi(x)) \rightarrow (\forall x\varphi(x) \wedge \forall x\psi(x))$
8. $\vdash \exists x(\varphi(x) \vee \psi(x)) \rightarrow \exists x\varphi(x) \vee \exists x\psi(x)$
9. $\vdash \forall x(\neg\varphi(x) \vee \neg\psi(x)) \rightarrow \neg\exists x(\varphi(x) \wedge \psi(x))$
10. $\vdash \forall x\exists y P(x,y) \rightarrow \forall x'\exists y' P(x',y')$
11. $\vdash \exists x\forall y P(x,y) \rightarrow \forall y\exists x P(x,y)$

PROOFS USING SETS AND OPERATIONS ON SETS

12. See exercises 2 through 4 in homework # 5
13. See exercises 1(b) and 4 in homework # 6
14. See exercise 5 in homework 7

TRANSLATION FROM NATURAL LANGUAGE TO PREDICATE LOGIC

15. See exercise 2 in homework 6
16. See exercise 1(a) in homework 8

TRUTH AND MODELS

17. See exercise 3 in homework 6
18. See exercises 2, 3, and 4 in homework 7
19. See exercise 2.3(a) in homework 8.
20. Consider a model consisting of the positive natural numbers, including 0, with the natural ordering. You should picture such a model as having numbers 0, 1, 2, 3, 4, 5, etc. that are ordered in the standard way. Let *Greater-than* be a two-place predicate that is interpreted as the relation $>$, or more precisely, $I($*Greater-than*$) = \{\langle n, m\rangle | n > m\}$. Call this model N. Check whether the following hold or not. Explain your answers.

- $N \models \exists x \exists y (x = y)$
- $N \models \exists x \exists y \neg (x = y)$
- $N \models \forall x \exists y \textit{Greater-than}(y, x)$
- $N \models \exists x \forall y \textit{Greater-than}(y, x)$
- $N \models \exists x \forall y \textit{Greater-than}(x, y)$
- $N \models \forall x \forall y \exists z (\textit{Greater-than}(y, x) \rightarrow (\textit{Greater-than}(z, x) \wedge \textit{Greater-than}(y, z)))$

VALIDITY, LOGICAL CONSEQUENCE, SOUNDNESS, AND COMPLETENESS

21. See exercise 1 in homework 9

22. See exercise 3 in homework 9

23. Check whether the following hold, and if not, give a counterexample:

    - $\forall x P(x) \vee \forall x Q(x) \models \forall x (P(x) \vee Q(x))$
    - $\forall x (P(x) \vee Q(x)) \models \forall x P(x) \vee \forall x Q(x)$
    - $\exists x P(x) \vee \exists x Q(x) \models \exists x (P(x) \vee Q(x))$
    - $\exists x (P(x) \vee Q(x)) \models \exists x P(x) \vee \exists x Q(x)$
    - $\exists x P(x) \wedge \exists x Q(x) \models \exists x (P(x) \wedge Q(x))$
    - $\exists x (P(x) \wedge Q(x)) \models \exists x P(x) \wedge \exists x Q(x)$

24. Show with a semantic argument that the Barcan formula is valid provided $D_w = D_v$ for all possible worlds w and v.